Write your name here

Surname	Other names

Pearson Edexcel International GCSE

Centre Number	Candidate Number

Mathematics A

model answers

Level 1/2
Paper 1H

Higher Tier

Sample assessment material for first teaching September 2016
Time: 2 hours

Paper Reference
4MA1/1H

You must have:
Ruler graduated in centimetres and millimetres, protractor, compasses, pen, HB pencil, eraser, calculator. Tracing paper may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- **Calculators may be used.**
- You must **NOT** write anything on the formulae page. Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶

S51832A

1/2/1/

PEARSON

International GCSE Mathematics

Formulae sheet – Higher Tier

Arithmetic series

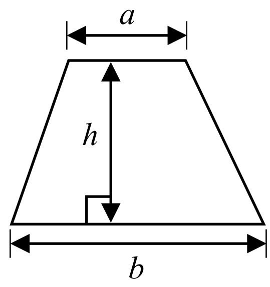
Sum to n terms, $S_n = \frac{n}{2}\,[2a + (n - 1)d]$

The quadratic equation

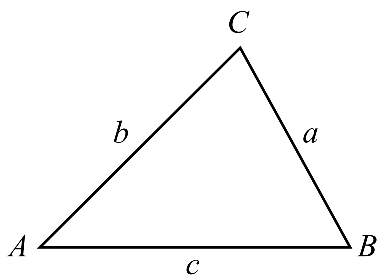
The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$ are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Area of trapezium $= \frac{1}{2}(a + b)h$



Trigonometry



In any triangle *ABC*

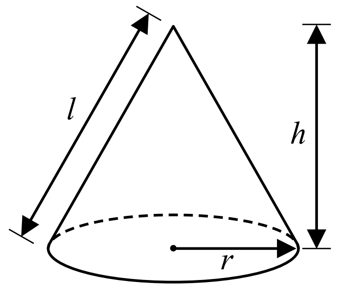
Sine Rule $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine Rule $a^2 = b^2 + c^2 - 2bc\cos A$

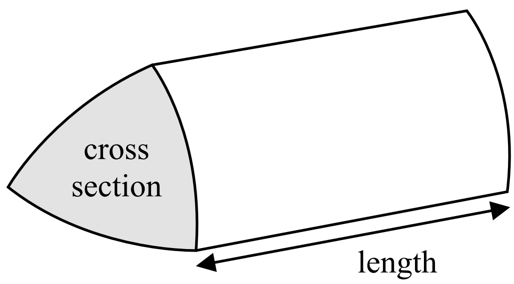
Area of triangle $= \frac{1}{2}ab\sin C$

Volume of cone $= \frac{1}{3}\pi r^2 h$

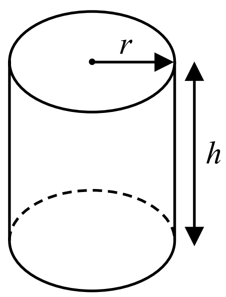
Curved surface area of cone $= \pi rl$



Volume of prism
= area of cross section × length

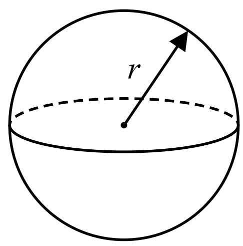


Volume of cylinder $= \pi r^2 h$
Curved surface area of cylinder $= 2\pi rh$



Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$



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Answer ALL TWENTY THREE questions.

Write your answers in the spaces provided.

You must write down all stages in your working.

1 Yoko flew on a plane from Tokyo to Sydney.
The plane flew a distance of 7800 km.
The flight time was 9 hours 45 minutes.

Work out the average speed of the plane in kilometres per hour.

average speed = $\dfrac{\text{total distance}}{\text{total time}}$

9hr 45min = 9.75hrs

(45 mins is 3/4 of an hour)

speed = $\dfrac{7800}{9.75}$ = 800

800 km/h

(Total for Question 1 is 3 marks)

2 Penny, Amjit and James share some money in the ratios 3 : 6 : 4
Amjit gets $28 more than James.

Work out the amount of money that Penny gets.

A - J

6 - 4 = 2 , so 2 parts = $28

1 part = $14

P : A : J

3 : 6 : 4

×14

42 : 84 : 56

$ 42

(Total for Question 2 is 3 marks)

3 A factory has 60 workers.

The table shows information about the distances, in km, the workers travel to the factory each day.

Distance (d km)	**Frequency** (f)	midpoint (x)	fx
$0 < d \leqslant 5$	12	2.5	30
$5 < d \leqslant 10$	6	7.5	45
$10 < d \leqslant 15$	4	12.5	50
$15 < d \leqslant 20$	6	17.5	105
$20 < d \leqslant 25$	14	22.5	315
$25 < d \leqslant 30$	18	27.5	495

(a) Write down the modal class.

highest frequency

$25 < d \leqslant 30$

(1)

(b) Work out an estimate for the mean distance travelled to the factory each day.

$$\text{mean} = \frac{\text{sum of } fx}{\text{total freq.}} = \frac{30+45+50+105+315+495}{60}$$

$$= \frac{1040}{60} = 17.333..$$

told in q

17.3 km

(4)

One of these workers is chosen at random.

(c) Write down the probability that this worker travels more than 20 km to the factory each day.

no. workers who travel more than 20km $= 18+14 = 32$

$$\frac{32}{60} = \frac{8}{15}$$

total

$8/15$

(2)

(Total for Question 3 is 7 marks)

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4 Nigel bought 12 boxes of melons.
He paid $15 for each box.
There were 12 melons in each box.

Nigel sold $\frac{3}{4}$ of the melons for $1.60 each.

He sold all the other melons at a reduced price.

He made an overall profit of 15%

Work out how much Nigel sold each reduced price melon for.

total melons : 12 x 12 = 144

money spent : 12 x 15 = $180

profit : 15% = 1.15 as a decimal multiplier

180 x 1.15 = $207

money made from full price melons : 144 x 3/4 = 108 sold for $1.60.

108 x 1.6 = $172.80

money made from reduced melons : 207 − 172.8 = $34.20

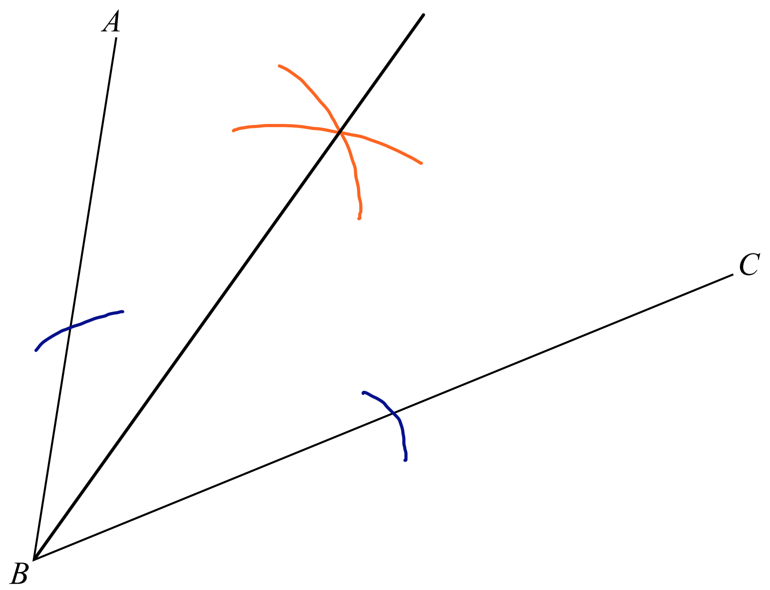
1/4 x 144 = 36 melons sold.

$\frac{\$34.20}{36}$ = $0.95 per reduced melon

$ ....

(Total for Question 4 is 5 marks)

5 Use ruler and compasses to construct the bisector of angle ABC.
You must show all your construction lines.



(Total for Question 5 is 2 marks)

6 (a) Factorise fully $18e^3f + 45e^2f^4$

$= 9e^2f(2e + 5f^3)$

$= 18e^3f + 45e^2f^4$

$9e^2f(2e + 5f^3)$

(2)

(b) Solve $x^2 - 4x - 12 = 0$
Show clear algebraic working.

$x^2 - 4x - 12 = 0$

factorise

$(x-6)(x+2) = 0$

set each bracket to zero

$x - 6 = 0$ $\quad$ $x + 2 = 0$

$x = 6$ $\quad$ $x = -2$

$x = 6,\ x = -2$

(3)

(Total for Question 6 is 5 marks)

7

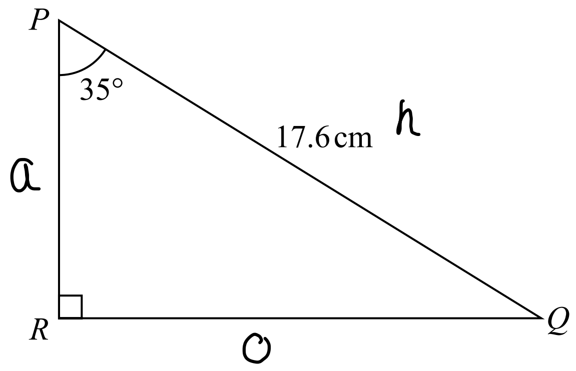


Diagram **NOT** accurately drawn

Calculate the length of *PR*.
Give your answer correct to 3 significant figures.

$\cos Q = a/h$

$\cos 35 = \frac{PR}{17.6}$

$\cos 35 \times 17.6 = PR$ $PR = 14.4\text{cm } (3sf)$

14.4 .... cm

(Total for Question 7 is 3 marks)

8 In a sale, all normal prices are reduced by 15%
The normal price of a mixer is reduced by 22.50 dollars.

Work out the normal price of the mixer.

$15\% = 22.5$

$\div 15$ $1\% = 1.5$ $\div 15$

$\times 100$ $100\% = 150$ $\times 100$

150 .... dollars

(Total for Question 8 is 3 marks)

9 The table shows the diameters, in kilometres, of five planets

Planet	Diameter (km)
Venus	1.2×10^4
Jupiter	1.4×10^5
Neptune	5.0×10^4
Mars	6.8×10^3
Saturn	1.2×10^5

(a) Write 1.4×10^5 as an ordinary number.

140000

(1)

(b) Which of these planets has the smallest diameter?

Mars

(1)

(c) Calculate the difference, in kilometres, between the diameter of Saturn and the diameter of Neptune.
Give your answer in standard form.

Saturn : $1 \cdot 2 \times 10^5 = 120000$

Neptune : $5 \times 10^4 = 50000$

$120000 - 50000 = 70000 = 7 \times 10^4$

7×10^4 km

(2)

The diameter of the Moon is 3.5×10^3 km.
The diameter of the Sun is 1.4×10^6 km.

(d) Calculate the ratio of the diameter of the Moon to the diameter of the Sun.
Give your ratio in the form $1 : n$

M : S

$\div 3 \cdot 5 \times 10^3$ $\quad 3 \cdot 5 \times 10^3 : 1 \cdot 4 \times 10^6 \quad$ $\div 3 \cdot 5 \times 10^3$

$1 : 400$

$1 : 400$

(2)

(Total for Question 9 is 6 marks)

10

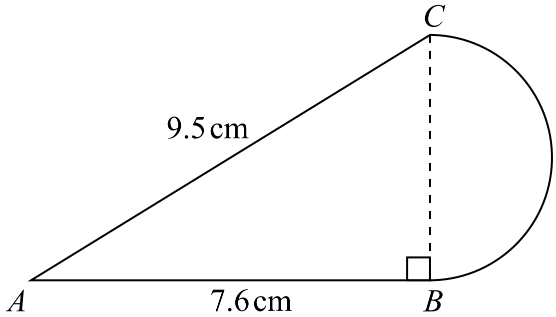


Diagram **NOT** accurately drawn

The diagram shows a shape made from triangle *ABC* and a semicircle with diameter *BC*.
Triangle *ABC* is right-angled at *B*.

Pythagoras' theorem: $a^2+b^2=c^2$

AB = 7.6 cm and *AC* = 9.5 cm.

so, $AB^2 + BC^2 = AC^2$

$7.6^2 + BC^2 = 9.5^2$

$BC^2 = 32.49$

$BC = 5.7cm$

Calculate the area of the shape.
Give your answer correct to 3 significant figures.

area of triangle $= \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 7.6 \times 5.7$

$= 21.66 cm^2$

diameter of semicircle = 5.7, so, radius = 2.85

area of semicircle $= \frac{1}{2} \times \pi r^2$

$= \frac{1}{2} \times \pi \times 2.85^2 = 12.76cm^2$

total area $= 21.66 + 12.76$

$= 34.42cm^2$

$= 34.4 cm^2$ (3sf)

34.4 cm^2

(Total for Question 10 is 5 marks)

11 Expand and simplify $(x + 5)(x - 3)(x + 3)$

$(x+5)(x-3) = x^2 - 3x + 5x - 15$
$= x^2 + 2x - 15$

$(x^2+2x-15)(x+3)$
$= x^3 + 2x^2 - 15x + 3x^2 + 6x - 45$
$= x^3 + 5x^2 - 9x - 45$

$x^3 + 5x^2 - 9x - 45$

(Total for Question 11 is 3 marks)

12 Here are the points that Carmelo scored in his last 11 basketball games.

23 20 14 23 17 24 24 18 16 22 21

(a) Find the interquartile range of these points. first, order numbers

14, 16, 17, 18, 20, 21, 22, 23, 23, 24, 24

lower quartile: $11+1/4 = 3 \Rightarrow$ 3rd position : 17

upper quartile: $3 \times 3 = 9 \Rightarrow$ 9th position : 23

IQR : $23 - 17 = 6$

6

(3)

Kobe also plays basketball.
The median number of points Kobe has scored in his last 11 games is 18.5
The interquartile range of Kobe's points is 10

(b) Which of Carmelo or Kobe is the more consistent points scorer?
Give a reason for your answer.

Carmelo, as his IQR is smaller, implying his scores were closer together.

(1)

(Total for Question 12 is 4 marks)

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13 (a) Find an equation of the line that passes through the points (−3, 5) and (1, 2)
Give your answer in the form $ax + by = c$ where a, b and c are integers.

gradient $= \frac{y_1 - y_2}{x_1 - x_2} = \frac{5-2}{-3-1} = \frac{3}{-4}$

$y = mx + c$

sub in values using gradient and unknown point

$2 = -3/4(1) + c$

$2 = -3/4 + c$

$c = 11/4$

sub back into $y = mx + c$

$y = -3/4x + 11/4$

$\times 4$

$4y = -3x + 11$

$3x + 4y = 11$

$3x + 4y = 11$

(4)

Line $\mathbf{L}_1$ has equation $y = 3x + 5$
Line $\mathbf{L}_2$ has equation $6y + 2x = 1$

(b) Show that $\mathbf{L}_1$ is perpendicular to $\mathbf{L}_2$

$y = mx + c$

m. of line $L_1 = 3$

$6y + 2x = 1$

$6y = -2x + 1$

$y = -2/6x + 1/6$

$y = -1/3x + 1/6$

m. of line $L_2 = -1/3$

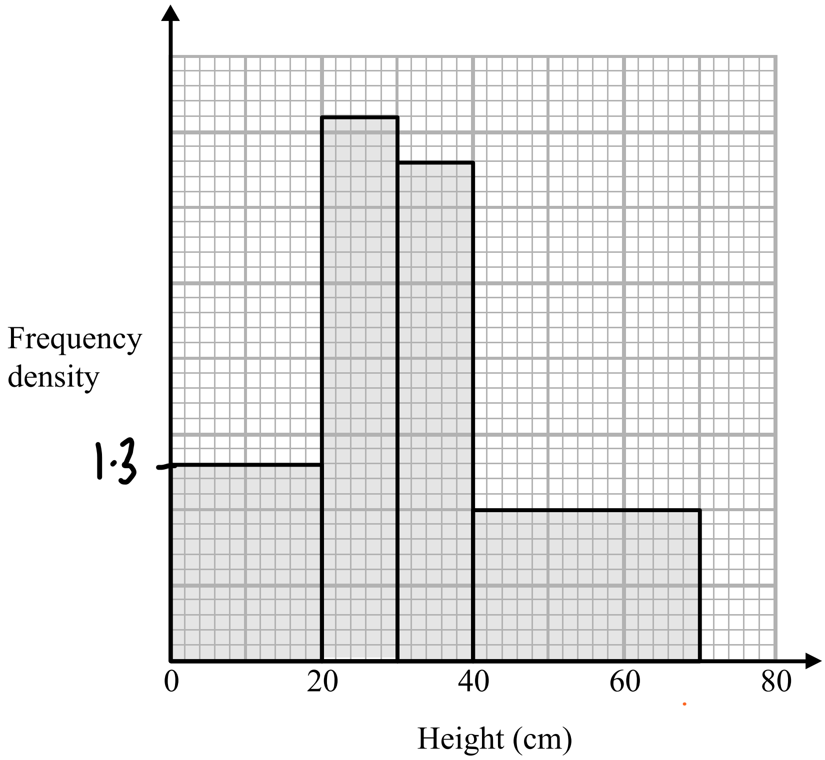
$3 \times -1/3 = -1$

So, the gradients are negative reciprocals, meaning the lines are perpendicular.

(2)

(Total for Question 13 is 6 marks)

14 The histogram shows information about the heights of some tomato plants.



26 plants have a height of less than 20 cm.

Work out the total number of plants.

frequency density = $\frac{\text{frequency}}{\text{class width}}$

$f.d = \frac{26}{20} = 1.3$

using this we can understand the scale on the y axis

20-30cm: freq = 3.6 x 10 = 36

30-40cm: freq = 3.3 x 10 = 33

40-70cm: freq = 1 x 10 = 10

26 + 36 + 33 + 10 = 125

125

(Total for Question 14 is 3 marks)

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15 A rectangular lawn has a length of $3x$ metres and a width of $2x$ metres.
The lawn has a path of width 1 metre on three of its sides as shown in the diagram.

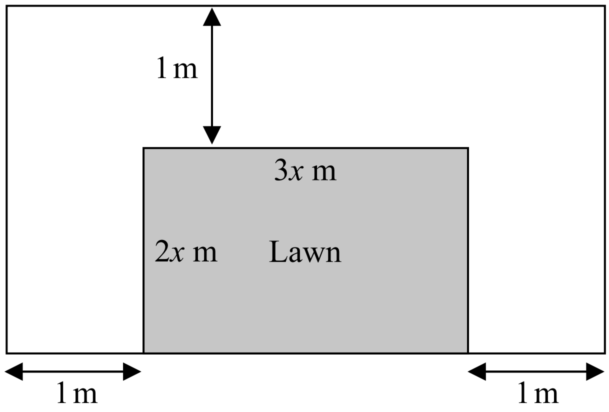


Diagram **NOT** accurately drawn

The total area of the lawn and the path is $100\,\text{m}^2$

(a) Show that $6x^2 + 7x - 98 = 0$

length of plot = $3x+2$
width of plot = $2x+1$
area = $(3x+2)(2x+1)$ expand
$100 = 6x^2+3x+4x+2$
-100 $0 = 6x^2 + 7x - 98$ -100

(2)

(b) Calculate the area of the lawn.
Show clear algebraic working.

$ax^2+bx+c=0$
$6x^2+7x-98=0 \quad a=6 \quad b=7 \quad c=-98$
sub into quadratic formula
$x = \dfrac{-7 \pm \sqrt{7^2-(4\times 6\times -98)}}{2\times 6}$

$x = 3.5$ or $x = -14/3$
x must be positive
area of lawn = $3x \times 2x = 6x^2$
area = $6 \times 3.5^2 = 73.5\ \text{m}^2$

73.5 m²

(5)

(Total for Question 15 is 7 marks)

16

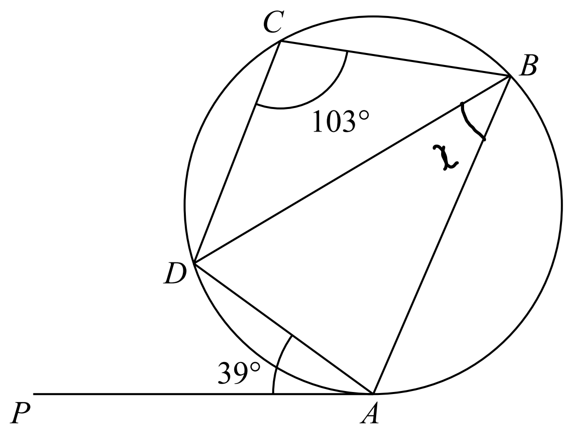


Diagram **NOT** accurately drawn

A, *B*, *C* and *D* are points on a circle.
PA is a tangent to the circle.
Angle *PAD* = 39°
Angle *BCD* = 103°

Calculate the size of angle *ADB*.
Give a reason for each stage of your working.

$\angle DAB = 180 - 103 = 77°$
opposite angles in a cyclic quadrilateral sum 180°.

$\angle BDA = \angle PAD = 39°$ alternate segment theorem.

$\angle ADB = 180 - 77 - 39 = 64°$ angles in a triangle sum 180°

64 °

(Total for Question 16 is 5 marks)

17 $y = \dfrac{2a}{b - c}$

a = 42 correct to 2 significant figures.
b = 24 correct to 2 significant figures.
c = 14 correct to 2 significant figures.

Work out the lower bound for the value of y.
Give your answer correct to 2 significant figures.
Show your working clearly.

lower $y = \dfrac{2 \times \text{lower } a}{\text{upper } b - \text{lower } c}$

LB $a = 41.5$

smallest number which rounds to 42.

UB $b = 24.5$ first number that doesn't round to 25

LB $c = 13.5$ Smallest number which rounds to 14.

$y = \dfrac{2 \times 41.5}{24.5 - 13.5} = \dfrac{83}{11} = 7.545 \rightarrow 7.5 \ (2\text{ sf})$

7.5

(Total for Question 17 is 3 marks)

18 Show that $3 - (x - 1) \div \left(\dfrac{x^2 - 1}{3x + 2}\right)$ can be written as $\dfrac{a}{x + b}$ where a and b are integers.

B
I
D
M
A
S

×must do division first

$\dfrac{x-1}{1} \div \dfrac{x^2-1}{3x+2} = \dfrac{x-1}{1} \times \dfrac{3x+2}{x^2-1}$

$= \dfrac{(x-1)(3x+2)}{x^2-1} = \dfrac{\cancel{(x-1)}(3x+2)}{(x+1)\cancel{(x-1)}} = \dfrac{3x+2}{x+1}$

$3 - \dfrac{3x+2}{x+1} = \dfrac{3(x+1)}{x+1} - \dfrac{3x+2}{x+1} = \dfrac{3(x+1) - 3x - 2}{x+1}$

$= \dfrac{3x + 3 - 3x - 2}{x+1}$

$= \dfrac{1}{x+1}$

(Total for Question 18 is 4 marks)

19

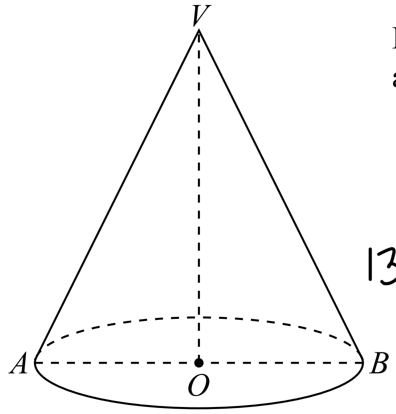


Diagram **NOT** accurately drawn

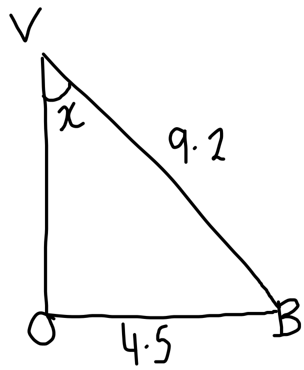
curved area = $\pi r l$

$130 = \pi \times 4.5 \times l$

$l = 9.2\text{cm}$ (3sf)

The diagram shows a solid cone.
The base of the cone is a horizontal circle, centre O, with radius 4.5 cm.
AB is a diameter of the base and OV is the vertical height of the cone.
The curved surface area of the cone is 130 cm^2

Calculate the size of the angle AVB.
Give your answer correct to 1 decimal place.



$\sin \theta = o/h$

$\sin x = 4.5/9.2$

$x = \sin^{-1}\left(\frac{4.5}{9.2}\right)$

$x = 29.3°$

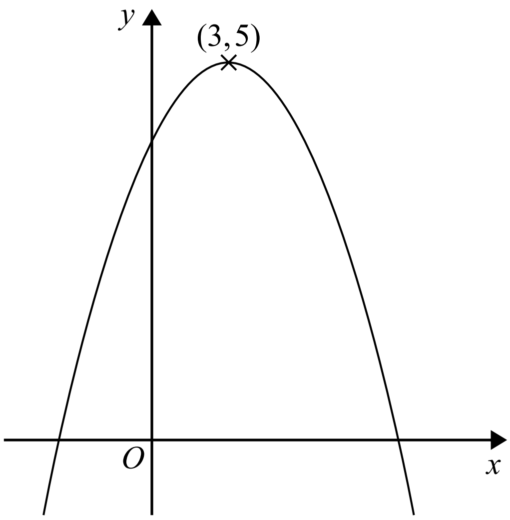
$\angle AVB = 2 \times \angle OVB = 2x$

$= 58.6°$

58.6 °

(Total for Question 19 is 4 marks)

20



The diagram shows part of the curve with equation $y = f(x)$
The coordinates of the maximum point of the curve are (3, 5)

(a) Write down the coordinates of the maximum point of the curve with equation

(i) $y = f(x + 3)$

3 left
-3 from x coordinate

(0 , 5)
(1)

(ii) $y = 2f(x)$

y coordinate x2

(3 , 10)
(1)

(iii) $y = f(3x)$

x coordinate ÷3

(1 , 5)
(1)

The curve with equation $y = f(x)$ is transformed to give the curve with equation $y = f(x) - 4$

(b) Describe the transformation.

translation $\begin{pmatrix} 0 \\ -4 \end{pmatrix}$

(1)

(Total for Question 20 is 4 marks)

21 The curve with equation $y = 8x^2 + \dfrac{2}{x}$ has one stationary point.

Find the co-ordinates of this stationary point.
Show your working clearly.

$y = 8x^2 + \dfrac{2}{x}$

differentiate

$\dfrac{dy}{dx} = 16x - 2x^{-2}$

gradient is 0 at the stationary point

$0 = 16x - 2x^{-2}$

$\times x^2$ $\times x^2$

$0 = 16x^3 - 2$

$2 = 16x^3$

$\dfrac{1}{8} = x^3$

sub into eqn for curve

$\dfrac{1}{2} = x$

$y = 8\left(\dfrac{1}{2}\right)^2 + \dfrac{2}{\frac{1}{2}}$

$= 2 + 4$

$= 6$

$(\frac{1}{2}, 6)$

($\frac{1}{2}$, 6)

(Total for Question 21 is 5 marks)

22

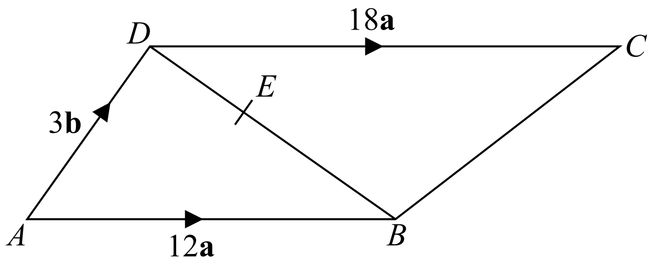


Diagram **NOT** accurately drawn

ABCD is a trapezium.
AB is parallel to *DC*.

$\overrightarrow{AB} = 12\mathbf{a}$

$\overrightarrow{AD} = 3\mathbf{b}$

$\overrightarrow{DC} = 18\mathbf{a}$

E is the point on the line *DB* such that $DE : EB = 1 : 2$

Show by a vector method that *BC* is parallel to *AE*.

$\vec{BC} = \vec{BA} + \vec{AD} + \vec{DC}$
$= -12a + 3b + 18a = 6a + 3b$

$\vec{AE} = \vec{AD} + \vec{DE}$

$\vec{DE} = \frac{1}{3} DB$

$\vec{DB} = \vec{DA} + \vec{AB} = -3b + 12a$

$\vec{DE} = \frac{1}{3}(-3b + 12a) = -b + 4a$

$\vec{AE} = 3b - b + 4a$
$= 2b + 4a$

$\vec{BC}$ is a multiple of $\vec{AE}$, so they are parallel.

(Total for Question 22 is 5 marks)

23 The 4th term of an arithmetic series is 17
The 10th term of the same arithmetic series is 35

Find the sum of the first 50 terms of this arithmetic series.

4th term = $a + 3d = 17$
10th term = $a + 9d = 35$
$-6d = -18$
$6d = 18$
$d = 3$

$a + 3(3) = 17$
$a = 8$

sub in to find a

sum of 50 terms:
$n = 50 \quad a = 8 \quad d = 3$

$S_{50} = \frac{50}{2}\left(2(8) + 49(3)\right) = 25 \times 163$
$= 4075$

(Total for Question 23 is 5 marks)

TOTAL FOR PAPER IS 100 MARKS

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