Level 1 / Level 2 GCSE (9 – 1)

MATHEMATICS

Paper 3 (Calculator)

Higher Tier

Time : 1 hour 30 minutes **Paper : 1 MA1 / 3H**

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided *– there may be more space than you need.*
- You must **show all your working.**
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- **Calculators may be used.**
- If your calculator does not have a π button, take the value of π to be 3.142 unless the question instructs otherwise.

Information

- The total mark for this paper is 80.
- The marks for **each** question are shown in brackets *– use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

Answer ALL questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1. a. Simplify $2a^2b \times 3ab$

for partial simplification e.g. a^3 or b^2 (1)

for $6a^3b^2$ (1)

....

(2)

b. Simplify $\frac{p^5}{p^2}$

correct answer only p^3 (1)

....

(1)

c. Solve $\frac{2x}{3} < 5$

$\frac{2x}{3} < 5$

$2x < 15$ (1)

$x < 7.5$ (1)

....

(2)

(Total for Question 1 is 5 marks)

2. Mr Ben drives 45 km from work at a speed of 60 km/h.
Mrs Ben drives 60 km from work at a speed of 75 km/h.
They both leave work at the same time.
Who arrives home first?

$\text{time} = \frac{\text{distance}}{\text{speed}}$

Mr Ben: $\frac{45}{60} = 0.75$ hours (1)

Mrs Ben: $\frac{60}{75} = 0.8$ hours (1)

Mr Ben arrives home first.

....

(Total for Question 2 is 2 marks)

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3. A number, y, is rounded to 1 significant figure.
The result is 800.
Write down the error interval for y.

$750 \leq y < 850$

(1) (1)

....................$750 \leq y < 850$..................

(Total for Question 3 is 2 marks)

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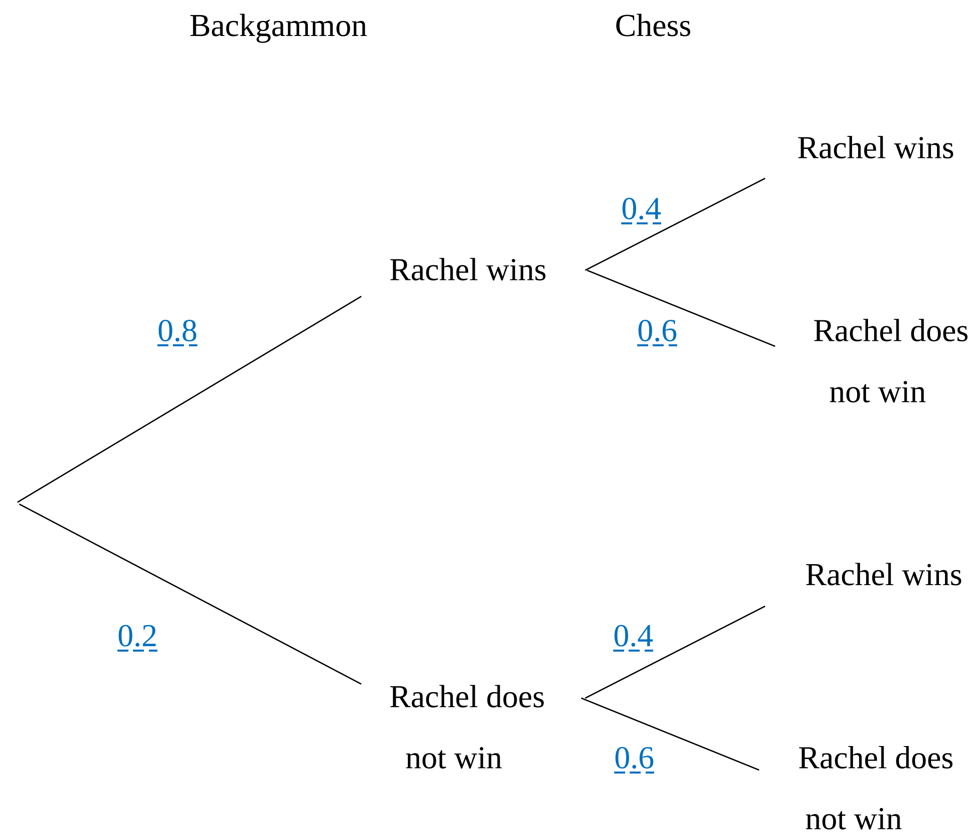
4. Rachel is going to play one game of backgammon and one game of chess.

The probability she will win the game of backgammon is 0.8
The probability that she will the game of chess is 0.4

a. Complete the probability tree diagram.



at least 2 correct probabilities (1)

six fully correct probabilities (1)

(2)

b. Work out the probability that Rachel will win both games.

$P(\text{Win, Win}) = 0.8 \times 0.4 = 0.32$

(1) (1)

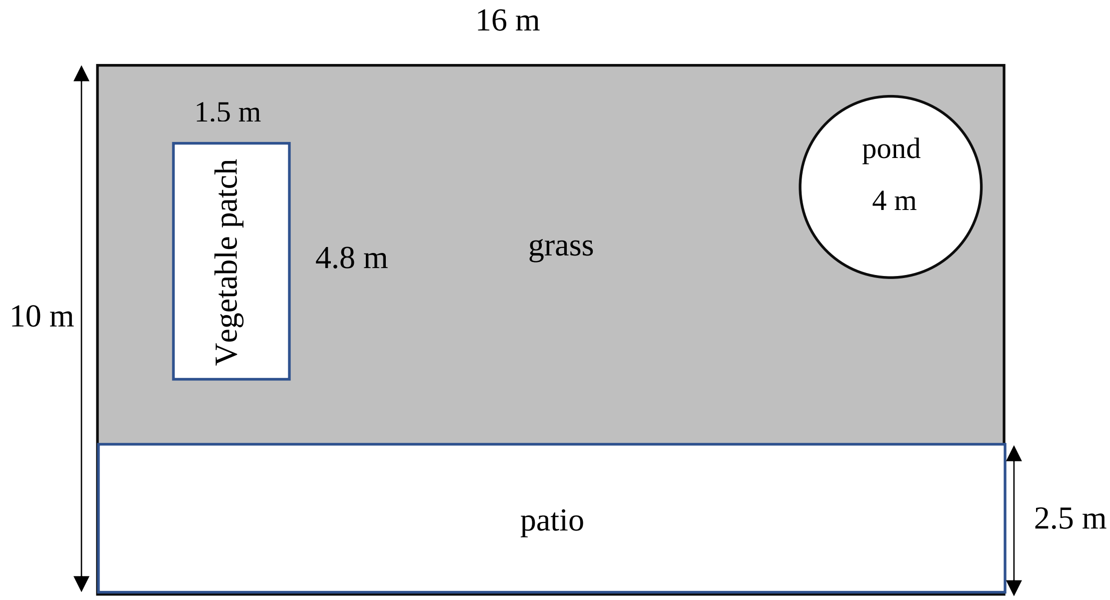
....
(2)

(Total for Question 4 is 4 marks)

5. Mr Kingdom's garden is in the shape of a rectangle.

In the garden, there is a patio in the shape of rectangle, a vegetable patch that is 4.8 m long and 1.5 m wide and a pond in the shape of a circle with diameter 4 m.

The rest of the garden is grass.



Mr Kingdom wants to spread fertiliser over all the grass.

One box of fertilizer costs £4.95 and will cover 6 m by 5 m of grass.

Work out the total cost to fertilise the grass.

Area of entire garden:

$$10 \times 16 = 160\text{m}^2$$

Area of patio:

$$16 \times 2.5 = 40\text{m}^2$$

Area of vegetable patch:

$$1.5 \times 4.8 = 7.2\text{m}^2$$

Area of pond:

$$\pi \times 2^2 = 12.5664m^2$$

(1) For an attempt at finding non-grass areas
(1) For all non-grass areas correct

Area of grass:

$$160 - (40 + 7.2 + 12.5664) = 100.2336$$

(1)

Boxes of fertiliser needed:

$$100.2336 \div (6 \times 5) = 3.3411 \quad (1)$$

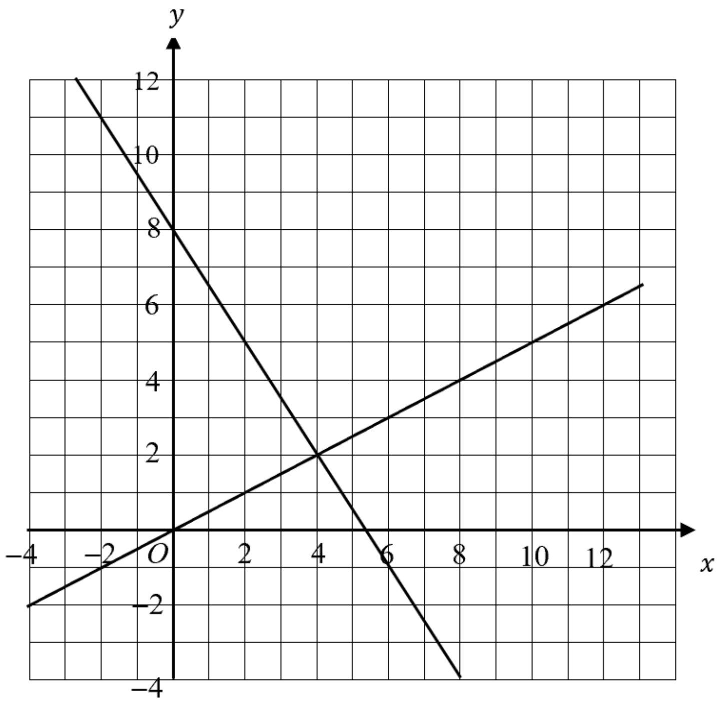
Mr Kingdom will need 4 boxes of fertiliser, which will cost:

$$4 \times £4.95 = £19.80 \qquad (1)$$

....

(Total for Question 5 is 5 marks)

6.



a. Use this graph to solve the simultaneous equation:

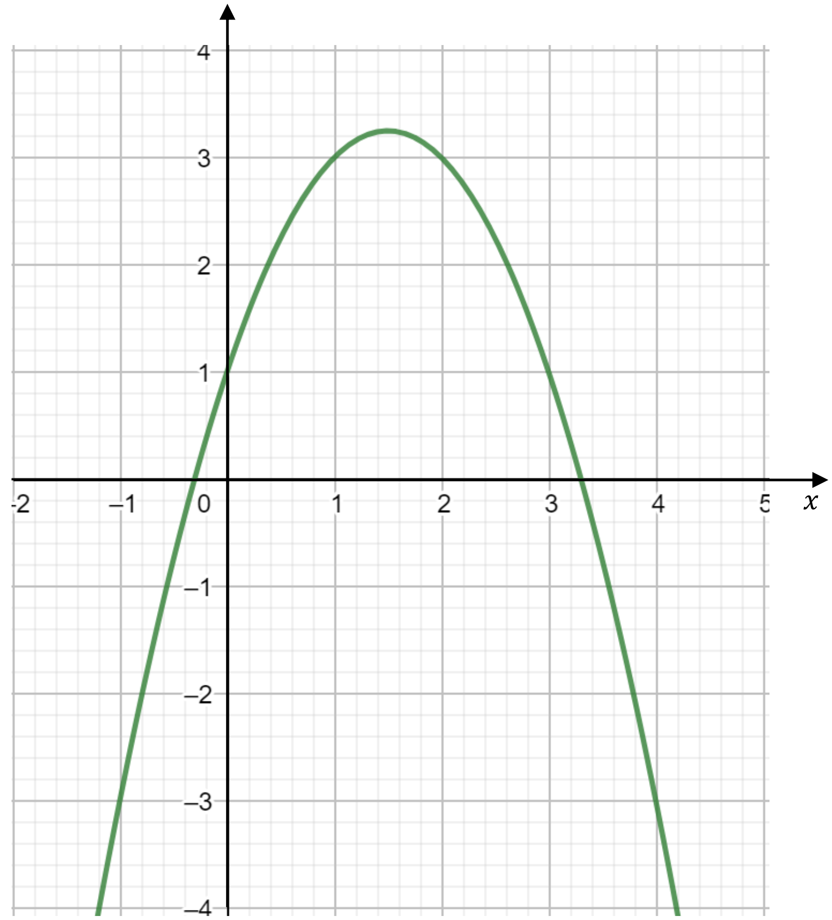
$$3x - 2y = 16$$
$$y = \frac{1}{2}x$$

The graph shows the lines $3x - 2y = 16$ and $y = \frac{1}{2}x$. The solution to these equations is found at the point where these lines cross, which is $(4,2)$.

$$x = 4, y = 2 \quad (1)$$

(1)

Here is a graph of $y = 3x + 1 - x^2$



b. i. Write down the turning point of the graph $y = 3x + 1 - x^2$

correct answer only (1.5 , 3.25) (1)

(1)

ii. Use the graph to find an estimate of a solution to the equation $x^2 = 3x + 1$

$0 = 3x + 1 - x^2 \Rightarrow x^2 = 3x + 1$

The solutions to this equation are given by the intercepts with the x −axis. These can be read off the graph: $(3.3, 0)$, $(-0.3, 0)$.

for correct method (1)

for answers in the range -0.25 to -0.35 and 3.25 to 3.35 (1)

....

(2)

(Total for Question 6 is 4 marks)

7. The students in Year 9A and Year 9B take the same examination.
There are 24 students in Year 9A and 26 students in Year 9B.
The mean score for all students in both Years 9 is 78.
The mean score for the students in Year 9B is 75.
Work out the mean score for the students in Year 9A.

Total scores of all Year 9s:

$$78 \times (24 + 26) = 3900$$

Mean score for students in 9A:

$$\frac{3900 - 1950}{24} = 81.25$$

Method to find total of scores of Years 9 (1)

for complete method (1)
correct answer (1)

....

(Total for Question 7 is 3 marks)

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8. A biased spinner can land on 1, 2 or 3.
The table shows the probability that the spinner will land on 2.

Number	1	2	3
Probability		0.32	

The probability that the spinner will land on 1 is three times the probability that spinner will land on 3.

a. Complete the table.

$P(1) + P(3) = 1 - 0.32 = 0.68$ (1)

$P(1) = 3x, P(3) = x$

$3x + x = 0.68 \Rightarrow x = 0.17$ (1)

$P(1) = 0.51$

$P(3) = 0.17$

(2)

Kelly is going to spin the spinner 200 times.

Work out an estimate for the number of times the spinner will land on 1.

$200 \times 0.51 = 102$ (1)

(1)

(Total for Question 8 is 3 marks)

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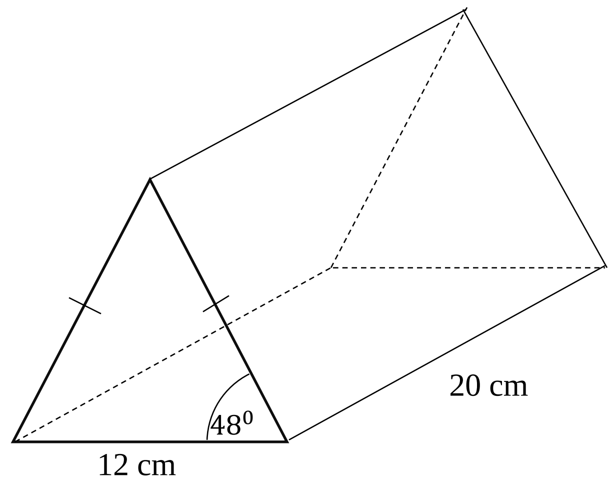
9. The diagram shows a triangular prism.



The cross-section of the prism is an isosceles triangle.
Calculate the volume of the prism.
Give your answer correct to three significant figures.

Height of triangle:

$$h = 6\tan 48 = 6.66367$$

Area of triangle:

$$\frac{1}{2} \times 12 \times 6.66367 = 39.982$$

Volume of prism:

$$\text{Volume=area of cross section} \times \text{length}$$
$$= 39.982 \times 20 = 799.642$$
$$= 800cm^3 \text{ to 3.s.f}$$

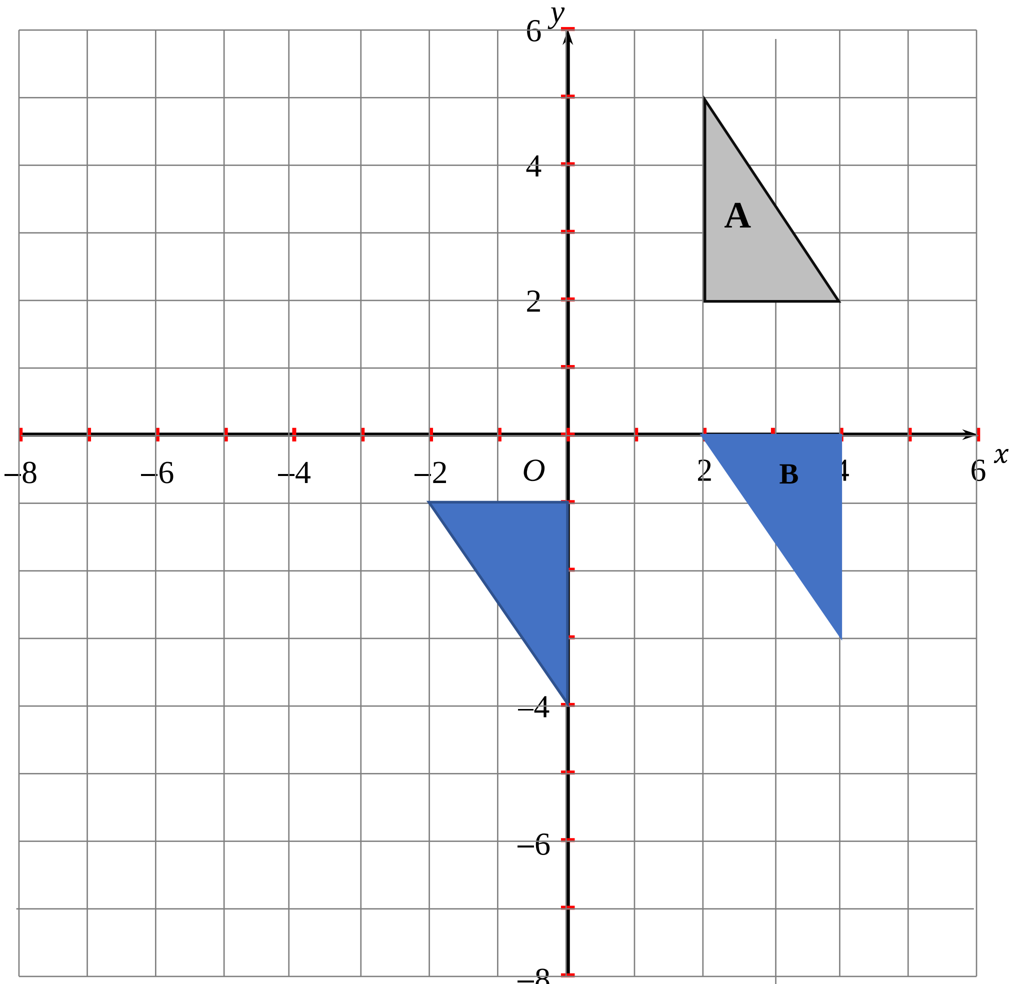
Start of process to find the height of triangle or slant height (1)
Complete process to find height of triangle or slant height (1)

Start of process to find volume of prism (1)
Correct answer only (1)

....

(Total for Question 9 is 4 marks)

10.



Triangle **A** is transformed by the combined transformation of a rotation of 180^0 about the point (1,0) followed by a translation with vector $\begin{pmatrix}4\\2\end{pmatrix}$ to a triangle **B**.

Describe fully the single transformation that maps triangle **A** onto triangle **B**.

Finding intermediate shape of triangle (at $(0,-2),(0,-5),(-2,-2)$)
or for method to find position of B (at $(2,0),(4,0),(4,-3)$)
or for method to translate their intermediate shape of triangle correctly. (1)
Complete transformation:
for rotation of 180^0 about (3,1)
or enlargement by scale factor -1, centre (3,1) (1)

(2)

Under the transformation that maps triangle **A** onto triangle **B**, the point P is invariant.

b. Write down the coordinates of P.

for (3,1) (1)

(1)

(Total for Question 10 is 3 marks)

11. The distance between Earth and Sun is 150 million kilometres.
Light travels at a speed of 3×10^5 km/s

a. Calculate the time, in seconds, it takes for light to travel from Earth to Sun.
Give your answer in standard form.

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

$$= \frac{150 \times 10^6}{3 \times 10^5} = 5 \times 10^2 \text{ seconds}$$

Process to find time (1)
Correct answer in standard form with units (1)

……………………………..
(2)

The distance travelled by light in a vacuum during one year is equal to 9.454×10^{12} km.
b. How far does light travel in 1 second? (1 year = 365 days)
Give your answer in standard form correct to 1 significant figure.

$$\frac{9.454 \times 10^{12}}{365 \times 24 \times 3600} = 299784.3734$$ (1)

$= 3 \times 10^5$ to 1.s.f (1)

………………………………………………………
(2)

(Total for Question 11 is 4 marks)

12. a. Express $\frac{2y}{y+4} - \frac{y}{y-2}$ as a single fraction in its simplest form.

$$\frac{2y}{y+4} - \frac{y}{y-2}$$

$$= \frac{2y(y-2)}{(y+4)(y-2)} - \frac{y(y+4)}{(y+4)(y-2)} \quad (1)$$

$$= \frac{2y(y-2) - y(y+4)}{(y+4)(y-2)} \quad (1)$$

$$= \frac{y^2 - 8y}{(y+4)(y-2)} \; or \; \frac{y^2 - 8y}{y^2 + 2y - 8} \quad (1)$$

Method to identify a common denominator (1)
Method to combine the fraction (1)
Correct answer (1)

……………………………………..

(3)

b. Expand and simplify $(2x+3)(x-4)^2$.

Show your working clearly.

$(2x+3)(x-4)(x-4) = (2x+3)(x^2 - 8x + 16)$

$= 2x^3 - 16x^2 + 32x + 3x^2 - 24x + 48$

$= 2x^3 - 13x^2 + 8x + 48$

Method to find the product of two linear expressions, 3 correct terms or 4 terms (ignoring signs) (1)

Complete method to obtain all terms, half of which are correct (1)

Correct answer only (1)

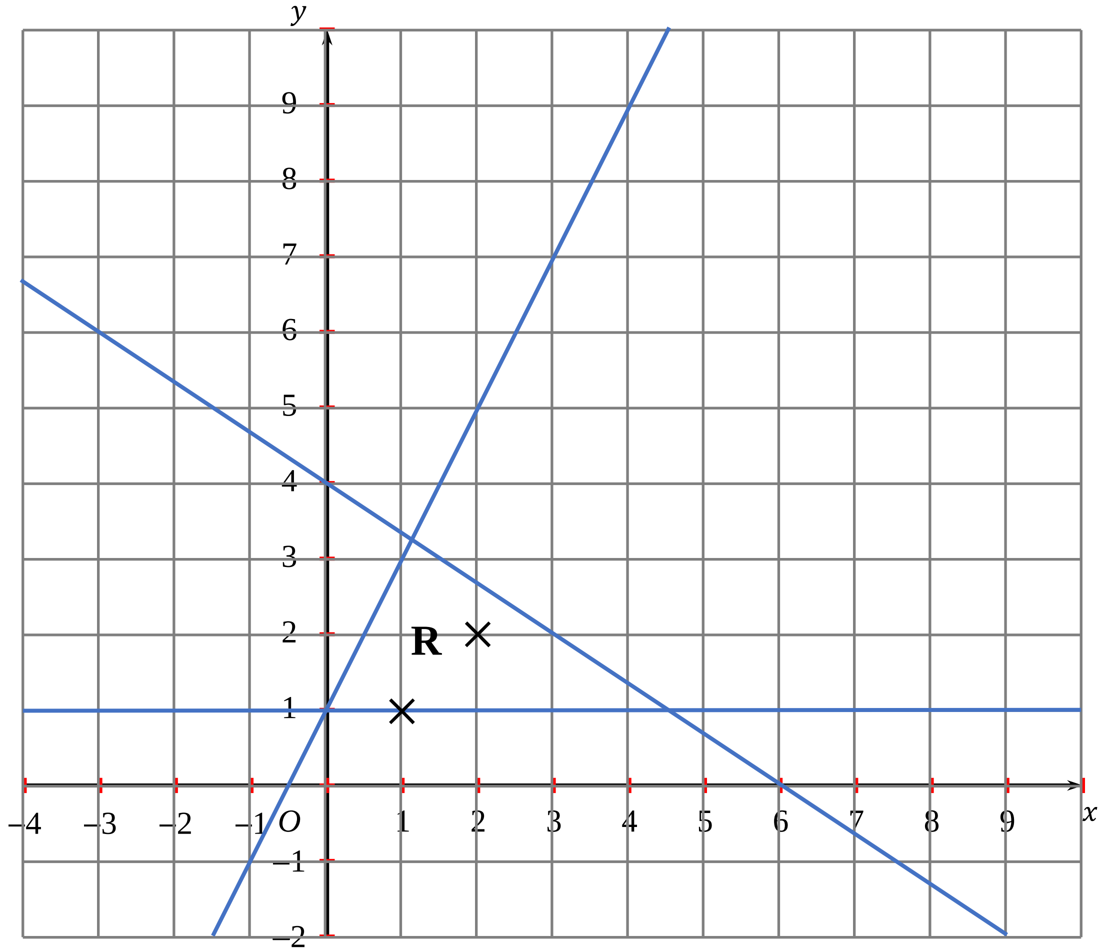
……………………………………..

(3)

(Total for Question 12 is 6 marks)

13. a. On the grid, shade the region that satisfies all these inequalities.

$y \leq 2x + 1$ $3y + 2x \leq 12$ $y \geq 1$



for 2 of the lines $y = 2x + 1$, $3y + 2x = 12$, $y = 1$ correctly drawn (1)
for all 3 lines $y \leq 2x + 1$, $3y + 2x \leq 12$, $y \geq 1$ correctly drawn (1)
for correct region identified (1)

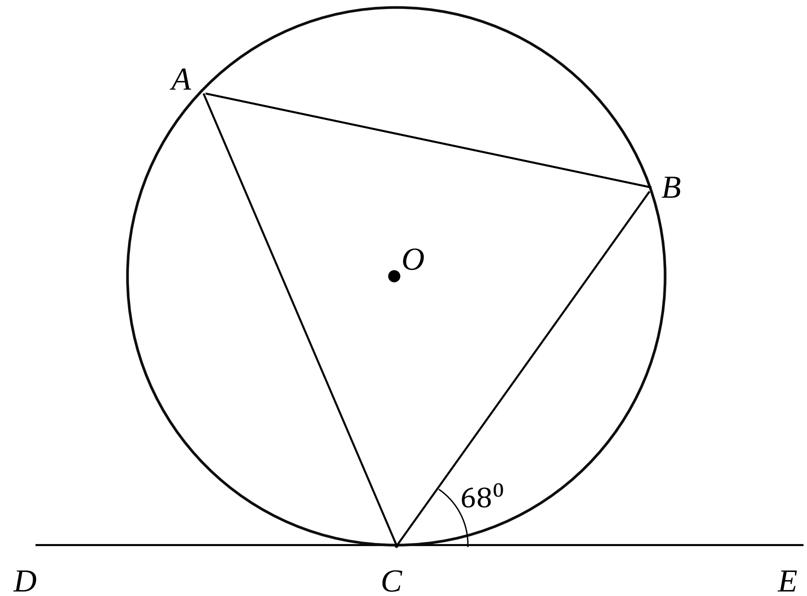
x and y are both integers.

b. Mark with a cross (×) the points in the region that satisfy $y = x$

A cross at $(1,1)$ and $(2,2)$ (1)

(Total for Question 13 is 4 marks)

14.



A and *B* are points on the circumference of a circle, centre *O*.
DCE is a tangent to the circle.

Angle $BCE = 68°$

a. Find the size of angle *CAB*, giving a reason for your answer.

Angle $CAB = 68°$ (1)

Due to the alternate segment theorem (1)

……………………………°**(2)**

b. Find the size of angle *COB*, giving a reason for your answer.

Angle $COB = 136°$ (1)

Because the angle at the centre of the circle is twice that of the angle at the circumference. $68 \times 2 = 136$. (1)

………………………………°**(2)**

(2)

(Total for Question 14 is 4 marks)

15. Prove algebraically that the recurring decimal $0.0\dot{2}$ can be written as $\frac{1}{45}$.

$$100x = 2.222\dot{2}$$
$$-10x = 0.222\dot{2}$$
$$90x = 2$$
$$x = \frac{2}{90} \Rightarrow x = \frac{1}{45}$$

For finding difference that would lead to a terminating decimal **(1)**

For completing algebra to reach $\frac{1}{45}$ **(1)**

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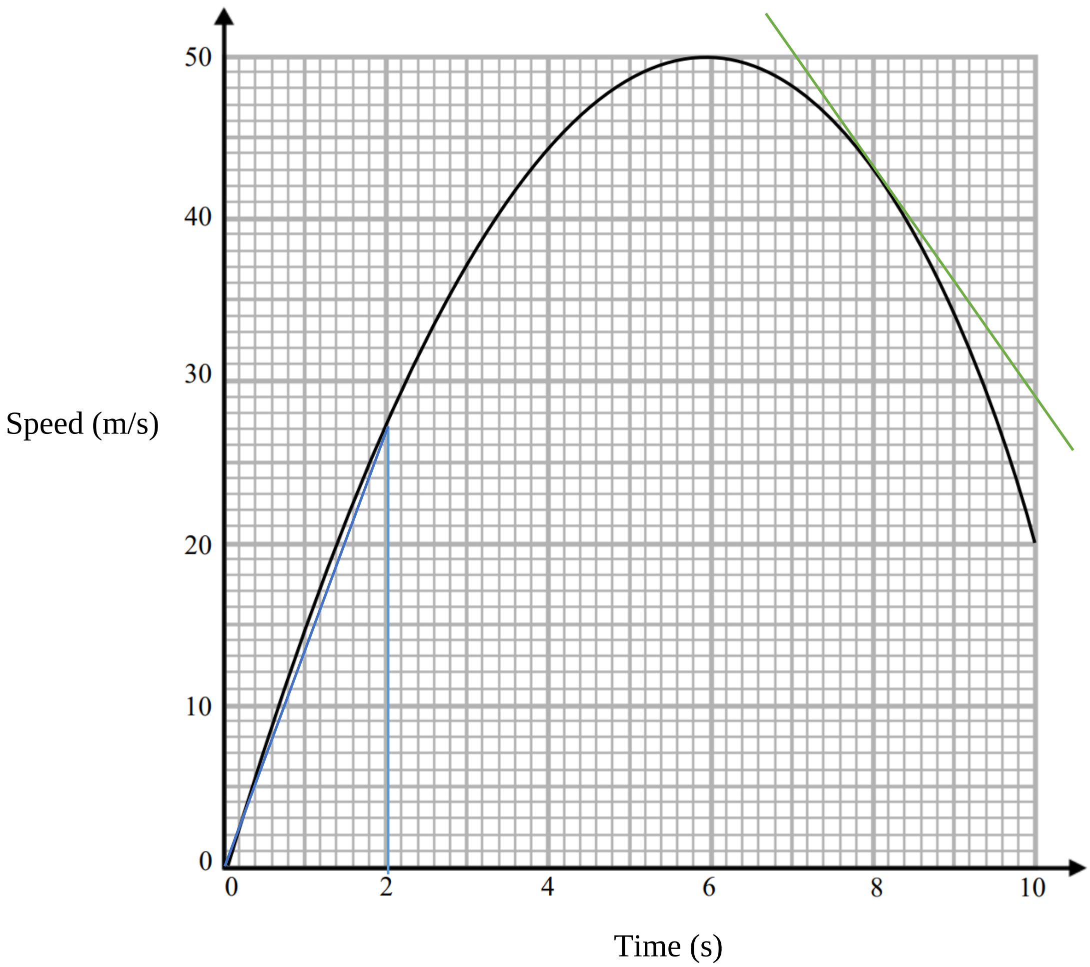
(Total for Question 15 is 2 marks)

16. Here is a speed-time graph.



a. Work out an estimate for the distance travelled in the first 2 seconds.

Estimate the area under the curve using the blue triangle drawn on the graph.

Area$= 0.5 \times 2 \times 27$ (1)

$= 27$ (1)

.... m

(2)

b. Is your answer to part (a) an underestimate or an overestimate of the actual distance? Give a reason for your answer.

for underestimate and appropriate reason linked to their method. e.g. area between triangle and curve not included (1)

(1)

c. Work out an estimate for the deceleration when $t = 8$

Estimate can be found by working out gradient at $t = 8$, given by the green line on the graph.

gradient $= \frac{43-30}{8-10}$ (1)

$= 6.5$ (1)

....

(2)

(Total for Question 16 is 5 marks)

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17. $P = 3 \times 5^{43}$

$Q = 27 \times 5^{41}$

Find the highest common factor (HCF) of P and Q.

$$P = 3 \times 5^{41} \times 5^2$$

$$Q = 3 \times 3^2 \times 5^{41}$$

$$HCF = 3 \times 5^{41}$$ (1)

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(Total for Question 17 is 1 mark)

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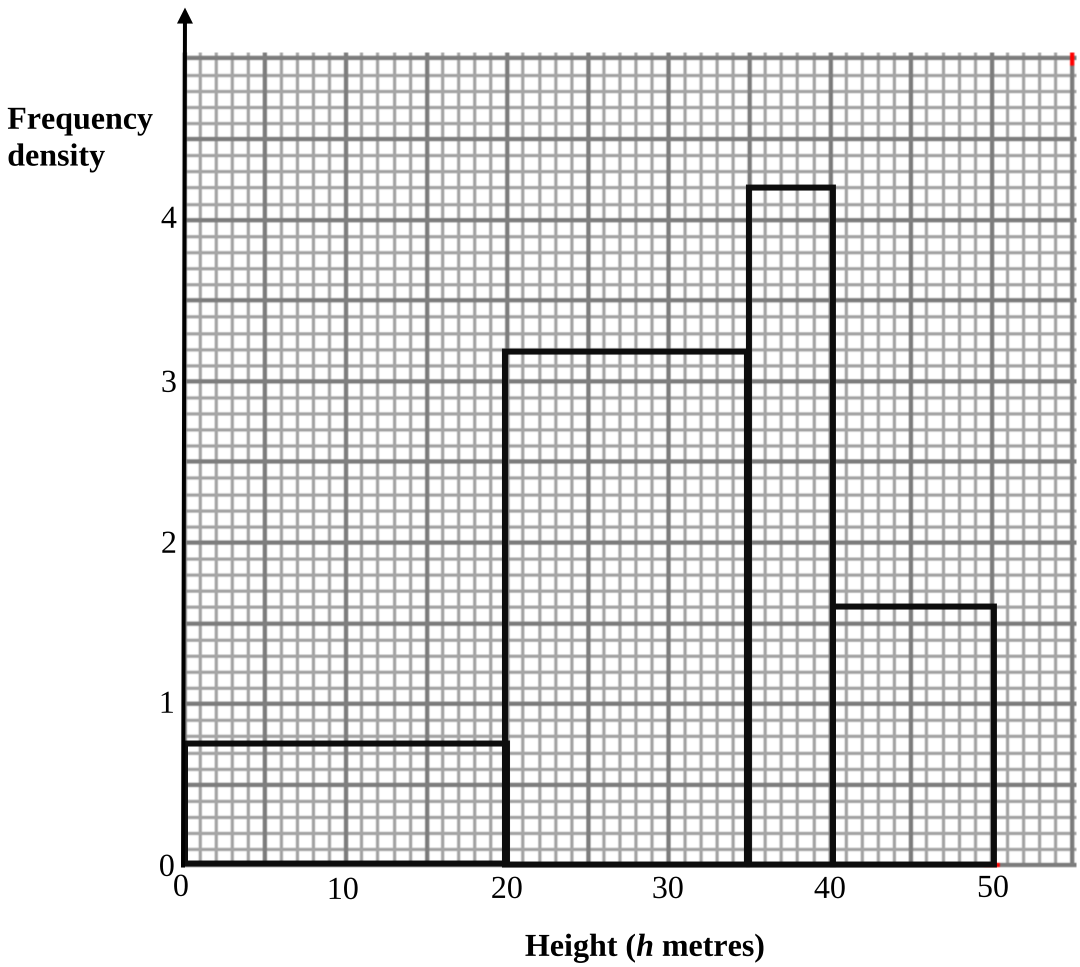
18. The histogram shows the heights of some trees.



a. Complete the grouped frequency table for the data.

Height (h metres)	Frequency
$0 < h \leq 20$	15
$20 < h \leq 35$	48
$35 < h \leq 40$	21
$40 < h \leq 50$	16

for a correct method to find at least 2 frequencies from bars of different widths.

$$\text{Frequency} = \text{Frequency density} \times \text{class width}$$
$$15 \times 3.2 = 48$$
$$5 \times 4.2 = 21$$
$$10 \times 1.6 = 16$$

Attempt to find frequency by finding area (1)

All correct frequencies found (1)

(2)

b. Work out an estimate for the mean height of some trees.

$$(10 \times 15 + 27.5 \times 48 + 37.5 \times 21 + 45 \times 16) \div 100 = 29.775$$

(1) (1)

(2)

c. Work out an estimate for the number of these trees with a height greater than 30 cm.

for process to find total number of trees with a height greater than 30 cm (1)

Estimate for $30 < h \leq 35 = 48 \div 3 = 16$
$30 < h \leq 50 = 16 + 21 + 16 = 53$ (1)

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(2)

(Total for Question 18 is 6 marks)

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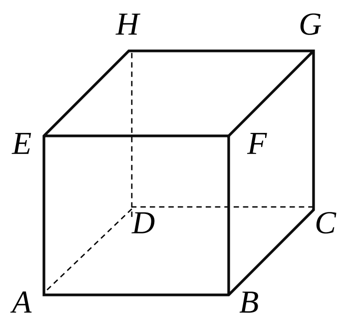
19. The diagram shows a cube.



The volume of a cube is 130 cm^3 correct to the nearest 10 cm^3. Calculate the lower bound for the length of diagonal *AG*.
You must show all your working.

Lower bound for volume $= 125\text{cm}^3$ (1)

$a^3 = 125 \quad \Rightarrow a = 5$ (1)

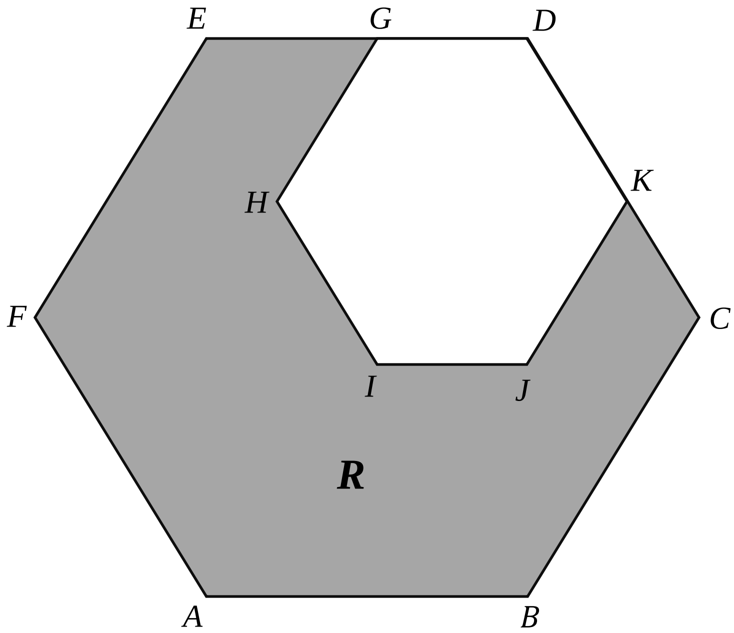
$AC = \sqrt{5^2 + 5^2} = 5\sqrt{2}$ (1)

$AG = \sqrt{(5\sqrt{2})^2 + 5^2} = 5\sqrt{3}$ (1)

....

(Total for Question 19 is 4 marks)

20.



DGHIJK is a regular hexagon with sides of length 2 cm.
This hexagon is enlarged, centre *D*, by scale factor *p* to give hexagon *DEFABC*.

The area of the shaded region, ***R***, in the diagram is $\frac{15\sqrt{3}}{2}$.

Find the value of *p*.

Area of *DGHIJK:*
Area can be split into 6 equilateral triangles of length 2cm.

Area $= 6 \times \frac{1}{2} \times 2 \times 2 \times \sin 60 = 6\sqrt{3}$ (1)

Area of *DEFABC*.
Area can be split into 6 equilateral triangles of length $2 \times p$.

Area $= 6 \times \frac{1}{2} \times 2p \times 2p \times \sin 60 = 6\sqrt{3}\, p^2$ (1)

Area of shaded region R

$6\sqrt{3}p^2 - 6\sqrt{3} = \frac{15\sqrt{3}}{2}$ (1)

$\Rightarrow \; p = \frac{3}{2}$ (1)

....

(Total for Question 20 is 4 marks)

21 $\frac{7p}{2} + q = 2p + 5q$

a. Find the ratio $p : q$

$\frac{7p}{2} + q = 2p + 5q \Rightarrow \frac{7}{2}p - 2p = 5q - q \Rightarrow \frac{3}{2}p = 4q \Rightarrow 3p = 8q$ (1)

$p : q = 3 : 8$ (1)

……………………………..

(2)

$10a^2 = 7ab + 12b^2$ where $a > 0$ and $b > 0$

b. Find the ratio $a : b$

$10a^2 - 7ab - 12b^2 = 0$

Substituting a value $a > 0$ into the equation:

$a = 2 \Rightarrow 40 - 14b - 12b^2 = 0$ (1)

Using the quadratic formula, $b = \frac{4}{3}$ *or* $b = -\frac{5}{2}$

As $b > 0$, we choose $b = \frac{4}{3}$. (1)

$a : b = 2 : \frac{4}{3} = 3 : 2$ (1)

…………………………………

(3)

(Total for Question 21 is 5 marks)

TOTAL FOR PAPER IS 80 MARKS