MODEL SOLUTIONS

![AQA]

GCSE
Mathematics
Specification (8300/3H)

Paper 3 Higher tier

H

Date	Morning	1 hour 30 minutes

Materials

For this paper you must have:
- a calculator
- mathematical instruments.

Instructions
- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the bottom of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book.
- In all calculations, show clearly how you work out your answer.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

Please write clearly, in block capitals, to allow character computer recognition.

Centre number ☐☐☐☐☐ Candidate number ☐☐☐☐

Surname ☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐

Forename(s) ☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐☐

Candidate signature _____

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2

Answer **all** questions in the spaces provided.

1   Work out the square root of 100 million.

Circle your answer.

[1 mark]

1000   (10 000)   100 000   1 000 000

2 $\mathbf{a} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$ $\underline{a} - \underline{b} = \begin{pmatrix} 5 \\ -2 \end{pmatrix} - \begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 5 - -2 \\ -2 - 3 \end{pmatrix} = \begin{pmatrix} 7 \\ -5 \end{pmatrix}$

Circle the vector $\mathbf{a} - \mathbf{b}$

[1 mark]

$\begin{pmatrix} -3 \\ -5 \end{pmatrix}$ $\begin{pmatrix} 7 \\ 1 \end{pmatrix}$ $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ $\left(\begin{pmatrix} 7 \\ -5 \end{pmatrix}\right)$

3   Circle the decimal that is closest in value to $\dfrac{2}{3}$

[1 mark]

0.6   0.66   (0.667)   0.67

$\dfrac{2}{3} = 0.\dot{6}$ so 0.667 is closest
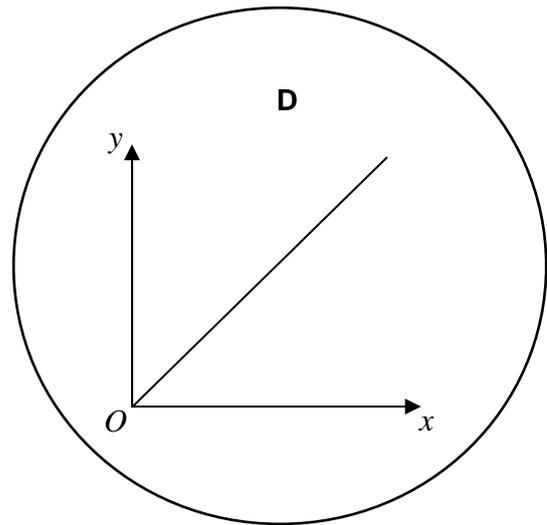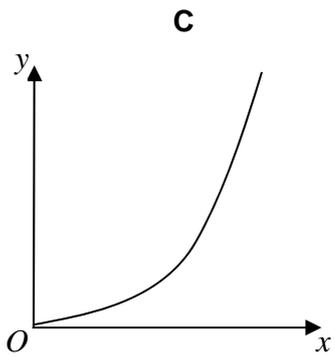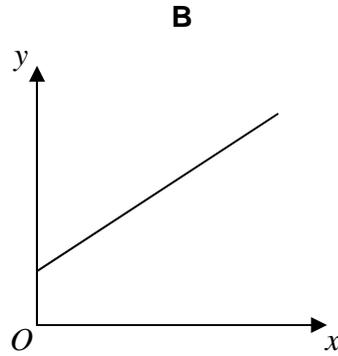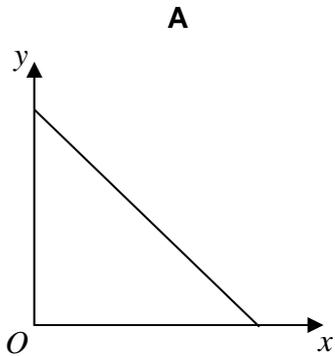
4 *y* is directly proportional to *x*.

y ∝ *x* so *y* = K*x*

Which graph shows this?
Circle the correct letter.

(graph is a straight line through (0,0))

[1 mark]

A

B

C

D

Turn over for the next question

5 In 1999 the minimum wage for adults was £3.60 per hour.

In 2013 it was £6.31 per hour.

Work out the percentage increase in the minimum wage.

[3 marks]

increase = 6.31 − 3.60 = £2.71

% increase = $\frac{2.71}{3.60}$ × 100 = 75.3%

denominator is original wage

Answer _____75.3_____ %

6 A bag contains counters that are red, blue, green or yellow.

	red	blue	green	yellow
Number of counters	9	$3x$	$x - 5$	$2x$

A counter is chosen at random.

The probability it is **red** is $\frac{9}{100}$

Work out the probability it is green.   probability of red = $\frac{9}{100}$ **[4 marks]**

and there are 9 red counters

100 counters in total

100 = 9 + 3x + (x − 5) + 2x
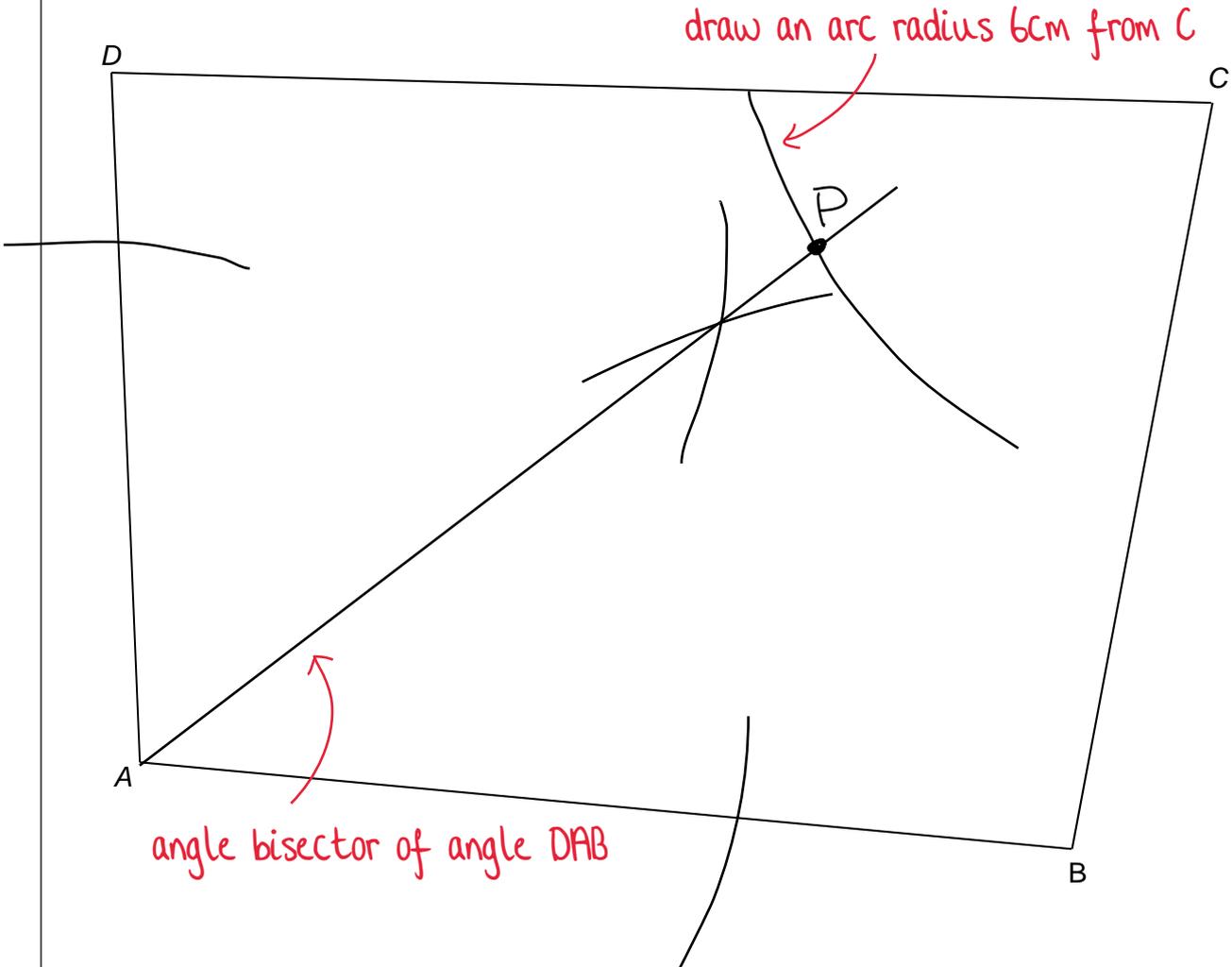
100 = 4 + 6x

96 = 6x

16 = x

probability of green = $\frac{x-5}{100}$ = $\frac{16-5}{100}$ = $\frac{11}{100}$

Answer _____$\frac{11}{100}$_____

7 Use ruler and compasses to answer this question.

Point *P* is

- the same distance from *AB* and *AD*
- 6 cm from *C*.

draw an arc radius 6cm from C

D

C

P

A

angle bisector of angle DAB

B

Show the position of *P* on the diagram.

[3 marks]

P is the point on the line of the angle bisector that crosses the arc from C.

Turn over for the next question

8 (a) Use your calculator to work out $19.42^2 - \sqrt[3]{1006} \div 4.95$

Write down your full calculator display.

[1 mark]

$$\frac{19.42^2 - \sqrt[3]{1006}}{4.95} = 375.1121656$$

Answer _____ 375.1121656 _____

8 (b) Use approximations to check that your answer to part (a) is sensible.
You **must** show your working.

[2 marks]

$$\left.\begin{array}{l} 19.42 \approx 20 \\ 1006 \approx 1000 \\ 4.95 \approx 5 \end{array}\right\} \text{1 sf}$$

$$\frac{20^2 - \sqrt[3]{1000}}{5} = 398$$

9 The exterior angle of a regular polygon is 45°
Circle the name of the regular polygon.

[1 mark]

pentagon   hexagon   (octagon)   decagon

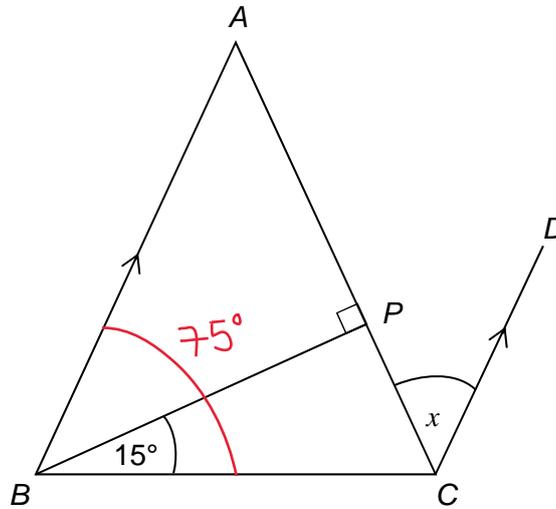$$\frac{360}{45} = 8 \text{ sides, so an octagon}$$

exterior angles add to 360°

10   *ABC* is a triangle with *AB* = *AC*

BA is parallel to *CD*.

Not drawn
accurately

A

D

$75°$

P

x

15°

B   C

Show that angle $x = 30°$

[3 marks]

angle PCB = 180 - 90 - 15 = 75°

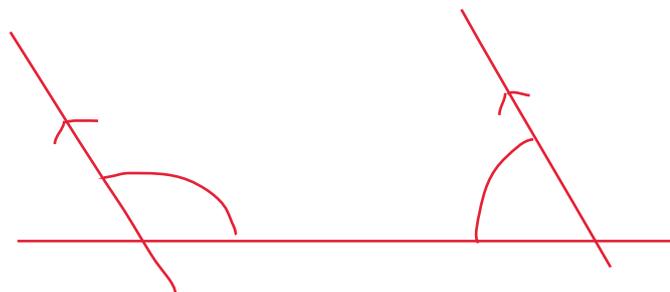angles in a triangle add to 180°

angle ABP = 75 - 15 = 60°

because AC = AB so triangle ABC is isosceles.

180 = 75 + (75 + x) Base angles in an isosceles triangle are

180 - 150 = x   equal.

x = 30°

interior angles add to 180°

Turn over ▶

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11 The pressure at sea level is 101 325 Pascals.

Any rise of 1 km above sea level decreases the pressure by 14%

For example,

at 3 km above sea level the pressure is 14% less than at 2 km

Work out the pressure at 4 km above sea level.

Give your answer to 2 significant figures.

[4 marks]

$1 - 0.14 = 0.86$ (pressure is 86% of pressure 1Km lower)

$101325 \times 0.86^4 = 55425.6$ Pascals

$= 55000$ Pascals (2 sf)

4Km above sea level

Answer _____ 55000 _____ Pascals

12   Tick whether each statement is true or false.

Give a reason for your answer.

12 (a)   When $x^2 = 16$ the **only** value that x can be is 4

[1 mark]

True ☐   False ☑

Reason   <u>x could be -4.</u> $(-4)^2 = 16$

12 (b)   When n is a positive integer, the value of $2n$ is **always** a factor of the value of $20n$.

[1 mark]

True ☑   False ☐

Reason $\dfrac{20n}{2n} = 10$

2n always goes into 20n

12 (c)   When y is positive, the value of y^2 is **always** greater than the value of y.
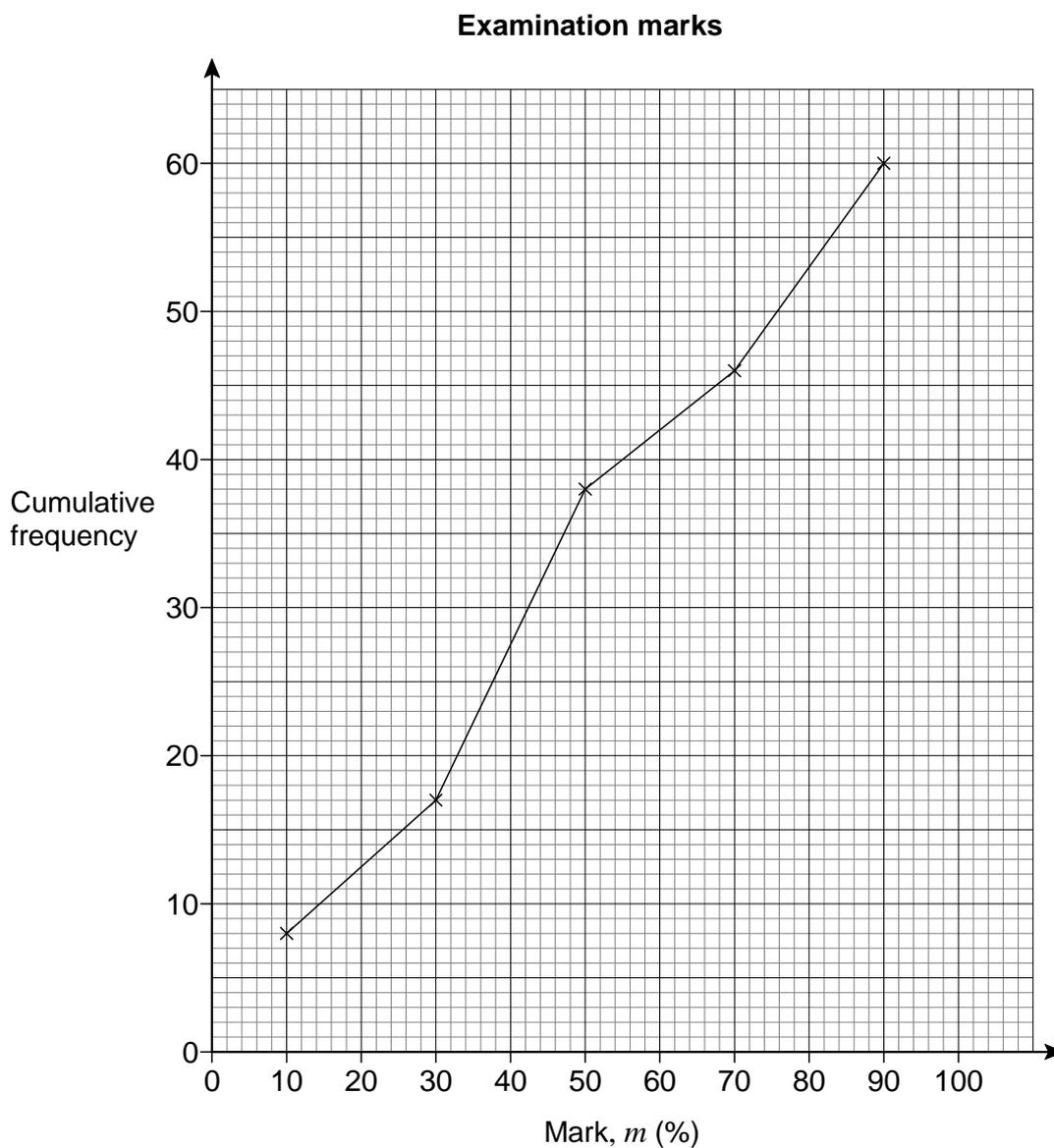
[1 mark]

True ☐   False ☑

Reason   <u>if y is between 1 and 0.</u>

eg. $0.5^2 = 0.25$ and $0.25 < 0.5$

Turn over ▶

13 Here are the examination marks for 60 pupils.

Mark, m (%)	Frequency
$0 \leqslant m < 20$	8
$20 \leqslant m < 40$	9
$40 \leqslant m < 60$	21
$60 \leqslant m < 80$	10
$80 \leqslant m < 100$	12

Molly drew this cumulative frequency graph to show the data.

Examination marks

Make **two** criticisms of Molly's graph.

[2 marks]

Criticism 1   <u>Points should be plotted at the end of class intervals.</u>

Criticism 2   <u>Middle point plotted incorrectly. Should be cumulative</u>
<u>frequency = 48, not 46.</u>

Turn over for the next question

14 (a) The nth term of a sequence is $2^n + 2^{n-1}$

Work out the 10th term of the sequence. $\underline{\quad} \ n = 10$

[1 mark]

$$2^{10} + 2^{10-1} = 2^{10} + 2^9 = 1024 + 512 = 1536$$

Answer _____ 1536 _____

14 (b) The nth term of a different sequence is $4(2^n + 2^{n-1})$

Circle the expression that is equivalent to $4(2^n + 2^{n-1})$

[1 mark]

$\boxed{2^{n+2} + 2^{n+1}}$ $\qquad\qquad$ $2^{2n} + 2^{2(n-1)}$

$8^n + 8^{n-1}$ $\qquad\qquad$ $2^{n+2} + 2^{n-1}$

$4 = 2^2$ $\qquad$ *add powers to expand bracket*

$2^2 (2^n + 2^{n-1}) = 2^{n+2} + 2^{n-1+2}$

$\qquad\qquad\qquad = 2^{n+2} + 2^{n+1}$

15 The diagram shows a design for a zipwire.

The zipwire will run between the top of two vertical posts, *AB* and *CD*.

Not drawn accurately

A

H

O

6.5 m

A

D

95°

2.3 m

B

C
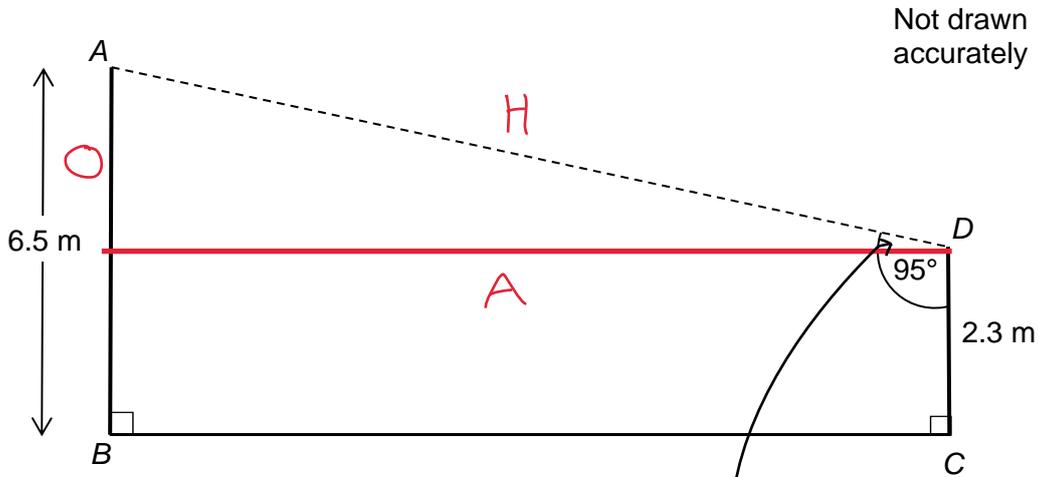
Work out the distance *AD*.

[4 marks]

$O = 6.5 - 2.3 = 4.2 cm$ $95 - 90 = 5$

SOH CAH TOA so $\sin x = \frac{O}{H}$

$\sin(5) = \frac{4.2}{AD}$

$AD = \frac{4.2}{\sin 5} = 48.2m$

Answer _____ 48.2 _____ m

Turn over ▶

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16 During a game, players can win and lose counters.

At the start of the game

Rob, Tim and Zak share the counters in the ratio 5 : 6 : 7

At the end of the game

Rob, Tim and Zak share the **same number** of counters in the ratio 7 : 9 : 8

Show that Rob ends the game with more counters than he started with.

[3 marks]

$$R : T : Z$$

Start: $5 : 6 : 7$

$5 + 6 + 7 = 18$ parts. Rob has

End: $7 : 9 : 8$

$7 + 9 + 8 = 24$ parts. Rob has

$$\frac{5}{18} = \frac{20}{72} \qquad \frac{7}{24} = \frac{21}{72}$$

×4 ×3

At the end, Rob has a greater proportion of the counters, so he ends with more counters than he started with.

17 Factorise $3x^2 + 14x + 8$

[2 marks]

$$1 + 4 = +12$$
$$3 \times 2 = + 2$$
$$\overline{\hspace{1.5cm}}$$
$$14 \longrightarrow (x + 4)(3x + 2)$$

→ check by expanding:
$$3x^2 + 2x + 12x + 8$$

Answer _____ $(x + 4)(3x + 2)$ _____

18 Here is some information about the number of books read by a group of people in 2014

One of the frequencies is missing.

Number of books	Frequency	Midpoint	
0 – 4	16	2	
5 – 9		7	
10 – 14	20	12	
15 – 19	10	17	

Midpoints are used to work out an estimate for the mean number of books read.

The answer is 8.5

Work out the missing frequency.

[5 marks]

$$\text{mean} = \frac{16 \times 2 + F \times 7 + 20 \times 12 + 10 \times 17}{16 + F + 20 + 10}$$

$$8.5 = \frac{32 + 7F + 240 + 170}{46 + F}$$

$$8.5(46 + F) = 442 + 7F$$
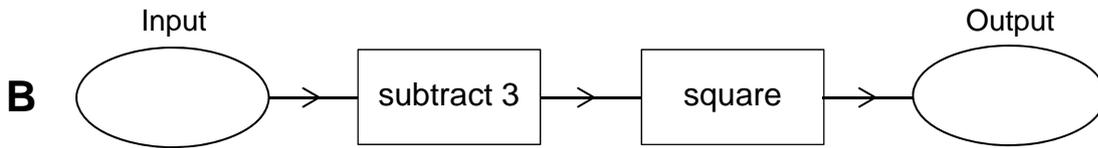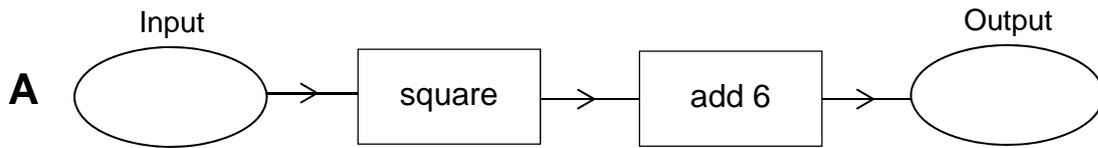$$391 + 8.5F = 442 + 7F$$
$$8.5F - 7F = 442 - 391$$
$$1.5F = 51$$
$$F = \frac{51}{1.5} = 34$$

Answer _____ 34

19 Here are two function machines, **A** and **B.**

Input   Output

A (Input) → [square] → [add 6] → (Output)

Input   Output

B (Input) → [subtract 3] → [square] → (Output)

Both machines have the same input.

Work out the range of input values for which

the output of **A** is **less** than the output of **B**.

[4 marks]

input = x

A: $x^2 + 6$

B: $(x - 3)^2$

$(x - 3)(x - 3)$

$= x^2 - 3x - 3x + 9$

want   output A < output B
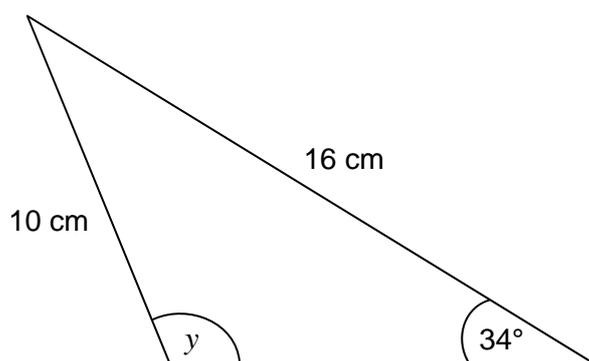
$= x^2 - 6x + 9$

$x^2 + 6 < (x - 3)^2$

$x^2 + 6 < x^2 - 6x + 9$

$6x < 9 - 6$

$6x < 3 \longrightarrow x < \frac{1}{2}$

Answer $\qquad x < \frac{1}{2} \qquad$

20 In the triangle, angle y is obtuse.

Not drawn accurately

16 cm

10 cm

y 34°

Work out the size of angle y.

[3 marks]

$$\frac{\sin y}{16} = \frac{\sin 34}{10}$$

$$\sin y = \frac{16 \sin 34}{10} = 0.8947$$

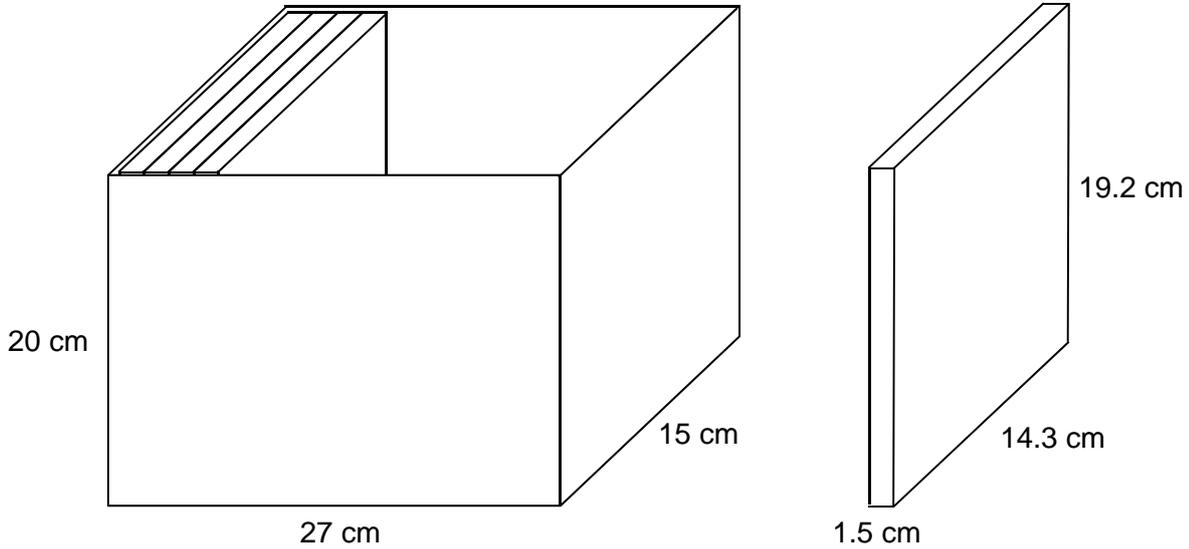$$y = \sin^{-1}(0.8947) = 63.5°$$

but y is obtuse $(90° < x < 180°)$

$$y = 180 - 63.5 = 116.5°$$

Answer _____116.5_____ degrees

Turn over for the next question

21 A box is a cuboid with dimensions 27 cm by 15 cm by 20 cm

These dimensions are to the nearest **centimetre**.

DVD cases are cuboids with dimensions 1.5 cm by 14.3 cm by 19.2 cm

These dimensions are to the nearest **millimetre**.

20 cm

27 cm   15 cm

19.2 cm

14.3 cm

1.5 cm

Show that 17 DVD cases, stacked as shown, will definitely fit in the box.

[4 marks]

box $26.5 \leqslant$ width < 27.5

$14.5 \leqslant$ width < 15.5

$19.5 \leqslant$ height < 20.5

CD case $1.45 \leqslant$ width < 1.55 check that 17 will fit according to width

min number of CDs that will fit = $\dfrac{\text{min width of box}}{\text{max width of CD}}$ = $\dfrac{26.5}{1.55}$

So 17 will definitely fit.

22   Bag X contains 9 blue balls and 18 red balls.

Bag Y contains 7 blue balls and 14 red balls.

Liz picks a ball at random from bag X.

She puts the ball into bag Y.

Mike now picks a ball at random from bag Y.

Show that

P (Liz picks a blue ball) = P (Mike picks a blue ball)

[4 marks]

$$P \text{ (Liz picks a blue ball) } = \frac{9}{9+18} = \frac{9}{27} = \frac{1}{3}$$

bag Y now contains 8 blue balls and 14 red balls

$$\text{Mike picks a blue ball: } \frac{1}{3} \times \frac{8}{8+14} = \frac{8}{3 \times 22} = \frac{4}{33}$$

$$\text{or: } \frac{2}{3} \times \frac{7}{8+14} = \frac{14}{3 \times 22} = \frac{7}{33}$$

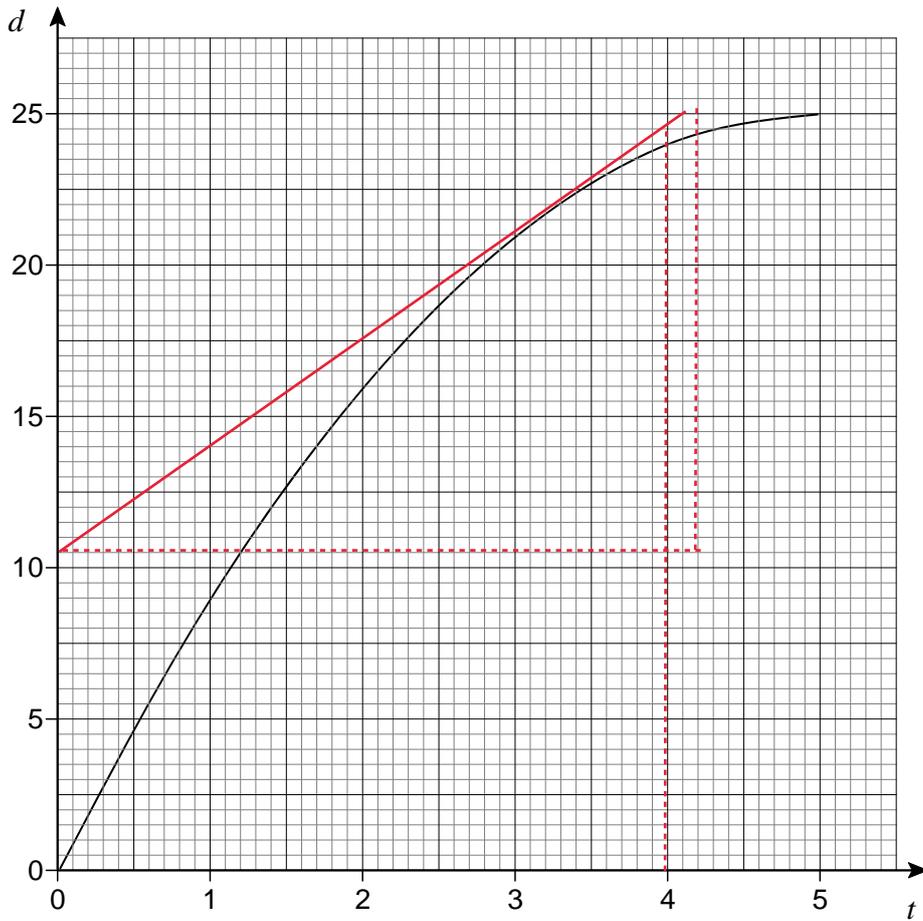$$P \text{ (Mike picks a new ball) } = \frac{4}{33} + \frac{7}{33} = \frac{11}{33} = \frac{1}{3}$$

$$P \text{ (Liz picks a blue ball) } = P \text{ (Mike picks a blue ball)}$$
$$\text{as } \quad \frac{1}{3} = \frac{1}{3}$$

Turn over ▶

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23   A container is filled with water in 5 seconds.

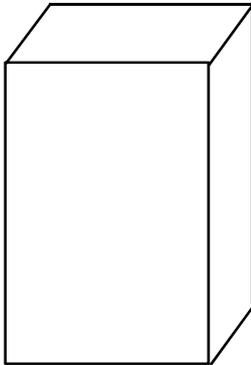The graph shows the depth of water, d cm, at time t seconds.

23 (a) The water flows into the container at a constant rate.
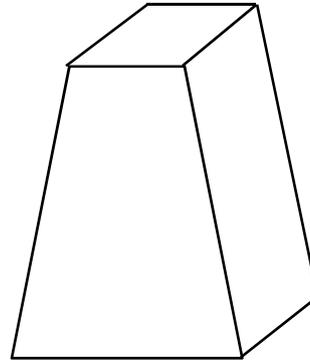
Which diagram represents the container?

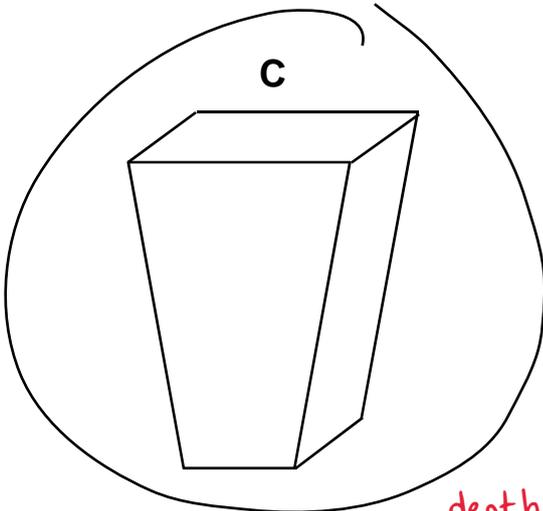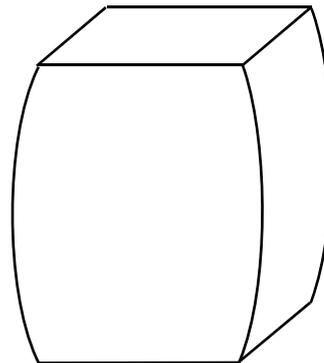Circle the correct letter.

[1 mark]

A

B

C

D

depth decreases more slowly towards the
end because the container becomes wider

23 (b) Use the graph to estimate the rate at which the depth of water is increasing at 3 seconds.

You **must** show your working.

[2 marks]

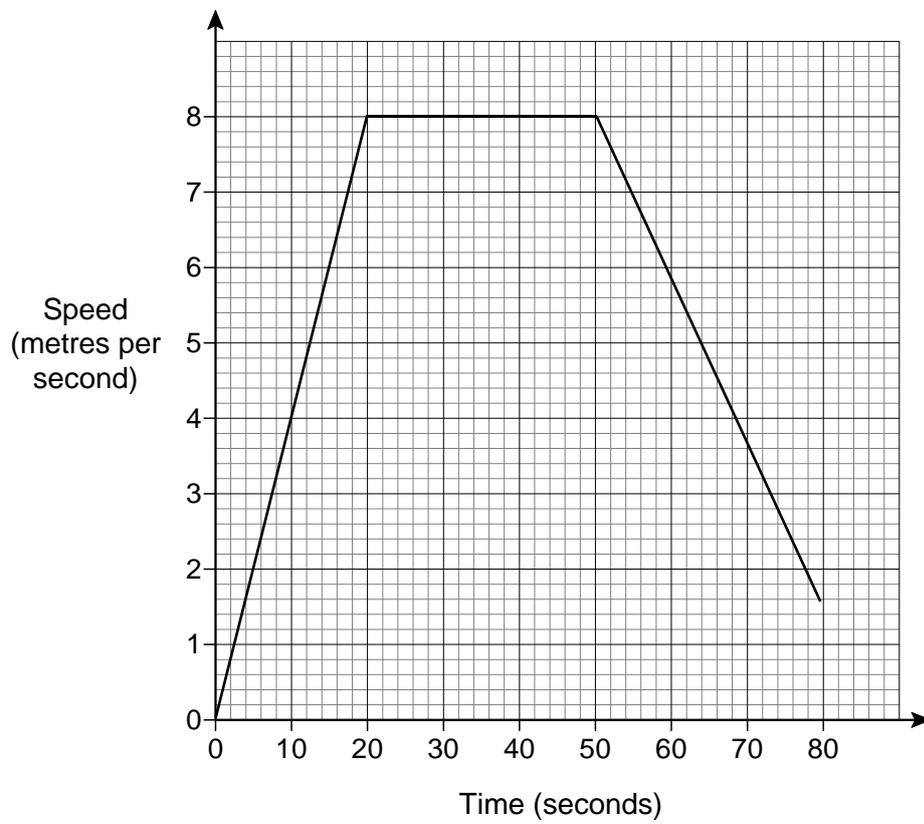$$\text{gradient of tangent} = \frac{26-11}{4} = \frac{15}{4} = 3.75$$

gradient = change in y ÷ change in x

Answer _____ 3.75 _____ cm/s

24   Amina and Ben had a cycle race.

Here is Amina's speed-time graph from the start of the race.

24 The distance of the race was 400 metres.

Ben cycled the 400 metres in 64 seconds.

Who won the race?

You **must** show your working.

distance travelled = area under graph

[4 marks]

area 0 to 20s = $\frac{20 \times 8}{2}$ = 80

area 20 to 50s = 30 × 8 = 240

— Ben finishes

area 50 to 64s = $\frac{8+5}{2}$ × 4 = 91 — $\frac{a+b}{2} \times h$

80 + 240 + 91 = 411m

Amina had already finished the 400m race at 64s, so Amina won.

Answer _____ Amina _____
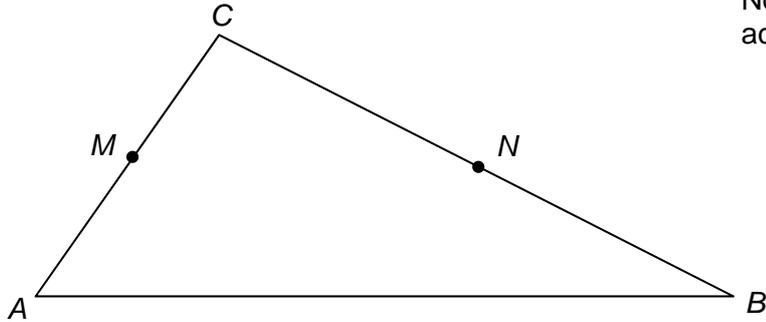
Turn over for the next question

25 In triangle ABC

M is the midpoint of AC

N is the point on BC where $BN : NC = 2 : 3$

$\overrightarrow{AC} = 2\mathbf{a}$

$\overrightarrow{AB} = 3\mathbf{b}$

Not drawn
accurately

25 (a) Work out $\overrightarrow{MN}$ in terms of **a** and **b**.

Give your answer in its simplest form.

[3 marks]

$\overrightarrow{MN} = \overrightarrow{MA} + \overrightarrow{AB} + \overrightarrow{BN}$ $\qquad \overrightarrow{BN} = \frac{2}{5}\overrightarrow{BC}$

$\overrightarrow{BC} = 2\underline{a} - 3\underline{b}$, so $\overrightarrow{BN} = \frac{2}{5}(2\underline{a} - 3\underline{b})$

← M is midpoint, so $\frac{2a}{2} = a$

$\overrightarrow{MN} = -\underline{a} + 3\underline{b} + \frac{2}{5}(2\underline{a} - 3\underline{b})$

$= \frac{4}{5}\underline{a} - \underline{a} + 3\underline{b} - \frac{6}{5}\underline{b}$

$= -\frac{1}{5}\underline{a} + \frac{9}{5}\underline{b}$

Answer $\quad -\frac{1}{5}\underline{a} + \frac{9}{5}\underline{b}$

25 (b) Use your answer to part (a) to explain why MN is **not** parallel to AB.

[1 mark]

$\overrightarrow{MN}$ is not a multiple of $\overrightarrow{AB}$

26 An approximate solution to an equation is found using this iterative process.

$$x_{n+1} = \frac{(x_n)^3 - 3}{8} \quad \text{and} \quad x_1 = -1$$

26 (a) Work out the values of x_2 and x_3

[2 marks]

$$x_2 = \frac{(-1)^3 - 3}{8} = -0.5$$

$$x_3 = \frac{(-0.5)^3 - 3}{8} = -0.390625$$

$x_2 = $ _____ -0.5 _____

$x_3 = $ _____ -0.390625 _____

26 (b) Work out the solution to 6 decimal places.

[1 mark]

$x_4 = -0.382451$ $x_8 = -0.381966$

$x_5 = -0.381993$ So solution to 6dp = -0.381966

$x_6 = -0.381967$

$x_7 = -0.381966$

$x = $ _____ -0.381966 _____

27 The curve with equation $y = x^2 - 5x + 2$ is reflected in the x-axis.

Circle the equation of the reflected curve.

[1 mark]

$y = x^2 - 5x - 2$ $\qquad\qquad\qquad\qquad$ $y = -x^2 + 5x + 2$
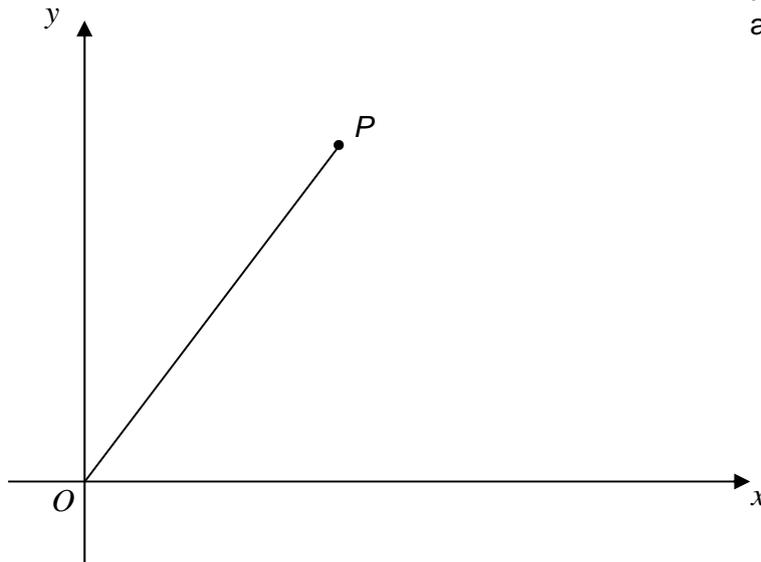
$\boxed{y = -x^2 + 5x - 2}$ $\qquad\qquad\qquad\qquad$ $y = x^2 + 5x + 2$

— reflect in x-axis.

$y = -(x^2 - 5x + 2)$

$y = -x^2 + 5x - 2$

28   The diagram shows a line joining O to P.

Not drawn
accurately

The gradient of the line is 2

The length of the line is $\sqrt{2645}$

Work out the coordinates of P.

[4 marks]

gradient is 2 and goes through (0,0)

$m = 2$

$y = mx + c \rightarrow 0 = 0 + c$, therefore $c = 0$, so $y = 2x$

$x^2 + y^2 = (\sqrt{2645})^2$ ← Pythagoras' Theorem

$x^2 + (2x)^2 = 2645$ $x = \frac{2645}{5} = 529$

$x^2 + 4x^2 = 2645$ $x = 23$, $y = 46$, P(23, 46)

$5x^2 = 2645$ only +ve value as P is at positive
                                        x value

Answer (_____23_____ , _____46_____)

END OF QUESTIONS

There are no questions printed on this page

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED

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