Level 1 / Level 2 GCSE (9 – 1)

MATHEMATICS

Paper 2 (Calculator)

Higher Tier

Time : 1 hour 30 minutes   **Paper : 1 MA1 / 2H**

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You must **show all your working.**
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- **Calculators may be used.**
- If your calculator does not have a π button, take the value of π to be 3.142 unless the question instructs otherwise.

Information

- The total mark for this paper is 80.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

Answer ALL questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1. a. Write 48 as a product of its prime factors.

for a complete method to find prime factors, could be shown on a factor tree with no more than one arithmetic error   or for 2,2,2,2,3   (1)

$2 \times 2 \times 2 \times 2 \times 3$ or equivalent   (1)

……………………………..
(2)

b. Find the highest common factor of 40 and 48.

$48 = 2 \times 2 \times 2 \times 2 \times 3$

$40 = 2 \times 2 \times 2 \times 5$ (1)
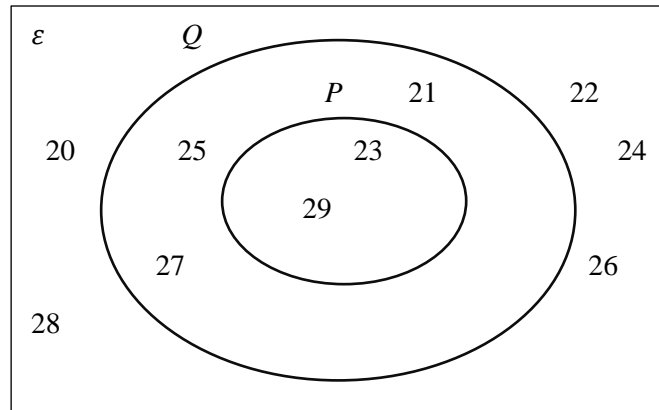
$HCF = 2 \times 2 \times 2 = 8$ (1)

…………………………….
(2)

(Total for Question 1 is 4 marks)

2. $\varepsilon = \{20, 21, 22, 23, 24, 25, 26, 27, 28, 29\}$
$P = \{\text{prime numbers}\}$
$Q = \{\text{odd numbers}\}$

a.   Complete the Venn diagram for this information.

for correct numbers in at least two regions   (1)

for all regions correct   (1)

(2)

A number is chosen at random from the universal set ε.
b.   Find the probability that this number is in the set $P' \cap Q$

$P' = \{20, 21, 22, 24, 25, 26, 27, 29\}$

$P' \cap Q = \{21, 25, 27\}$ (1)

$P(P' \cap Q) = \dfrac{3}{10}$ (1)

....

(2)

(Total for Question 2 is 4 marks)

3. Michelle puts chocolates into small chocolate gift boxes and large chocolate gift boxes.

She puts 12 chocolates into each small box.

She puts 30 chocolates into each large box.

Michelle puts a total of 4800 chocolates into boxes so that

  number of chocolates in small boxes : number of chocolates in large boxes = 2 : 3

Michelle says that more than 60% of the boxes filled with chocolates are small boxes.

Is Michelle correct?

You must show all your working.

$4800 \div (2 + 3) = 960$ (1)

Number of chocolates in small boxes: $960 \times 3 = 1920$

Number of chocolates in large boxes: $960 \times 3 = 2880$

Number of small boxes filled: $1920 \div 12 = 160$
Number of large boxes filled: $2880 \div 30 = 96$ (1)

Percentage of small boxes:
$\frac{160}{160+96} \times 100 = 62.5\%$ (1)

Michelle is correct as $62.5 > 60$. (1)

....

(Total for Question 3 is 5 marks)

4. a. Complete the table of values for $y = x^3 + 2$.

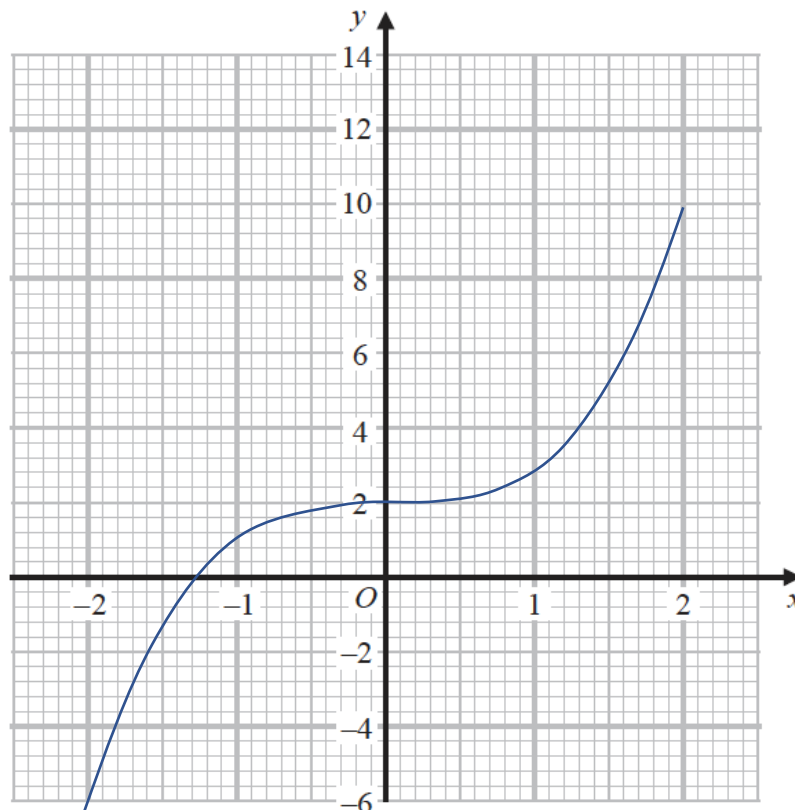| x	-2	-1	0	1	2
y	-6	1	2	3	10

for 2 or 3 correct values   (1)

for all 4 correct values   (1)

(2)

c. On the grid below, draw the graph of $y = x^3 + 2$ for values of x for –2 to 2

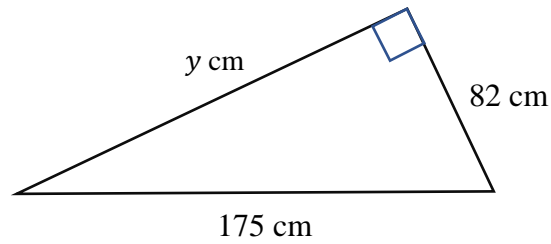for plotting at least 4 of the points from their table correctly   (1)
for a fully correct curve drawn   (1)

(2)

…………………………………………….

(Total for Question 4 is 4 marks)

5.

Work out the value of *y*.

Give your answer correct to 3 significant figures.

Using Pythagoras' theorem:

$y^2 = 175^2 - 82^2$ (1)

$y^2 = 23901$

$y = 154.5994$

$= 155$ to 3s.f. (1)

……………………………………………….

(Total for Question 5 is 2 marks)

6. $\mathbf{p} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ $\qquad \mathbf{q} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$

Find $3\mathbf{p} - 2\mathbf{q}$ as a column vector.

$3\begin{pmatrix} -1 \\ 2 \end{pmatrix} - 2\begin{pmatrix} 4 \\ -3 \end{pmatrix} = \begin{pmatrix} -11 \\ 12 \end{pmatrix}$
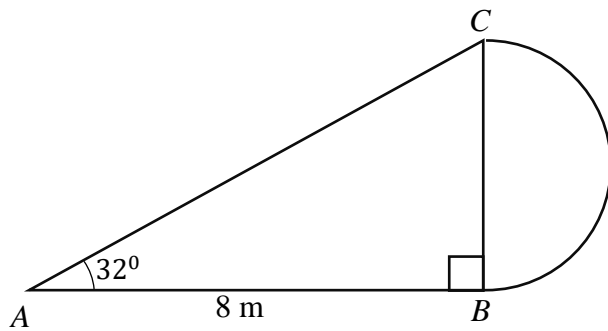
(1)   (1)

……………………………………………….

(Total for Question 6 is 2 marks)

7. The diagram shows a right-angled triangle and a semicircle.

The right-angled triangle ABC has angle $ABC = 90^0$.
$AB = 8$ m and angle $CAB = 32^0$
BC is the diameter of a semicircle.

Work out the area of the semicircle.
Give your answer correct to 3 significant figures.
You need to show all your working.

$\tan 32 = \dfrac{BC}{8}$ (1)

$BC = 4.99895 \ldots$ (1)

$\text{radius} = \dfrac{BC}{2} = \dfrac{4.99895}{2} = 2.499477$

$\text{Area} = \pi \times (\text{radius})^2 \div 2 = \pi \times (2.499477)^2 \div 2$ (1)

        $= 9.81$ (1)

.……………………………….

(Total for Question 7 is 4 marks)

8. In a sale, normal prices are reduced by 15%.

The sale price of the jacket is £68.
Work out the normal price of the jacket.

$x \times 0.85 = 68$ (1)
$x = 68 \div 0.85 = £80$ (1)

……………………………………………….

(Total for Question 8 is 2 marks)

9. Thomas invests $8000 in a saving account for 3 years.

The account pays compound interest at a rate of 1.2% per year for the first 2 years.

The compound interest rate changes for the third year.

At the end of 3 years, there is a total of $8316 in the account.

Work out the compound interest rate for the third year.

Give your answer correct to 1 decimal place.

Balance after 2 years:
$8000 \times (1.012)^2 = 8193.152$ (1)
Balance after 3 years:
$8193.152 \times x = 8316$
$x = \frac{8316}{8193.152} = 1.014994$ (1)
This means that the interest rate is 1.5% (1)

……………………………………………….

(Total for Question 9 is 3 marks)

10. Barbie is going to a party.

There are 8 night dresses, 6 pairs of shoes in her wardrobe and 5 pieces of jewellery in her jewellery box.

Barbie is going to choose one night dress, 1 pair of shoes and a piece of jewellery.
She puts each item on after she has picked it.
In how many ways can she put on a full outfit?

$8 \times 6 \times 5 = 240$, for process of using product rule for counting **(1)**
for correct answer only   240 **(1)**

....

(Total for Question 10 is 2 marks)

11. a. Find the value of x in $250^x = 1$

$x = 0$ **(1)**

....

(1)

b. Simplify $\frac{4(y+2)}{(y+2)^2}$

$\frac{4}{y+2}$ **(1)**

....

(1)

c. Simplify $(16p^4q^6)^{\frac{1}{2}}$

for two of $4, \ p^2, q^3$ in a product   **(1)**

correct answer only $4p^2q^3$ **(1)**
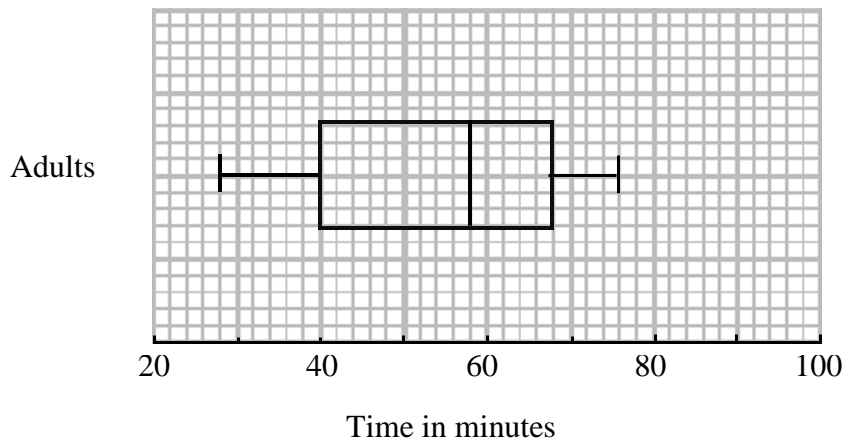
....

(2)

(Total for Question 11 is 4 marks)

12. James records the times, in seconds, taken for a group of children and a group of adults to complete puzzle.

The box plot shows information about the times taken for the adults to complete the puzzle.

Adults

20   40   60   80   100

Time in minutes

Find the interquartile range of the times taken for the adults to complete the puzzle.
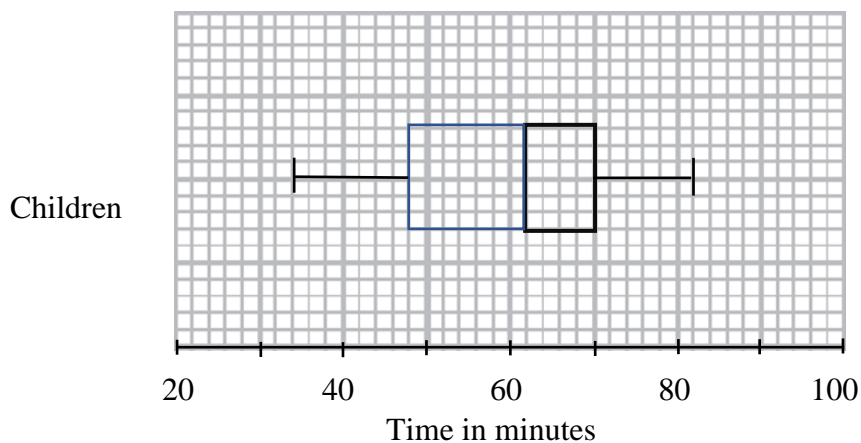
$$68 - 40 = 28$$

(1)   (1)

(2)

The table and part of the box plot shows some information about the times, in seconds, taken for the children to complete the puzzle.

Interquartile range	22
Upper quartile	70
Median	62
Greatest height	82
Range	48

Children

20   40   60   80   100

Time in minutes

Use the above information to complete the box plot.

for showing lower quartile or lowest height correctly on box plot   (1)
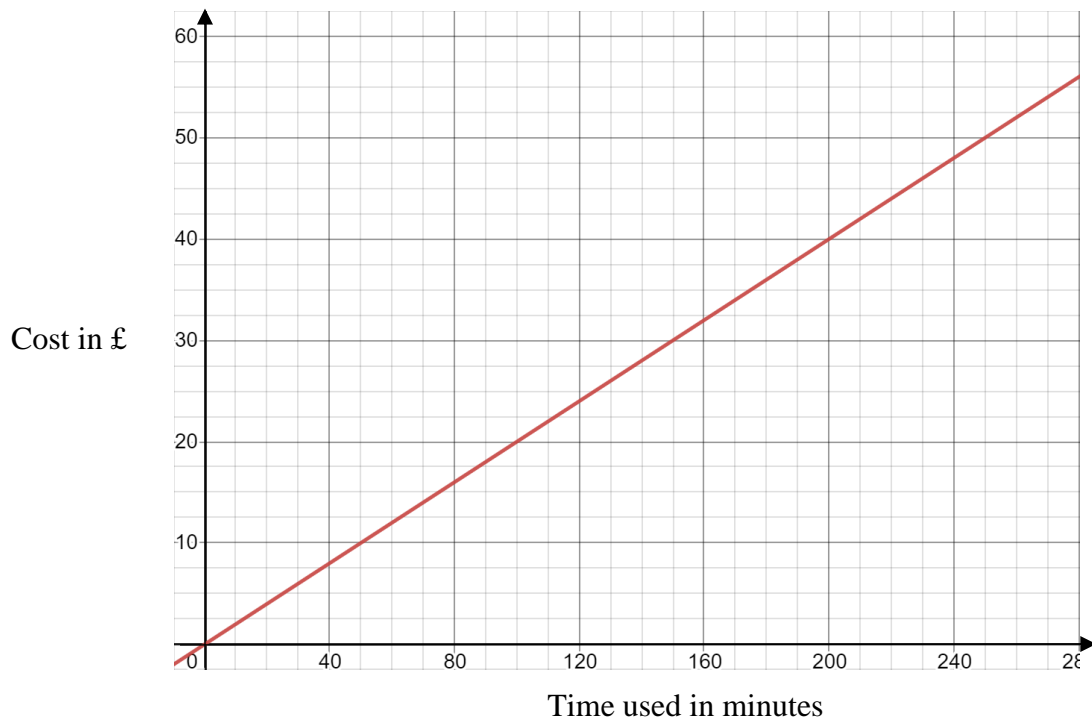
for showing lower quartile and lowest height correctly on box plot   (1)

(2)

(Total for Question 12 is 4 marks)

13. The graph shows the cost of using internet broadband for one month with tariff 1.

Cost in £

Time used in minutes

a. Find the gradient of the graph.

for suitable method to find gradient   e.g. 40 ÷ 200   (1)
for answer 0.20 or $\frac{1}{5}$ or equivalent   (1)

…………………………..
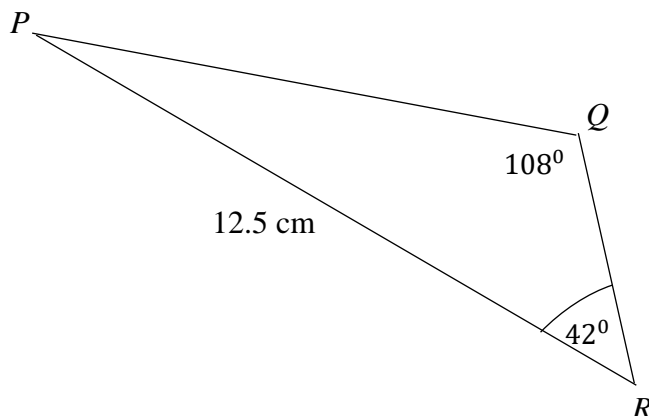**(2)**

b. Interpret what the gradient of the graph represents.

every minute the cost of using the internet broadband is 20 pence or £0.20 (or equivalent) (1)

…………………………..
(1)

(Total for Question 13 is 3 marks)

14.

Work out the perimeter of triangle PQR.

Give your answer to 3 significant figures.

The sine rule states:

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{12.5}{\sin 108} = \frac{PQ}{\sin 42} \Rightarrow PQ = 8.795$

Angles in a triangle total 180°, so $RPQ = 180 - (108 + 42) = 30$

Using the sine rule:

$\frac{QR}{\sin 30} = \frac{12.5}{\sin 108} \Rightarrow QR = 6.5716\ldots$
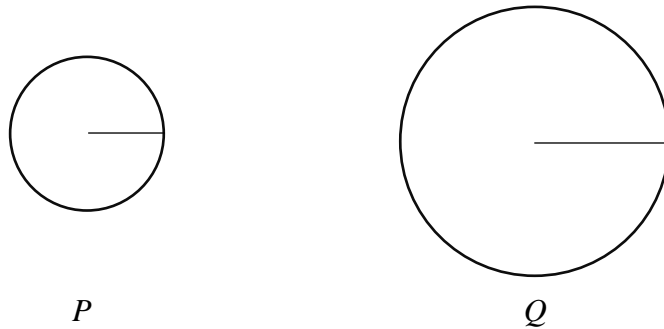
Perimeter of the triangle:

$12.5 + 8.795 + 6.5716 = 27.8666 = 27.9$ to 3 s.f.

There are lots of different methods to solve this problem using combinations of the sine and cosine rule.

....

(Total for Question 14 is 5 marks)

15. Here are two circles, P and Q.

P Q

The radius of circle P is 5 cm smaller than the radius of circle Q.
The area of circle Q is 50π cm^2 greater than the area of circle P.

Find the area of circle Q.
Give your answer correct to 3 significant figures.
You must show all your working.

We will denote the radii of the circles r_P and r_Q respectively. Similarly, we will denote the areas by A_P and A_Q.

$r_p + 5 = r_Q$

$A_P + 50\pi = A_Q$

$\Rightarrow \pi r_P^2 + 50\pi = \pi(r_P + 5)^2 \Rightarrow \pi r_P^2 + 50\pi = \pi\left(r_P^2 + 10r_p + 25\right)$ (1)

$\Rightarrow \pi r_P^2 + 50\pi = \pi r_p^2 + 10\pi r_P + 25\pi \Rightarrow 10r_P = 25 \Rightarrow r_P = 2.5$ (1)

$r_p = 2.5 \Rightarrow r_Q = 7.5$ (1)

$A_Q = \pi(7.5^2) = 176.715 = 176$ to 3.s.f. (1)

Setting up an equation   (1)

Process to reduce equation down to a linear equation ready to solve (1)

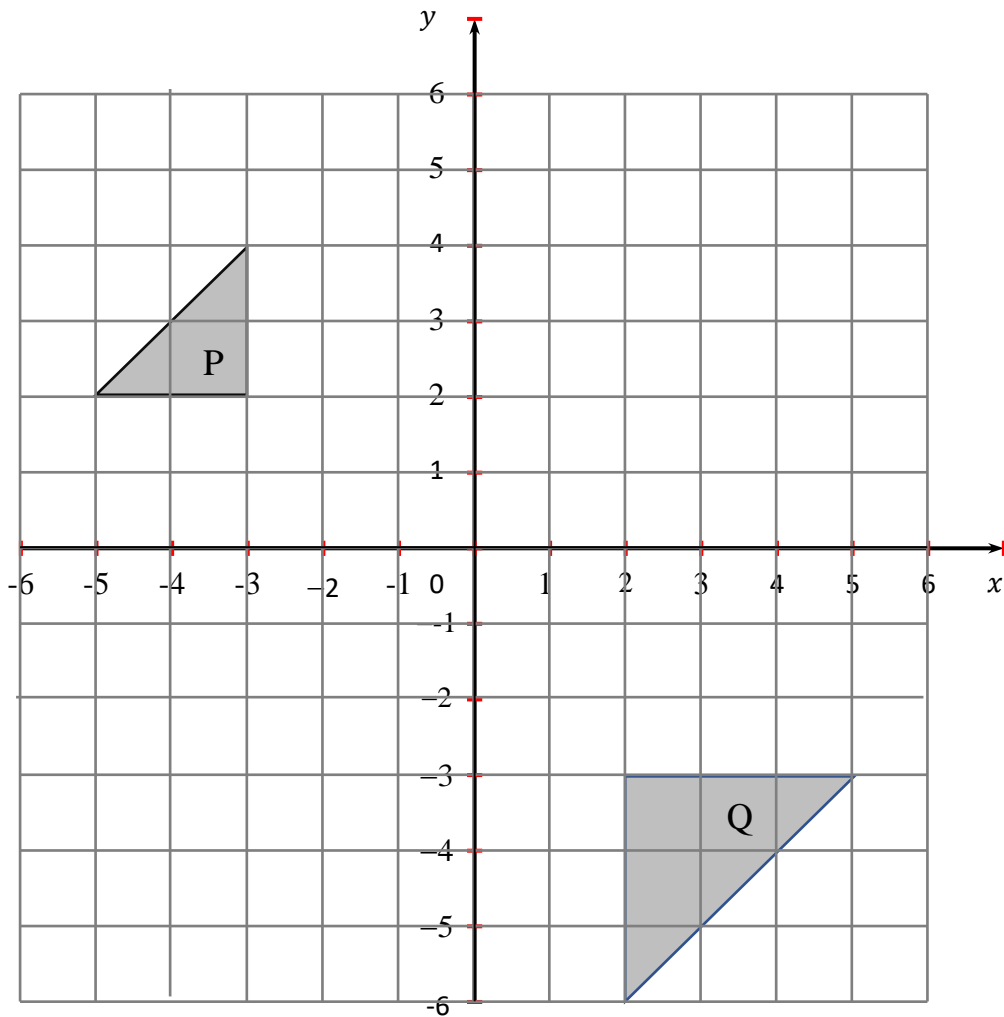for $r_P = 2.5$ or $r_Q = 7.5$ (1)

for finding the area of Q to 3.s.f. (1)

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(Total for Question 15 is 4 marks)

16.

Describe fully the single transformation that maps triangle **P** onto triangle **Q**.

for enlargement scale factor -1.5 OR enlargement centre $(-1,0)$ (1)
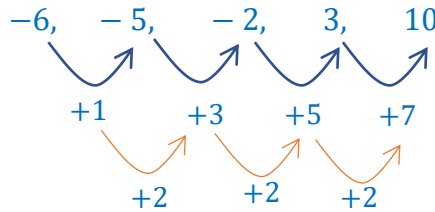for enlargement scale factor -1.5 AND centre $(-1,0)$ (1)

(Total for Question 16 is 2 marks)

17. A quadratic sequence starts

$$-6, -5, -2, 3, 10$$

a. Show that the nth term is $n^2 - 2n - 5$

$$-6, \quad -5, \quad -2, \quad 3, \quad 10$$

$$+1 \qquad +3 \qquad +5 \qquad +7$$

$$+2 \qquad +2 \qquad +2$$

A non-zero constant second difference of the sequence implies that there is an n^2 term:

$$n^2: 1, \quad 4, \quad 9, \quad 16, \quad 25$$

$$-6, \quad -5, \quad -2, \quad 3, \quad 10$$

Difference: $-7, \quad -9, \quad -11, \quad -13, \quad -15$

$$-2 \qquad -2 \qquad -2 \qquad -2$$

$$-2n: -2, \quad -4, \quad -6, \quad -8, \quad -10$$

$$-2n - 5: -7, -9, -11, -13, -15$$

n^{th} term of sequence: $n^2 - 2n - 5$

equal 2nd difference imply a term in n^2 (1)

working with n^2 (1)

for $n^2 - 2n - 5$ (1)

(3)

b. Hence find the term that has value 115.

$n^2 - 2n - 5 = 115 \Rightarrow n^2 - 2n - 120 = 0$ (1)

$(n - 12)(n + 10) = 0 \Rightarrow n = 12$ (1)

Obtaining a quadratic equation (1)
Factorising and solving a quadratic equation (1)

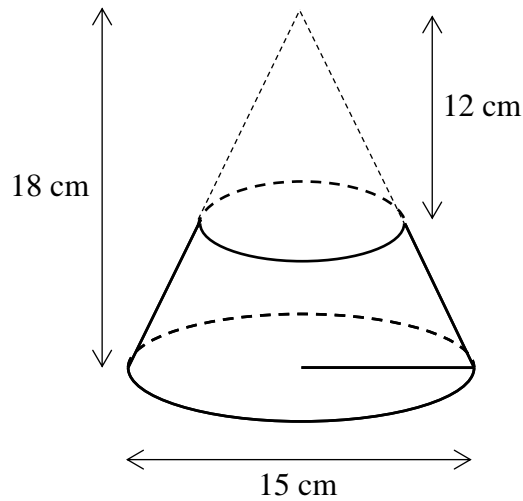...

(2)

(Total for Question 17 is 5 marks)

18. Write down the coordinates of the turning point on the graph $y = 8 - 2(x - 3)^2$

correct answer only (3,8) (1)

(................. , ...)
(Total for Question 18 is 1 mark)

19.

A frustum is made by removing a small cone from a similar large cone.

The height of the small cone is 12 cm.

The height of the large cone is 18 cm.

The diameter of the base of the large cone is 15 cm.

Work out the volume of the frustrum.

Give your answer correct to 3 significant figures.

Volume of big cone:
$\frac{1}{3}\pi \times (7.5)^2 \times 18 = \frac{675}{2}\pi = 1060.2875 ...$

The small cone similar to the big cone- shrunk with scale factor $\frac{12}{18} = \frac{2}{3}$

Radius of small cone $= \frac{2}{3} \times 7.5 = 5$.

Volume of small cone: $\frac{1}{3}\pi \times (5)^2 \times 12 = 100\pi$

Volume of frustrum= Volume of big cone− Volume of small cone
$$\frac{675}{2}\pi - 100\pi = \frac{475}{2}\pi$$

Process to find volume of big cone (1)
Process to find radius or diameter of the smaller cone (1)
Complete process (1)
Correct answer (1)

.………………………………………….cm^3

(Total for Question 19 is 4 marks)

20. In a bag there are n counters.

Four of these are blue and the rest of the counters are red.

Betty is going to remove two counters at random from the bag. She will not put them back.

Given that the probability that both counters she takes will be red is $\frac{1}{3}$.

Work out the number of red counters in the bag.

$\frac{n-4}{n} \times \frac{n-5}{n-1} = \frac{1}{3} \Rightarrow 3(n-4)(n-5) = n(n-1) \Rightarrow 2 - 26n + 60 = 0$

$2n^2 - 26n + 60 = 2(n-3)(n-10) \Rightarrow n = 3$ or $n = 10$

n cannot be 3 as we know that 4 counters are blue, so $n = 10$.

No. of red counters $10 - 4 = 6$

$\frac{n-4}{n}$ and $\frac{n-5}{n-1}$ or $\frac{n}{n+4}$ and $\frac{n-1}{n+3}$ (1)

Appropriate equation (1)

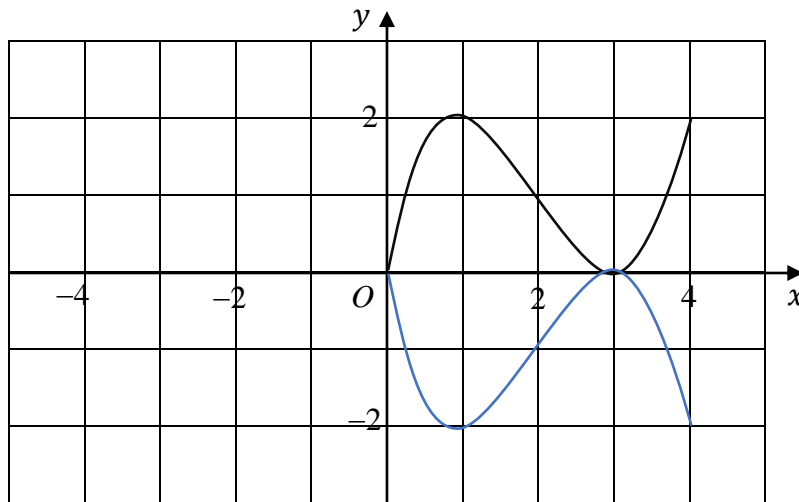Correctly finding a quadratic ready for solving (1)

process to solve quadratic equation with a 3 term quadratic factorizing, or using the quadratic formula, or completing the square (1)

correct answer only 6 (1)

....

(Total for Question 20 is 5 marks)

21. The graph of the curve with equation $y = f(x)$ is shown on the grid below.

a. On the grid above, sketch the graph of the curve with equation $y = -f(x)$.
   for a graph reflected in the y-axis or for a correct graph through four of the five
   key points   (1)

   for graph reflected in the x-axis   (1)

                                                                          (2)

b. The graph of $y = \cos x$ is translated by the vector $\begin{pmatrix} 60 \\ 0 \end{pmatrix}$.

   Write down the equation of the new graph.

   for $y = f(x - 60)$ (1)
   for $y = \cos(x - 60)$ (1)

   ……………………………….
                                                                          (2)

                                      (Total for Question 21 is 4 marks)

22. C is a circle with centre the origin.

A tangent to C crosses the x-axis at $\left(4\sqrt{3}, 0\right)$ and y-axis at $(0, -4)$.

Work out an equation of C

You must show all your working.

A circle of centre $(0,0)$ has equation $x^2 + y^2 = r^2$, where r is the radius.

Gradient of tangent: $\frac{-4-0}{0-4\sqrt{3}} = \frac{1}{\sqrt{3}}$

Equation of tangent: $y_1 = \frac{1}{\sqrt{3}}x - 4$

Gradient of normal to tangent (this will be the radius of the circle): $-\sqrt{3}$

Equation of radius: $y_2 = -\sqrt{3}x$

The radius and tangent intersect at $-\sqrt{3}x = \frac{1}{\sqrt{3}}x - 4 \Rightarrow 4 = \frac{4}{\sqrt{3}}x \Rightarrow x = \sqrt{3}$

$$x = \sqrt{3} \Rightarrow y = -3$$

Length of radius: $\sqrt{(\sqrt{3})^2 + (-3)^2} = 2\sqrt{3}$

Equation of circle: $x^2 + y^2 = 12$

for process to find gradient of tangent

OR start a method to find angle between tangent and x-axis   e.g. $\tan\alpha = \frac{4}{4\sqrt{3}}$

OR for $(4\sqrt{3})^2 + (-4)^2 = 64$ (1)

for process to find gradient of normal radius   e.g. $\frac{-1}{\frac{1}{\sqrt{3}}} = -\sqrt{3}$

OR complete process to find angle between tangent and x-axis.

e.g. $\alpha = \tan^{-1}\left(\frac{4}{4\sqrt{3}}\right) = 30$

OR for $\sqrt{(4\sqrt{3})^2 + (-4)^2} = 8$ (1)

for equation of tangent   e.g. $y = \frac{1}{\sqrt{3}}x - 4$

OR for equation of radius   e.g. $y = -\sqrt{3}x$

OR for using similar triangles   e.g. $\frac{r}{4} = \frac{4\sqrt{3}}{8}$

OR for $\sin 30 = \frac{r}{4\sqrt{3}}$ (1)

for process to find the x-coordinate   e.g. $\frac{1}{\sqrt{3}}x - 4 = -\sqrt{3}x \Rightarrow x = \frac{3}{\sqrt{3}}$

OR for $r = \frac{4\sqrt{3}}{8} \times 4$

OR $r = 4\sqrt{3} \times \sin 30$ (1)

Correct answer   (1)

....

(Total for Question 22 is 5 marks)

23. A scientist is studying a population of birds.
The number of birds, P_t, in the population, t years after the start of the study is modelled by the equation

$$P_{t+1} = 1.02P_t$$

Given that there were 240 birds at the start of Year 1, work out the number of birds at the start of year 4.

$1.02 \times 240 = 244.8$ or 249.696 or 254.68992 or 259.7837 (1)

correct answer only   255   (1)

....

(Total for Question 23 is 2 marks)
TOTAL FOR PAPER IS 80 MARKS