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**H**

**GCSE (9–1) Mathematics**  
**J560/04 Paper 4 (Higher Tier)**  
 Sample Question Paper

**Date – Morning/Afternoon**

Time allowed: 1 hour 30 minutes



**You may use:**

- A scientific or graphical calculator
- Geometrical instruments
- Tracing paper

*Model Solutions*



<b>First name</b>										
<b>Last name</b>										
<b>Centre number</b>						<b>Candidate number</b>				

**INSTRUCTIONS**

- Use black ink. You may use an HB pencil for graphs and diagrams.
- Complete the boxes above with your name, centre number and candidate number.
- Answer **all** the questions.
- Read each question carefully before you start to write your answer.
- Where appropriate, your answers should be supported with working. Marks may be given for a correct method even if the answer is incorrect.
- Write your answer to each question in the space provided.
- Additional paper may be used if required but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.

**INFORMATION**

- The total mark for this paper is **100**.
- The marks for each question are shown in brackets [ ].
- Use the  $\pi$  button on your calculator or take  $\pi$  to be 3.142 unless the question says otherwise.
- This document consists of **20** pages.

Answer **all** the questions

- 1 18 rice cakes weigh a total of 130 g.  
There are 329 calories in 100 g of rice cakes.

How many calories are there in one rice cake?

$$\frac{130}{18} = \frac{65}{9} \text{ g per rice cake}$$

$$\frac{329}{100} = 3.29 \text{ calories per gram}$$

$$3.29 \times \frac{65}{9} = \frac{4277}{180} \text{ calories in one rice cake}$$

$$= \underline{\underline{23.8}}$$

.....**23.8**..... calories [3]

- 2 A circular table top has radius 70 cm.

(a) Calculate the area of the table top in  $\text{cm}^2$ , giving your answer as a multiple of  $\pi$ .

$$\pi r^2 \rightarrow \pi \times 70^2 = \underline{\underline{4900 \pi \text{ cm}^2}}$$

(a) .....**4900 $\pi$** .....  $\text{cm}^2$  [2](b) The volume of the table top is  $17150\pi \text{ cm}^3$ .

Calculate the thickness of the table top.

$$\frac{17150\pi}{4900\pi} = \underline{\underline{3.5 \text{ cm}}}$$

(b) .....**3.5**..... cm [2]

- 3 The value of a car £V is given by

$$V = 20000 \times 0.9^t$$

where  $t$  is the age of the car in complete years.

- (a) Write down the value of  $V$  when  $t = 0$ .

$$\begin{aligned} V &= 20000 \times 0.9^0 \\ &= 20000 \times 1 \\ &= \underline{\underline{20000}} \end{aligned}$$

(a) £ 20,000 ..... [1]

- (b) What is the value of  $V$  when  $t = 3$ ?

$$\begin{aligned} V &= 20000 \times 0.9^3 \\ &= \underline{\underline{14580}} \end{aligned}$$

(b) £ 14,580 ..... [2]

- (c) After how many complete years will the car's value drop below £10 000?

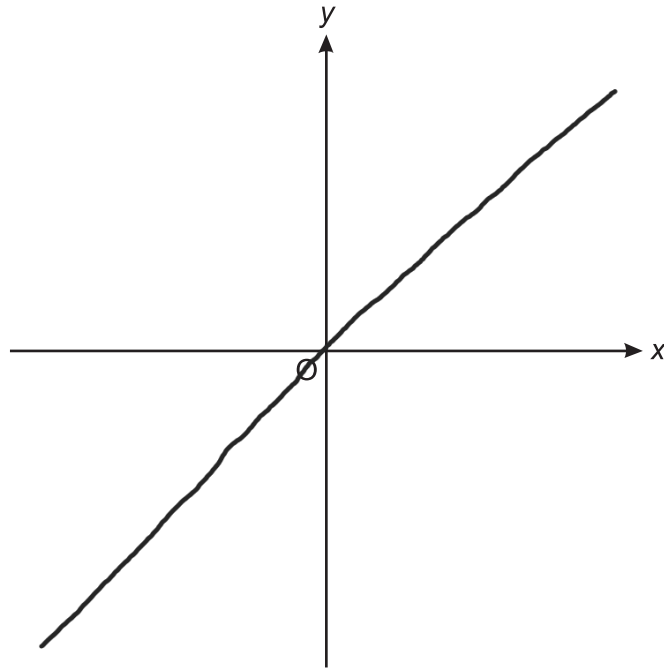
$$\begin{aligned} 10000 &> 20000 \times 0.9^t \\ 0.5 &> 0.9^t \end{aligned}$$

$$\text{When } t=6 \rightarrow 0.9^6 = 0.531441 > 0.5$$

$$\text{When } \underline{t=7} \rightarrow 0.9^7 = \underline{\underline{0.4782969}} < 0.5$$

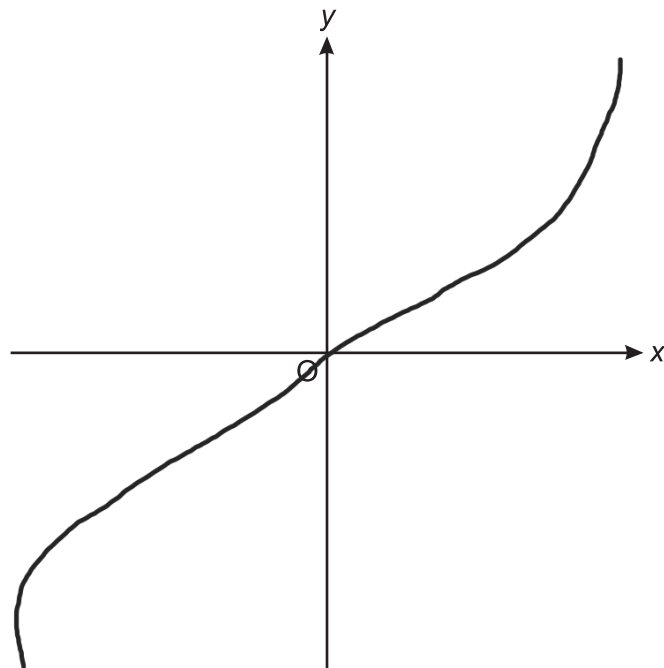
(c) ..... 7 ..... [2]

- 4 (a) (i) Sketch a graph on the axes below that shows that  $y$  is directly proportional to  $x$ .



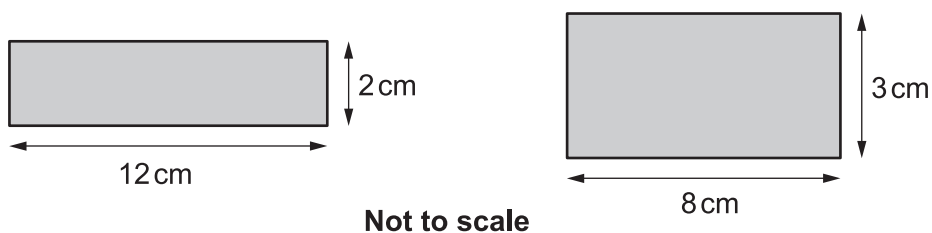
[2]

- (ii) Sketch a graph on the axes below that shows  $y = x^3$ .

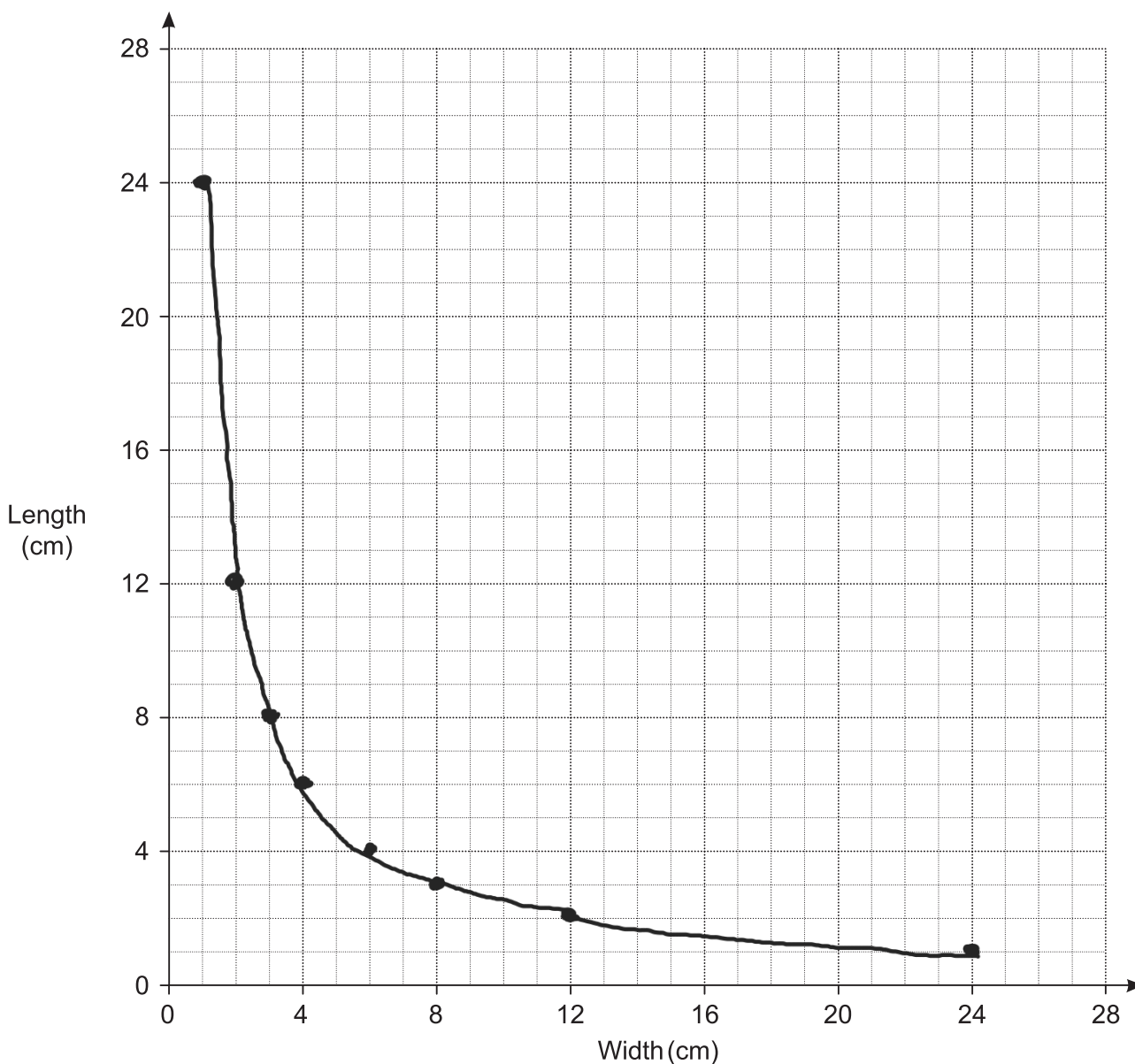


[2]

- (b) It is possible to draw many rectangles that have area  $24 \text{ cm}^2$ . Here are two of them.



- (i) Plot the dimensions of these two rectangles on the grid below. [1]
- (ii) Complete the graph to show the relationship between length and width for rectangles with area  $24 \text{ cm}^2$ . [3]



5 Kieran, Jermaine and Chris play football.

- Kieran has scored 8 more goals than Chris.
- Jermaine has scored 5 more goals than Kieran.
- Altogether they have scored 72 goals.

How many goals did they each score?

$$k = c + 8$$

$$J = k + 5 \quad \longrightarrow \quad \begin{array}{l} J = c + 8 + 5 \\ J = c + 13 \end{array}$$

$$k + J + c = 72$$

$$(c + 8) + (c + 13) + c = 72$$

$$3c + 21 = 72$$

$$3c = 51$$

$$\underline{\underline{c = 17}}$$

$$\begin{array}{l} \text{so } k = c + 8 \\ = 17 + 8 \\ = \underline{\underline{25}} \end{array}$$

$$\begin{array}{l} \text{so } J = c + 13 \\ = 17 + 13 \\ = \underline{\underline{30}} \end{array}$$

Kieran ...25.....

Jermaine ...30.....

Chris ...17.....

[5]

- 6 Peter makes a large amount of pink paint by mixing red and white paint in the ratio 2 : 3.

Red paint costs £80 per 10 litres.

White paint costs £5 per 10 litres.

Peter sells his pink paint in 10-litre tins for £60 per tin.

Calculate how much **profit** he makes for each tin he sells.

Profit  $\rightarrow$  income - cost

Income = £60

Cost  $\rightarrow$

Red = white

2 : 3  $\rightarrow$  total 5 parts of 10 litres  
 $10/5 = 2$  litres per part

So for red paint  $2 \times 2$  parts = 4 litres

so  $4/10 \times £80 = \underline{£32}$

£ ..... 25 ..... [5]

For white paint  $2 \times 3$  parts = 6 litres

so  $6/10 \times £5 = \underline{£3}$

Total cost  $\rightarrow$   $£32 + £3 = \underline{£35}$

Profit  $\rightarrow$   $£60 - £35 = \underline{\underline{£25}}$

- 7 Dan believes he knows what his brother Ethan is thinking. He carries out an experiment to test this.

Dan and Ethan sit back-to-back.

Ethan rolls an ordinary fair dice.

Ethan then thinks about the number on the dice while Dan tries to predict this number.

- (a) In 300 attempts, how many correct predictions would you expect Dan to make if he was just guessing?

$$300 \times 1/6 = \underline{\underline{50}}$$

(a) 50 ..... [2]

- (b) The results of the first 15 attempts are shown in the table.

Ethan's number	2	6	5	3	2	1	5	1	3	4	4	6	1	6	5
Dan's prediction	2	4	3	1	2	6	1	6	4	3	2	6	5	2	3
Matching pair	✓				✓							✓			

Estimate the probability of getting a matching pair using the results of

- (i) the first five attempts,

$$\underline{\underline{\frac{2}{5}}}$$

(b)(i) 2/5 ..... [1]

- (ii) all 15 attempts.

$$3/15 = \underline{\underline{1/5}}$$

(ii) 1/5 ..... [1]

- (c) Use answers from (a) and (b) to comment on Dan's belief that he knows what Ethan is thinking.

No evidence that Dan knows what Ethan is thinking as over the 15 trials the relative frequency of 1/5 is very close to the theoretical probability of 1/6. ..... [2]



8 (a) A function is represented by the following function machine.



- (i) A number is input into the machine.  
The output is used as a new input.  
The second output is 11.

Work out the number that was the **first input**.

Work backwards twice

$$11 - 5 = 6$$

$$6 \div 2 = 3 \quad \text{so 3 is second input / first output}$$

$$3 - 5 = -2$$

$$-2 \div 2 = -1 \quad \text{so -1 is first input}$$

(a)(i) ..... -1 ..... [2]

- (ii) A number is input into the machine.  
The output given is the same number.

Work out the number.

$$\text{input} = x$$

$$\text{output} = (x \times 2) + 5 = \underline{2x + 5}$$

$$x = 2x + 5$$

$$-x = 5$$

$$\underline{x = -5}$$

(ii) ..... -5 ..... [3]

(b) Another function machine is shown below.



- ① If the Input is 3, the Output is 5.
- ② If the Input is 7, the Output is 25.

Use this information to fill in the two boxes.

[3]

$$\text{So } \textcircled{1} \quad 3x - y = 5$$

$$\textcircled{2} \quad 7x - y = 25$$

$$-4x = -20$$

$$\underline{x = 5}$$

$$\text{Use } x = 5 \text{ in } \textcircled{1} \text{ for } y$$

$$3x - y = 5$$

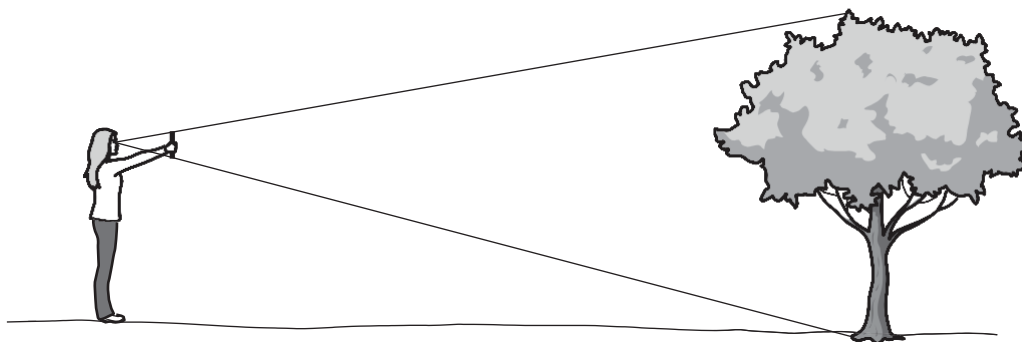
$$3(5) - y = 5$$

$$15 - y = 5$$

$$-y = -10$$

$$\underline{y = 10}$$

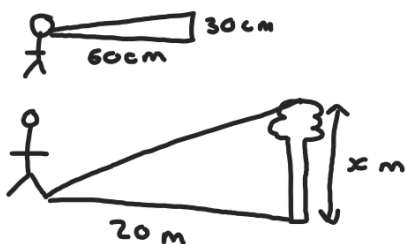
9 (a) Anna estimates the height of a tree.



Anna holds a ruler vertically so the height of the tree is exactly covered by the ruler.  
 She is 20 metres from the tree.  
 The ruler is 30 cm long.  
 The horizontal distance from her eyes to the ruler is 60 cm.

Calculate an estimate of the height of the tree.

*So the triangle made by ruler to her is similar to big triangle with tree.*



$$\frac{x}{20} = \frac{30}{60}$$

$$\frac{x}{20} = 0.5$$

$$x = 20 \times 0.5$$

$$x = 10 \text{ metres}$$

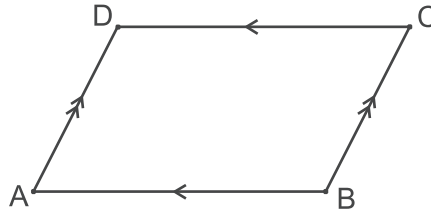
(a) .....10..... m [3]

(b) Give two reasons why this method may not be suitable to estimate the height of a very tall building.

- 1 *She would have to be very far from the building*
- 2 *The estimate is likely to be inaccurate due to the scale factors at the distances involved.*

[2]

10 ABCD is a parallelogram.



Prove that triangle ABD is congruent to triangle CDB.

[3]

BD is common to both triangles

$\angle ABD = \angle BDC$  as they are alternate angles

$AB = CD$  as they are parallel

They are congruent triangles as they follow SAS rule.

- 11 (a) Give **one** reason why 0 is an even number.

*Zero remainder when divided by 2.*

..... [1]

- (b) The lengths of the sides of a **right-angled** triangle are all integers.

Prove that if the lengths of the two shortest sides are even, then the length of the third side must also be even.

*$a^2 + b^2 = c^2$  as right angle triangle*

.....

*$b = 4n$*

*$a = 2n$*

*$(2n)^2 + (4n)^2 = c^2$*

*$4n^2 + 16n^2 = c^2$*

*$20n^2 = c^2$*

*$2(10n^2) = c^2 \rightarrow$  so  $c^2$  is even as its a multiple of 2.*

..... [3]

- 12 (a) Without using a calculator, show that  $\sqrt{20} = 2\sqrt{5}$ . [2]

*$\sqrt{20} = \sqrt{4 \times 5} = 2 \times \sqrt{5} = \underline{\underline{2\sqrt{5}}}$*

- (b) The point A is shown on the unit grid below.  
The point B is  $2\sqrt{5}$  units from A and lies on the intersection of two grid lines.

Mark **one** possible position for B.

*Use pythagoras*

*$a^2 + b^2 = (2\sqrt{5})^2$*

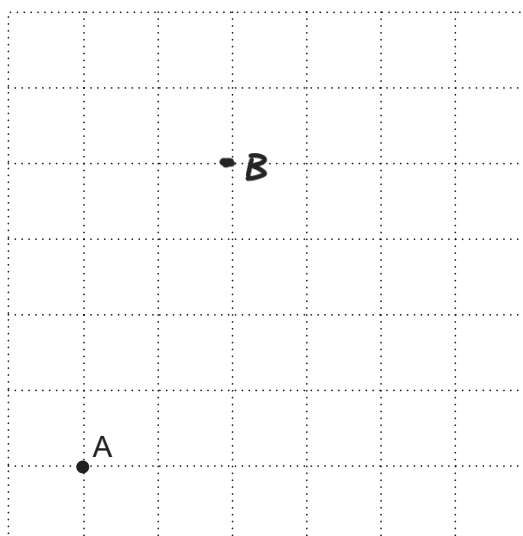
*$a^2 + b^2 = (\sqrt{20})^2$*

*$a^2 + b^2 = 20$*

*$2^2 + 4^2 = 20$*

*$4 + 16 = 20$*

*So 2 across and 4 up*



[3]

- 13 The volume of Earth is  $1.08 \times 10^{12} \text{ km}^3$ .  
The volume of Jupiter is  $1.43 \times 10^{15} \text{ km}^3$ .

How many times larger is the radius of Jupiter than the radius of Earth?  
Assume that Jupiter and Earth are both spheres.

[The volume  $v$  of a sphere with radius  $r$  is  $v = \frac{4}{3}\pi r^3$ .]

$$\frac{1.43 \times 10^{15}}{1.08 \times 10^{12}} = 1324.07 \text{ is how many times jupiter is bigger}$$

$$\text{Now radius} \rightarrow \sqrt[3]{1324.07} = 10.98 \\ = \underline{\underline{11}}$$

.....11..... [4]

14 The table shows the marks gained by 150 students taking an examination.

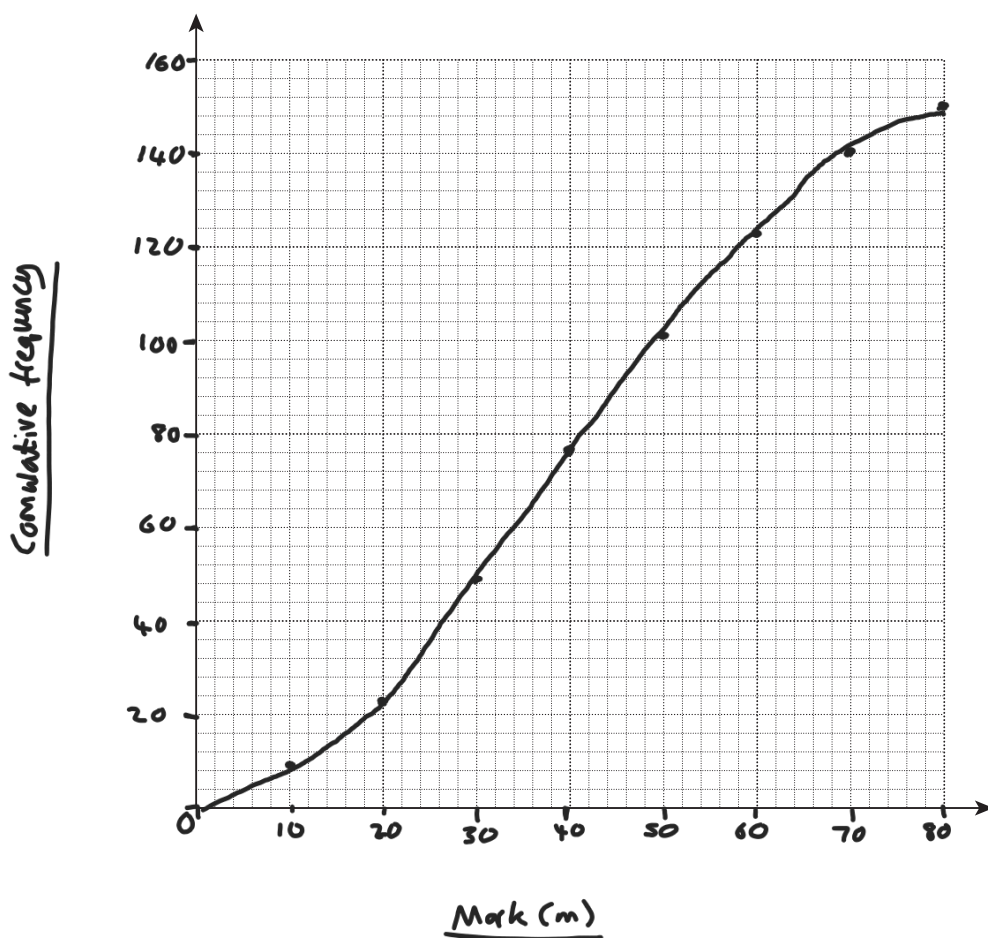
Mark ( $m$ )	$0 < m \leq 10$	$10 < m \leq 20$	$20 < m \leq 30$	$30 < m \leq 40$	$40 < m \leq 50$	$50 < m \leq 60$	$60 < m \leq 70$	$70 < m \leq 80$
Frequency	9	14	26	27	25	22	17	10

(a) (i) Construct a cumulative frequency table.

Mark ( $m$ )	$m \leq 10$	$m \leq 20$	$m \leq 30$	$m \leq 40$	$m \leq 50$	$m \leq 60$	$m \leq 70$	$m \leq 80$
Cumulative Frequency	9	23	49	76	101	123	140	150

[2]

(ii) Draw the cumulative frequency graph on the grid below.



[4]

- (b) Students are to be awarded Gold, Silver, Bronze or Fail.  
The students' teacher wishes to award the top 10% of students Gold, the next 60% Silver and the next 20% Bronze.

Use your graph to estimate the lowest mark that Silver will be awarded for.

$$\text{Gold} \rightarrow 10\% = 0.1$$

$$0.1 \times 150 = 15 \text{ students}$$

$$150 - 15 = 135 \text{ not gold}$$

$$60\% \text{ of } 150 = 90 \text{ students silver}$$

$$135 - 90 = 45^{\text{th}} \text{ mark lowest for silver}$$

$$45^{\text{th}} \text{ mark is at } \underline{28} \text{ on my graph}$$

(b) ..... 28 ..... [3]

- (c) Explain why the teacher's method will not necessarily award Gold to exactly 10% of the students.

The boundaries are set from approximations based on group data, not actual scores obtained by students. .... [1]

- 15 At a constant temperature, the volume of a gas  $V$  is inversely proportional to its pressure  $p$ .

By what percentage will the pressure of a gas change if its volume increases by 25%?

$$V \propto 1/p \rightarrow pV \propto 1 \text{ so } pV \text{ is constant}$$

$$p_1 V_1 = p_2 V_2 \text{ as this equals constant}$$

$$p_1 = 1 \quad p_2 = x$$

$$V_1 = 1 \quad V_2 = 1.25 = 1.25$$

$$1 \times 1 = 1.25 \times x$$

$$1 = 1.25x$$

$$0.8 = x$$

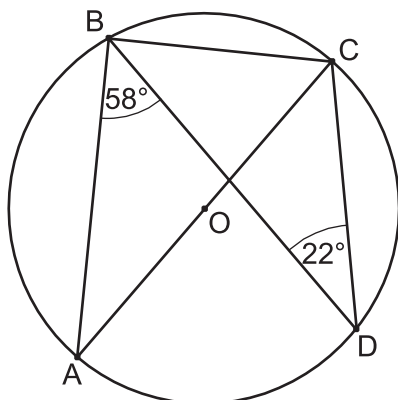
$$1 - 0.8 = 0.2$$

$$0.2 = 20\%$$

So 20% decrease

..... 20 ..... % [4]

- 16 A, B, C and D are points on the circumference of a circle, centre O.  
 AC is a diameter of the circle.  
 Angle ABD =  $58^\circ$ .  
 Angle CDB =  $22^\circ$ .



Not to scale

Work out the sizes of angle ACD and ACB, giving reasons for your answers.

(a) Angle ACD = 58..... $^\circ$

$\angle ACD = \angle ABD$   
as angles subtended from same arc are equal. [2]

(b) Angle ACB = 68..... $^\circ$

$180 - (58 + 22 + 32) = 68^\circ$   
Angles in triangle add to  $180^\circ$   
Also  $\angle DCB = 90 - 58 = 32^\circ$  as angles in semicircle add to  $90^\circ$ . [3]



17 A restaurant menu has 8 starters, 12 mains and 6 desserts.  
A customer can choose from the following meals

- a starter and a main,
- a main and a dessert,
- a starter, a main and a dessert.

Show that there are 744 different ways of choosing a meal at this restaurant.

[3]

Starter + Main

$$8 \times 12 = \underline{96 \text{ options}}$$

Main + Dessert

$$12 \times 6 = \underline{72 \text{ options}}$$

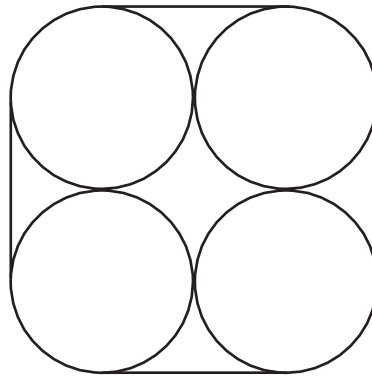
Starter + Main + Dessert

$$8 \times 12 \times 6 = \underline{576 \text{ options}}$$

Total options/combinations

$$96 + 72 + 576 = \underline{\underline{744}}$$

- 18 Four pencils are held together with a band.  
The figure below shows the bottom end of the pencils and the band.



Each of the pencils has diameter 9 mm.

Find the length of the band in this position.

Each corner =  $1/4$  of circle

So total 4 corners =  $4 \times 1/4 = 1$  circle arc length =  $\pi d = 9\pi \text{ mm} = \underline{28.3 \text{ mm}}$

Each straight part =  $2 \times \text{radius} = \text{diameter}$

So total straight parts =  $4 \times \text{diameter} = 4 \times 9 \text{ mm}$   
= 36 mm

Total  $\rightarrow 36 + 28.3 = \underline{64.3 \text{ mm}}$

..... 64.3 ..... mm [4]

19 A sequence is defined by the term-to-term rule  $u_{n+1} = u_n^2 - 8u_n + 17$ .

(a) Given that  $u_1 = 4$ , find  $u_2$  and  $u_3$ .

$$u_1 = 4$$

$$\begin{aligned} u_2 &\rightarrow (4)^2 - 8(4) + 17 \\ &= 16 - 32 + 17 \\ &= -16 + 17 \\ &= \underline{\underline{1}} \end{aligned}$$

$$u_2 = 1$$

$$\begin{aligned} u_3 &= (1)^2 - 8(1) + 17 \\ &= 1 - 8 + 17 \\ &= -7 + 17 \\ &= \underline{\underline{10}} \end{aligned}$$

(a)  $u_2 = 1$  .....  $u_3 = 10$  ..... [2]

(b) Given instead that  $u_1 = 2$ , find  $u_2$ ,  $u_3$  and  $u_{100}$ .

$$u_1 = 2$$

$$\begin{aligned} u_2 &= (2)^2 - 8(2) + 17 \\ &= 4 - 16 + 17 \\ &= -12 + 17 \\ &= \underline{\underline{5}} \end{aligned}$$

$$u_2 = 5$$

$$\begin{aligned} u_3 &= (5)^2 - 8(5) + 17 \\ &= 25 - 40 + 17 \\ &= -15 + 17 \\ &= \underline{\underline{2}} \end{aligned}$$

$$u_{\text{odd}} = 2$$

$$u_{\text{even}} = 5$$

(b)  $u_2 = 5$  .....  $u_3 = 2$  .....  $u_{100} = 5$  ..... [3]

$$100 \text{ is even so } u_{100} = \underline{\underline{5}}$$

20 (a) Express as a single fraction.

$$\frac{m+1}{n+1} - \frac{m}{n}$$

Simplify your answer.

$$\frac{m+1}{n+1} - \frac{m}{n} \rightarrow \frac{n(m+1) - m(n+1)}{n(n+1)}$$

$$\frac{mn+n - mn - m}{n(n+1)} \rightarrow \frac{n-m}{n(n+1)}$$

(a)  $\frac{n-m}{n(n+1)}$  ..... [2]

(b) Using your answer to part (a), prove that if  $m$  and  $n$  are positive integers and  $m < n$ , then

$$\frac{m+1}{n+1} - \frac{m}{n} > 0. \quad [2]$$

$$-m \left( \begin{array}{c} m < n \\ \hline 0 < n - m \end{array} \right) - m$$

Therefore the numerator is a positive integer

$n(n+1)$  is positive

$$\frac{\text{positive}}{\text{positive}} = \text{positive} > 0$$

so  $\frac{m+1}{n+1} - \frac{m}{n} > 0$

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