OCR
Oxford Cambridge and RSA

H

GCSE (9–1) Mathematics
J560/05 Paper 5 (Higher Tier)
Sample Question Paper

Date – Morning/Afternoon
Time allowed: 1 hour 30 minutes

Model Solutions

You may use:
- Geometrical instruments
- Tracing paper

Do not use:
- A calculator

First name	
Last name	

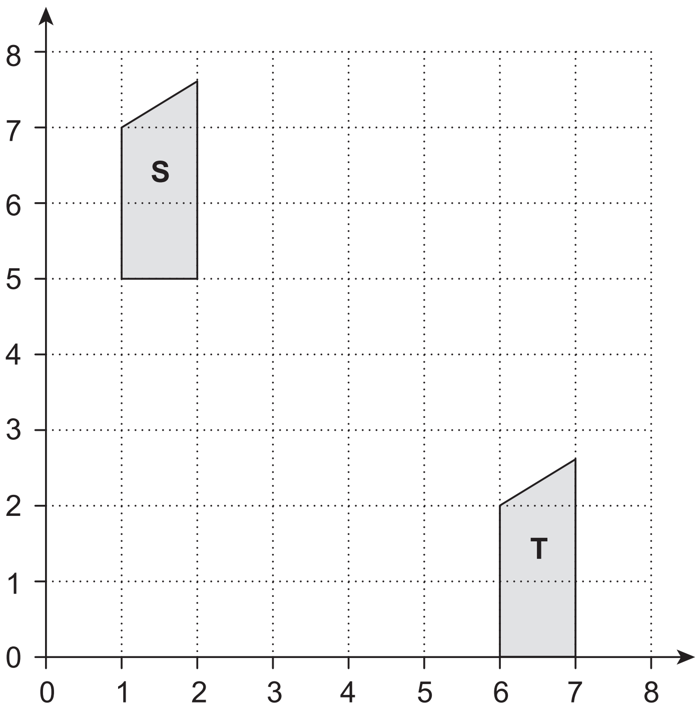
Centre number						Candidate number				

INSTRUCTIONS
- Use black ink. You may use an HB pencil for graphs and diagrams.
- Complete the boxes above with your name, centre number and candidate number.
- Answer **all** the questions.
- Read each question carefully before you start to write your answer.
- Where appropriate, your answers should be supported with working. Marks may be given for a correct method even if the answer is incorrect.
- Write your answer to each question in the space provided.
- Additional paper may be used if required but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document consists of **20** pages.

Answer **all** the questions

1 **(a)** Here is a coordinate grid.



Shape S is translated to Shape T using vector $\begin{pmatrix} p \\ q \end{pmatrix}$.

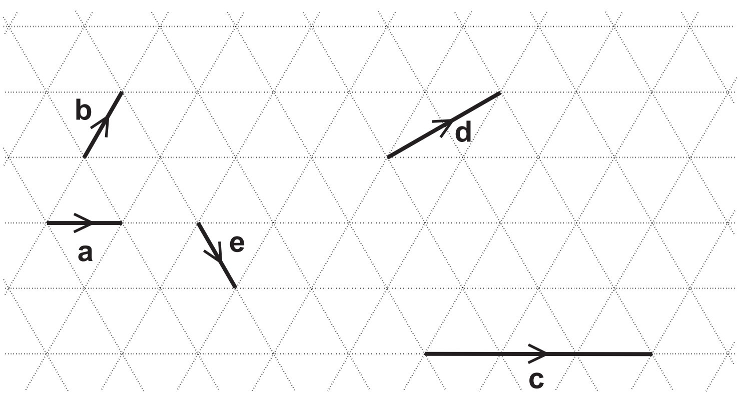
Write down the values of p and q.

Use bottom left corner of shapes.
Goes from (1,5) to (6,0)
So 5 right and 5 down → $\begin{pmatrix} 5 \\ -5 \end{pmatrix} = \begin{pmatrix} p \\ q \end{pmatrix}$

(a) p = 5

q = −5 **[2]**

(b) Vectors **a**, **b**, **c**, **d** and **e** are drawn on an isometric grid.



Write each of the vectors **c**, **d** and **e** in terms of **a** and/or **b**.

c = 3a

d = a + b

e = a − b

[3]

2 Sam and two friends put letters in envelopes on Monday.
The three of them take two hours to put 600 letters in envelopes.

(a) On Tuesday Sam has three friends helping.

Working at the same rate, how many letters should the **four** of them be able to put in envelopes in two hours?

$\frac{\text{600 envelopes}}{\text{3 people}}$ = 200 envelopes each

So 4 people × 200 = 800 envelopes

(a) ......800...... **[2]**

(b) Working at the same rate, how much longer would it take **four** people to put 1000 letters in envelopes than it would take **five** people?

4 people → $\frac{\text{1000 envelopes}}{\text{100 per hour per person}}$ = 10 hours

$\frac{\text{10 hours}}{4}$ = 2.5 hours if work together.

5 people → $\frac{\text{10 hours}}{\text{5 people}}$ = 2 hour if work together.

So 30 minutes more

(b) ......30 minutes...... **[4]**

(c) Sam says

> It took two hours for three people to put 600 letters in envelopes.
> If I assume they work all day, then in one day three people will put 7200 letters in envelopes because 600 × 12 = 7200.

Why is Sam's assumption not reasonable?
What effect has Sam's assumption had on her answer?

She has assumed by 'all day' it meant 24 hours but its not reasonable for them to work without a break. They cant work at that rate for that long so her answer is an over-estimate. **[2]**

3 Abi, Ben and Carl each drop a number of identical drawing pins, and count how many land with the pin upwards. The table shows some of their results.

	Number of pins dropped	Number landing 'pin up'
Abi	10	4
Ben	30	9
Carl	100	35

(a) Abi says

As a drawing pin can only land with its pin up or with its pin down, the probability of a drawing pin landing 'pin up' is $\frac{1}{2}$.

Criticise her statement.

Not correct as outcomes are not all equally likely

.... **[1]**

(b) Carl's results give the best estimate of the probability of a drawing pin landing 'pin up'. Explain why.

He carried out largest number of trials

.... **[1]**

(c) Two pins are dropped.

Estimate the probability that both pins land 'pin up'.

$$0.35 \times 0.35 = \left.\begin{matrix}(0.35 \times 0.3) \\ + \\ (0.35 \times 0.05)\end{matrix}\right\} \rightarrow \begin{matrix}0.105 \\ + \\ \frac{0.0350}{2}\end{matrix} \quad \begin{matrix}0.105 \\ + \\ 0.0175\end{matrix} = \begin{matrix}0.1050 \\ 0.0175 \\ \hline 0.1225\end{matrix}$$

(c) 0.1225 **[2]**

4 John is going to make chocolate squares to sell.

There are just three ingredients, chocolate, peanut butter and crisped rice, mixed in the ratio 4 : 2 : 3 respectively.

(a) How much of each ingredient will he need to make 900 g of mixture?

900 grams split into (2+3+4) = 9 parts

$\frac{900}{9} = 100g$ per part

C : PB : CR

4 : 2 : 3

400 : 200 : 300

(a) chocolate ....400.... g

peanut butter ....200.... g

crisped rice ....300.... g

[2]

(b) A bar of chocolate weighs 200 g and costs £2.50.
A jar of peanut butter contains 250 g and costs £1.70.
A packet of crisped rice contains 300 g and costs £2.00.

John makes 4.5 kg of mixture, from which he can cut 100 chocolate squares.
He charges 60p for each square and sells all 100 squares.

How much **profit** does he make?

1 kg = 1000 grams

4.5 kg = 4500 grams

Batches of mixture → $\frac{4500}{900} = 5$

Choclate needed → 5 × 400g = 2000g   cost → $\frac{2000g}{200g}$ = 10 bars → 10 × £2.50 = £25

Peanut Butter needed → 5 × 200g = 1000g   cost → $\frac{1000g}{250g}$ = 4 jars → 4 × £1.70 = £6.80

Crisped Rice needed → 5 × 300g = 1500g   cost → $\frac{1500g}{300g}$ = 5 packets → 5 × £2 = £10

Total money in → 100 squares × 60p = 6000p = £60

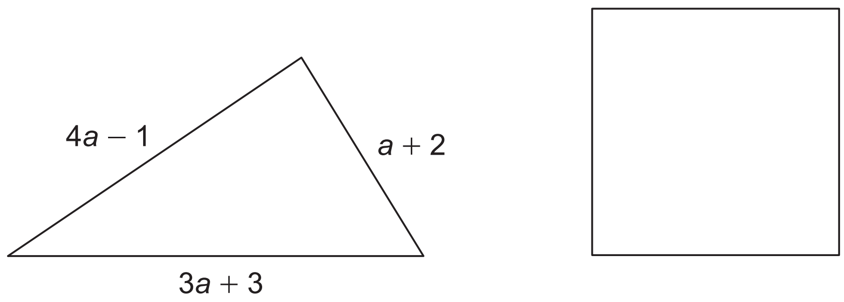
profit → £60 − (25 + 10 + 6.80)

£60 − £41.80

= £18.20

(b) £ ....18.20.... **[5]**

5 The perimeter of the triangle is the same length as the perimeter of the square.



Find an expression for the length of one side of the square in terms of a.
Give your answer in its simplest form.

Triangle perimeter → $4a-1+a+2+3a+3$
$8a+4$

Perimeter of square → $4 \times$ length

$4L = 8a+4$
$L = \frac{8a+4}{4} = 2a+1$

$2a+1$ .... **[4]**

6 A bag contains only red and blue marbles.

Yasmine takes one marble at random from the bag.
The probability that she takes a red marble is $\frac{1}{5}$.

Yasmine returns the marble to the bag and adds five more red marbles to the bag.
The probability that she takes one red marble at random is now $\frac{1}{3}$.

How many marbles of each colour were originally in the bag?

At start

$P(\text{red}) = 1/5$
$P(\text{blue}) = 1 - 1/5 = 4/5$

red : blue
$1/5 : 4/5$
$1 : 4$

Let there be x red marbles and $4x$ blue.

After adding more marbles

$x + 5$ red marbles
$4x$ blue marbles

Total $\rightarrow$ $x + 5 + 4x = 5x + 5$ marbles

$P(\text{red}) \rightarrow \frac{x+5}{5x+5} = \frac{1}{3}$ cross multiply

$3(x+5) = 5x+5$
$3x + 15 = 5x + 5$
$10 = 2x$
$x = 5$ red originally

$4x = 20$ blue originally

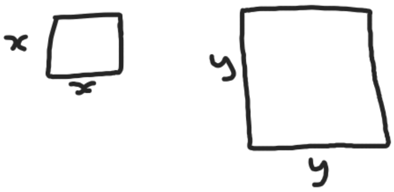
.................5................ red marbles

.................20................ blue marbles

[3]

7 The lengths of the sides of two squares are integers, when measured in cm.
The difference between the areas of the two squares is 36 cm^2.

Find the lengths of the sides of the two squares.



difference in areas =
$y^2 - x^2 = 36$
$100 - 64 = 36$
$10^2 - 8^2 = 36$

So $y = 10$, $x = 8$

Lengths of sides are 10cm and 8cm

..............8.............. cm

..............10.............. cm

[3]

8 Safety rules on a campsite require Sarah to set up her barbecue at least 4 m from her tent.
She decides to measure this distance using her stride length.
Sarah knows that her stride length is 0.8 m, rounded to the nearest 0.1 m.

Find the minimum number of strides Sarah will need to take to **guarantee** that her barbecue is a safe distance from her tent.

$0.75 \le \text{stride} < 0.85$
LB UB

Minimum strides $\rightarrow \frac{4m}{LB} = \frac{4m}{0.75} = \frac{16}{3} = 5.\dot{3}$
$= 6$ strides

..............6.............. **[3]**

9 A sculptor needs to lift a piece of marble.
It is a cuboid with dimensions 1 m by 0.5 m by 0.2 m.
Marble has a density of 2.7 g/cm^3.
The sculptor's lifting gear can lift a maximum load of 300 kg.

Can the lifting gear be used to lift the marble?
Justify your decision.

Cuboid volume → $1 \times 0.5 \times 0.2 = 100\text{cm} \times 50\text{cm} \times 20\text{cm}$
$= 100 \times 1000$
$= \underline{100{,}000 \text{ cm}^3}$

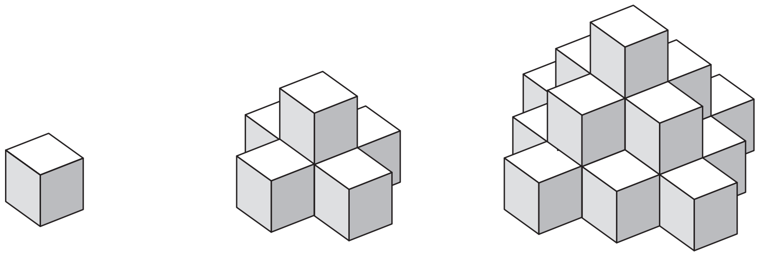
$\text{Density} = \dfrac{\text{mass}}{\text{volume}}$

Mass = density × volume
$= 2.7 \times 100000$
$= 270{,}000$ g
$= 270$ kg

270 kg < 300 kg. So, yes it can be used to lift marble

[4]

10 Here is a picture of three towers.
Not all the cubes can be seen in the towers.



Tower 1 Tower 2 Tower 3

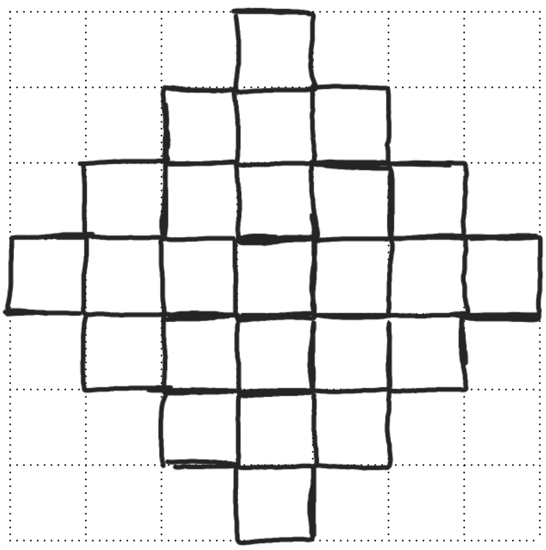
Edith uses 1 cube to build tower 1.
Edith uses 6 cubes to build tower 2. There are 5 cubes on the bottom layer.

(a) Write down the total number of cubes in tower 3.

Tower 3 = Tower 2 + new bottom layer.
= 6 + 13 = 19

(a) ....19........ **[1]**

(b) Draw a plan view of the arrangement of cubes Edith will use for the bottom layer of tower 4.



[1]

(c) Continue this sequence to show the number of cubes used for the bottom layer of each tower.

Tower 1	Tower 2	Tower 3	Tower 4
1	5	13	25

+4 +8 +12

+4 +4

[2]

(d) Find an expression for the number of cubes used in the bottom layer of tower *n*.

2nd difference = 4 so n^{th} term contains $\frac{4}{2}n^2 = 2n^2$

n	1	2	3	4
$2n^2$	2	8	18	32
to get to number in sequence	−1	−3	−5	−7

−2 −2 −2

So $-2x + n$ where n is 0^{th} term

0^{th} term → $-1 + 2 = 1$

So $-2n + 1$

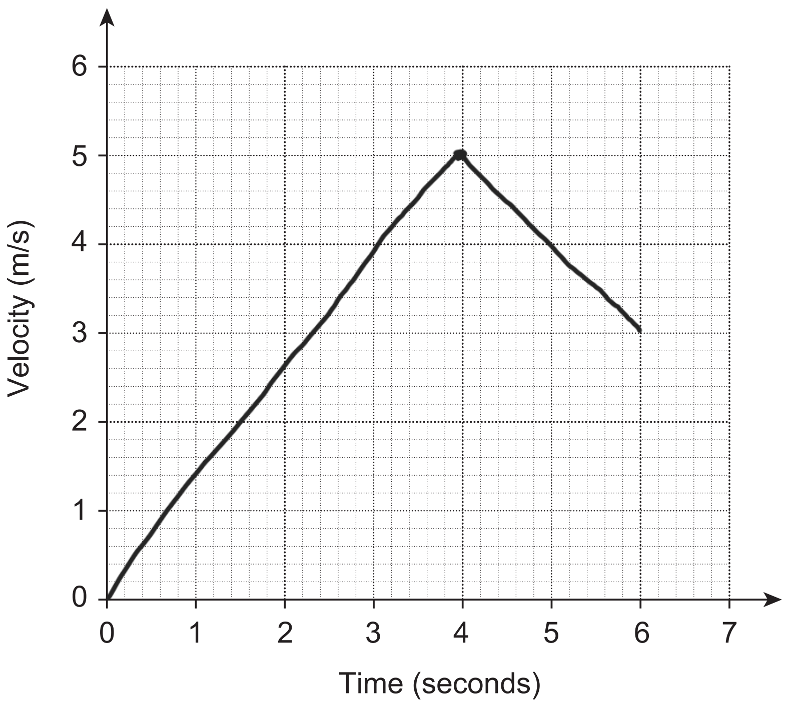
$2n^2 - 2n + 1$

(d) $2n^2 - 2n + 1$ **[4]**

11 A toy car is placed on the floor of a sports hall.

It moves in a straight line starting from rest.
It travels with constant acceleration for 4 seconds reaching a velocity of 5 m/s.
It then slows down with constant deceleration of 1 m/s^2 for 2 seconds.
It then hits a wall and stops.

(a) Draw a velocity-time graph for the toy car.



[3]

(b) Work out the total distance travelled by the toy car.

Distance = area under graph

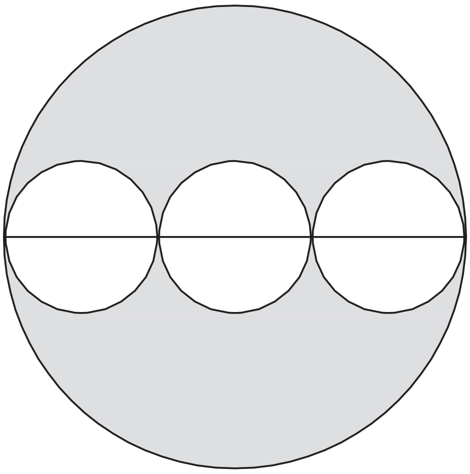
split into triangle and trapezium

$1/2\ b \times h$ $1/2(a+b)h$

area = $(1/2 \times 4 \times 5) + 1/2(5+3)2$
= 10 + 8 = 18m

(b) ....18....... m **[3]**

12 Three identical small circles are drawn inside one large circle, as shown in the diagram.
The centres of the small circles lie on the diameter of the large circle.



Find the fraction of the large circle that is shaded.

Diameter of large circle → 3 × diameter small circle.

Radius large circle → 3 × radius small circle

$R = 3r$

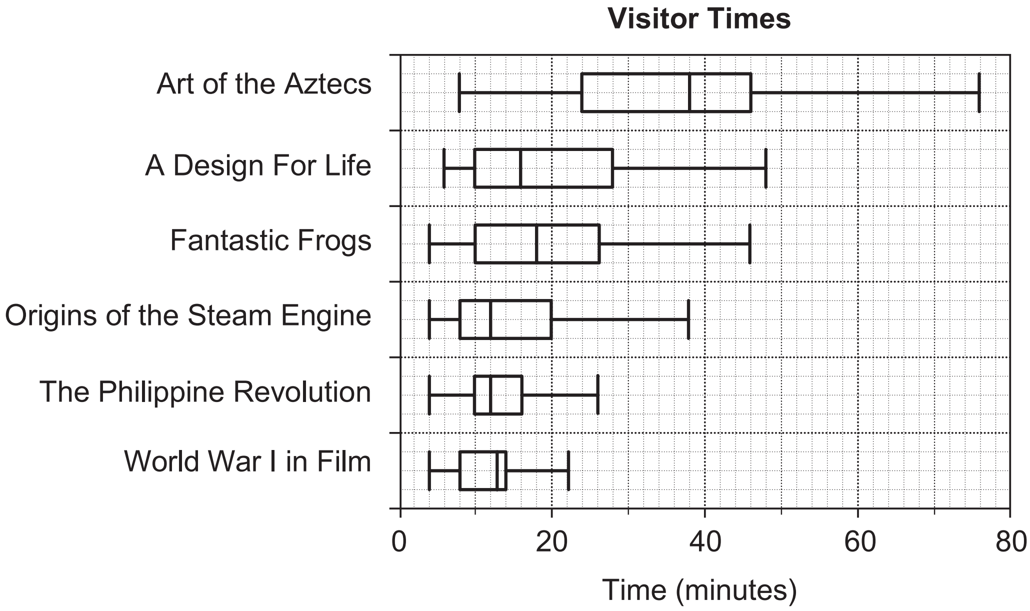
Area large circle → $\pi R^2 = \pi(3r)^2 = 9\pi r^2$

Area small circle → πr^2

Fraction of large circle shaded → $\dfrac{9\pi r^2 - \underbrace{3(\pi r^2)}_{\text{3 small circle}}}{9\pi r^2} = \dfrac{6\pi r^2}{9\pi r^2} = \dfrac{6}{9} = \dfrac{2}{3}$

.......2/3....... **[3]**

13 One day a museum monitored the time spent by visitors at six exhibitions.
The visitor times are summarised in the box plots below.



(a) Work out the **range** in visitor times at the **Fantastic Frogs** exhibition.

46 − 4 = 42

(a) ....42.......... **[2]**

(b) At which exhibition were visitor times the most consistent?
Give a reason for your answer.

World war 1 in film because this box plot had the smallest range

.... **[2]**

(c) Give one similarity and one difference between the **distributions** of the visitor times for **Origins of the Steam Engine** and **The Philippine Revolution**.

Similarity Same median value

Difference Range of values for 'The Philipine Revolution' was less so there was less variation in visitor time compared to 'Origins of the Steam Engine'.

[2]

(d) Is it possible to work out from the box plots which exhibition had the most visitors? Justify your answer.

No, because there is no indication of the numbers of visitors at each exibition.

[2]

14 Show that line $3y = 4x - 14$ is perpendicular to line $4y = -3x + 48$. **[4]**

$3y = 4x - 14$

$y = \frac{4}{3}x - \frac{14}{3}$

Gradient = 4/3

$4y = -3x + 48$

$y = \frac{-3}{4}x + \frac{48}{4}$

Gradient = −3/4

Two lines are perpendicular it the product of their gradients is −1

$\frac{4}{3} \times \frac{-3}{4} = \frac{-12}{12} = \underline{\underline{-1}}$ so these two lines are perpendicular.

15 **(a)** Write this list of numbers in order, smallest first.

$$\sqrt{35}, \quad \frac{20}{3}, \quad 2.5^2, \quad 6.83$$

$\sqrt{35} \rightarrow \sqrt{36} = 6$ so $\sqrt{35}$ is less than 6

$\frac{20}{3} = \frac{18}{3} + \frac{2}{3} = 6.\dot{6}$

$2.5^2 \rightarrow 2.5 \times 2.5 \rightarrow (2.5 \times 2) + (2.5 \times 0.5)$
$= (5) + (1.25) = 6.25$

$6.83 = 6.83$

(a) $\sqrt{35}$ .... 2.5^2 .... $\frac{20}{3}$ .... 6.83 .... **[2]**
smallest

(b) Write $(1 + \sqrt{3})^2$ in the form $a + b\sqrt{3}$.

$(1+\sqrt{3})(1+\sqrt{3}) \rightarrow 1 + \sqrt{3} + \sqrt{3} + 3$
$= 1 + 2\sqrt{3} + 3$
$= 4 + 2\sqrt{3}$

$a = 4 \quad b = 2$

(b) $4 + 2\sqrt{3}$ .... **[3]**

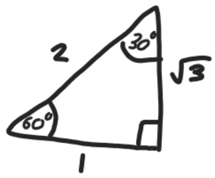
16 Bethany says that $(2x)^2$ is always greater than or equal to $2x$.

Decide whether she is correct or not.
Show your working to justify your decision. **[3]**

If $x = 0.1$
then $2x = 0.2$
and $(2x)^2 = (0.2)^2 = 0.04$
$0.2 > 0.04$, so $2x > (2x)^2$

This contradicts her statement so its not always true.

17 (a) Write down the exact value of tan 60°.

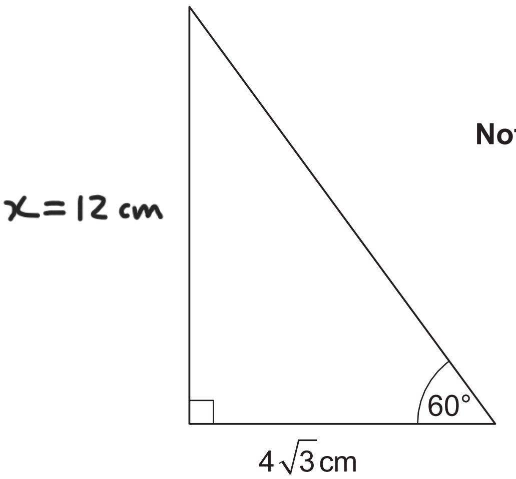


$\text{Tan } 60 = \frac{O}{A}$

$\text{Tan } 60 = \sqrt{3}/1 = \sqrt{3}$

(a) ....$\sqrt{3}$.... **[1]**

(b) Find the exact area of this triangle.



Not to scale

$\text{Tan } 60 = \frac{x}{4\sqrt{3}}$

$\underline{x} = 4\sqrt{3} \times \text{Tan } 60$

$= 4\sqrt{3} \times \sqrt{3} = 4 \times 3 = \underline{12\text{ cm}}$

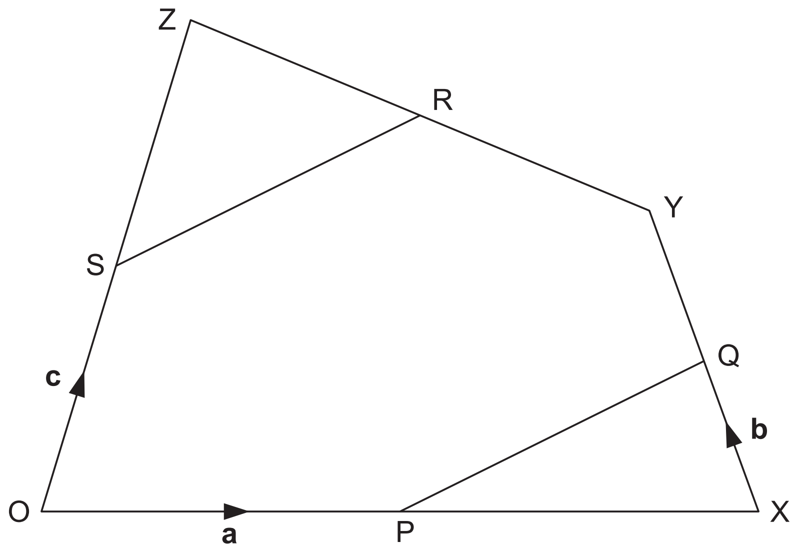
$\text{Area} = 1/2 \times b \times h$

$= 1/2 \times 12 \times 4\sqrt{3} = 6 \times 4\sqrt{3}$

$= \underline{\underline{24\sqrt{3}\text{ cm}^2}}$

(b) ....$24\sqrt{3}$.... cm^2 **[4]**

18 P, Q, R and S are the midpoints of OX, XY, YZ and OZ respectively.



$\overrightarrow{OP} = \mathbf{a}$, $\overrightarrow{XQ} = \mathbf{b}$ and $\overrightarrow{OS} = \mathbf{c}$.

Show that PQ is parallel to SR. **[5]**

$\overrightarrow{ZY} = \overrightarrow{ZO} + \overrightarrow{OX} + \overrightarrow{XY}$
$= 2(-c) + 2(a) + 2(b)$
$= \underline{-2c + 2a + 2b}$

$\overrightarrow{SR} = \overrightarrow{SZ} + \overrightarrow{ZR} = \overrightarrow{SZ} + 1/2\,\overrightarrow{ZY}$
$= c + 1/2(2a + 2b - 2c)$
$= c + a + b - c = \underline{a + b}$

$\overrightarrow{PQ} = \overrightarrow{PX} + \overrightarrow{XQ}$
$= \underline{a + b}$

$\overrightarrow{SR} = \overrightarrow{PQ}$
So they are parallel

19 The prices of two phones are in the ratio $x : y$.

When the prices are both increased by £20, the ratio becomes 5 : 2.
When the prices are both reduced by £5, the ratio becomes 5 : 1.

Express the ratio $x : y$ in its lowest terms.

$x : y$

$x+20 : y+20 = 5 : 2$

so $\frac{x+20}{y+20} = \frac{5}{2}$ cross multiply

$2(x+20) = 5(y+20)$

$2x+40 = 5y+100$

① $2x-5y=60$

$x : y$

$x-5 : y-5 = 5 : 1$

so $\frac{x-5}{y-5} = \frac{5}{1}$ cross multiply

$1(x-5) = 5(y-5)$

$x-5 = 5y-25$

② $x-5y=-20$

① $2x-5y=60$

② $x-5y=-20$

$x = 80$

sub $x=80$ in ① $\rightarrow 2x-5y=60$

$2(80)-5y=60$

$160-5y=60$

$-5y=-100$

$y=20$

$x : y$

$80 : 20$

$8 : 2$

$4 : 1$

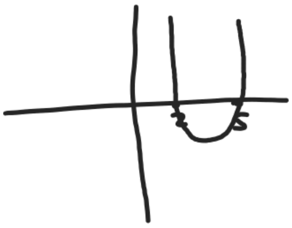
................4...... : ..1............ **[6]**

20 **(a)** Find the interval for which $x^2 - 7x + 10 \leqslant 0$.

$x^2 - 7x + 10 \leq 0$

$-5 \times -2 = 10$
$-5 + -2 = -7$

so $(x-5)(x-2) \leq 0$
$x=5$ $x=2$



$2 \leq x < 5$

(a) .... $\leqslant x \leqslant$ .... **[3]**

(b) The point (⁻3, ⁻4) is the turning point of the graph of $y = x^2 + ax + b$, where a and b are integers.

Find the values of a and b.

Write in complete square format

$(x+a)^2 - b \rightarrow (x+3)^2 - 4$
because min point $(-3,-4)$

$(x+3)(x+3) - 4$
$x^2 + 3x + 3x + 9 - 4$
$x^2 + 6x + 5$

$a=6$ $b=5$

(b) $a =$ ......6...... $b =$ ......5...... **[3]**