Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel International GCSE

Centre Number | Candidate Number

Time 2 hours

Paper reference **4PM1/02**

Further Pure Mathematics

PAPER 2

Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.
- Good luck with your examination.

Turn over ▶

P66027RRA

1/1/1/1/1/1/1

P 6 6 0 2 7 R R A 0 1 3 6

Pearson

International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere = $4\pi r^2$

Curved surface area of cone = $\pi r \times$ slant height

Volume of sphere = $\frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric series

Sum to n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

Calculus

Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

Trigonometry

Cosine rule

In triangle ABC: $a^2 = b^2 + c^2 - 2bc\cos A$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$ $\qquad$ $\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$ $\qquad$ $\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$ $\qquad$ $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$

Logarithms

$\log_a x = \frac{\log_b x}{\log_b a}$

BLANK PAGE

DO NOT WRITE ON THIS PAGE

Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 (a) Use the formula for $\cos(A + B)$ to show that $\cos 2A = 1 - 2\sin^2 A$

(2)

Given that $\sin A = \frac{x + 1}{2}$ and $\cos 2A = \frac{4 - y}{3}$

(b) show that $y = \frac{1}{2}(3x^2 + 6x + 5)$

(3)

Question 1 continued

(Total for Question 1 is 5 marks)

2 The finite region enclosed by the curve with equation $y = 4 - x^2$ and the line with equation $y = x + 2$ is rotated through 360° about the x-axis.

Use algebraic integration to find the exact volume of the solid formed.

(6)

Question 2 continued

(Total for Question 2 is 6 marks)

3

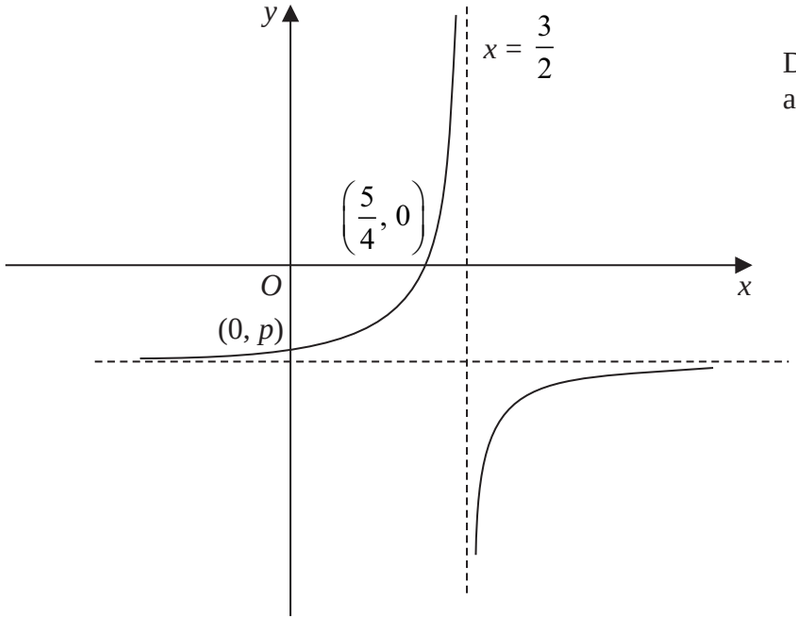


Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows a sketch of the curve with equation

$$y = \frac{a - bx}{cx - d} \qquad x \neq \frac{d}{c}$$

where a, c and d are prime numbers and b is an integer.

The asymptote to the curve that is parallel to the y-axis has equation $x = \frac{3}{2}$

(a) Write down the value of c and the value of d

(2)

The curve crosses the x-axis at the point $\left(\frac{5}{4}, 0\right)$

(b) Find the value of a and the value of b

(2)

The curve crosses the y-axis at the point $(0, p)$ where p is a rational number.

(c) Find the value of p

(2)

(d) Find an equation of the asymptote to the curve that is parallel to the x-axis.

(1)

.....

.....

.....

Question 3 continued

Question 3 continued

Question 3 continued

(Total for Question 3 is 7 marks)

4 The curve C has equation $y = 2x^2 + px + q$ where p and q are integers.

The curve C has a stationary point at $(3, -5)$

(a) Show that $p = -12$ and find the value of q

(4)

(b) State, giving a reason, the nature of the stationary point.

(1)

(c) Find an equation of the normal to C at the point on C where $x = 1$

Give your answer in the form $ax + by + c = 0$

(6)

Question 4 continued

(Total for Question 4 is 11 marks)

5 y and x vary so that $y = xe^{-2x}$

Given that the value of x increases by 3%, use calculus to find, in terms of x, an estimate for the percentage change in y
Give your answer in the form $a(b - cx)$ where a, b and c are integers.

(6)

Question 5 continued

(Total for Question 5 is 6 marks)

6 A particle P is moving along the x-axis. At time t seconds ($t \geqslant 0$) the displacement, s metres, of P from the origin O, is given by $s = t^3 - 4t^2 - 16t - 8$

(a) Find the distance of P from O when $t = 0$

(1)

(b) Find the value of t for which P is instantaneously at rest.

(4)

(c) Find the value of t for which P is accelerating at $10\,\text{m/s}^2$ in the positive x direction.

(3)

Question 6 continued

(Total for Question 6 is 8 marks)

7 A geometric series G has first term a and common ratio r

The sum of the first three terms of G is $\frac{61}{6}$

The sum to infinity of G is $\frac{125}{6}$

(a) (i) Show that $r = \frac{4}{5}$

(ii) Find the value of a

(6)

The sum of the first n terms of G is S_n

Given that $S_n > 19.8$

(b) show that $n\lg\left(\frac{4}{5}\right) < \lg\left(\frac{31}{625}\right)$

(2)

(c) Hence find the least value of n

(2)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 10 marks)

8 (a) Complete the table of values for $y = 2x + \frac{3}{x^2} - 3$ giving your answers to 2 decimal places where appropriate.

x	0.5	0.75	1	1.5	2	3	4	5
y	10		2				5.19	7.12

(2)

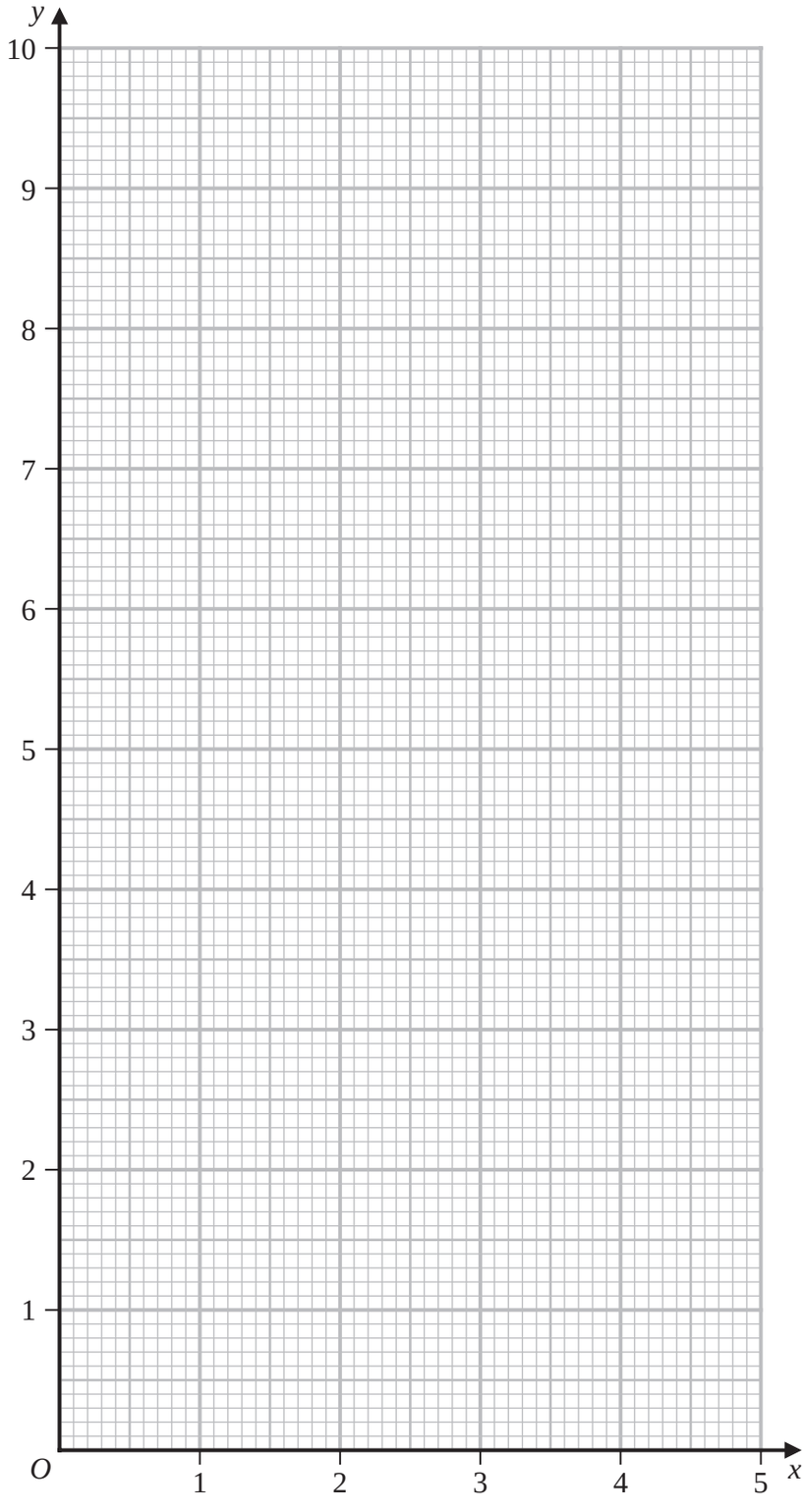
(b) On the grid opposite, draw the graph of $y = 2x + \frac{3}{x^2} - 3$ for $0.5 \leqslant x \leqslant 5$

(2)

(c) By drawing a suitable straight line on the grid, obtain estimates, to one decimal place, of the roots of the equation $4x^3 - 10x^2 + 3 = 0$ for $0.5 \leqslant x \leqslant 5$

(5)

Question 8 continued

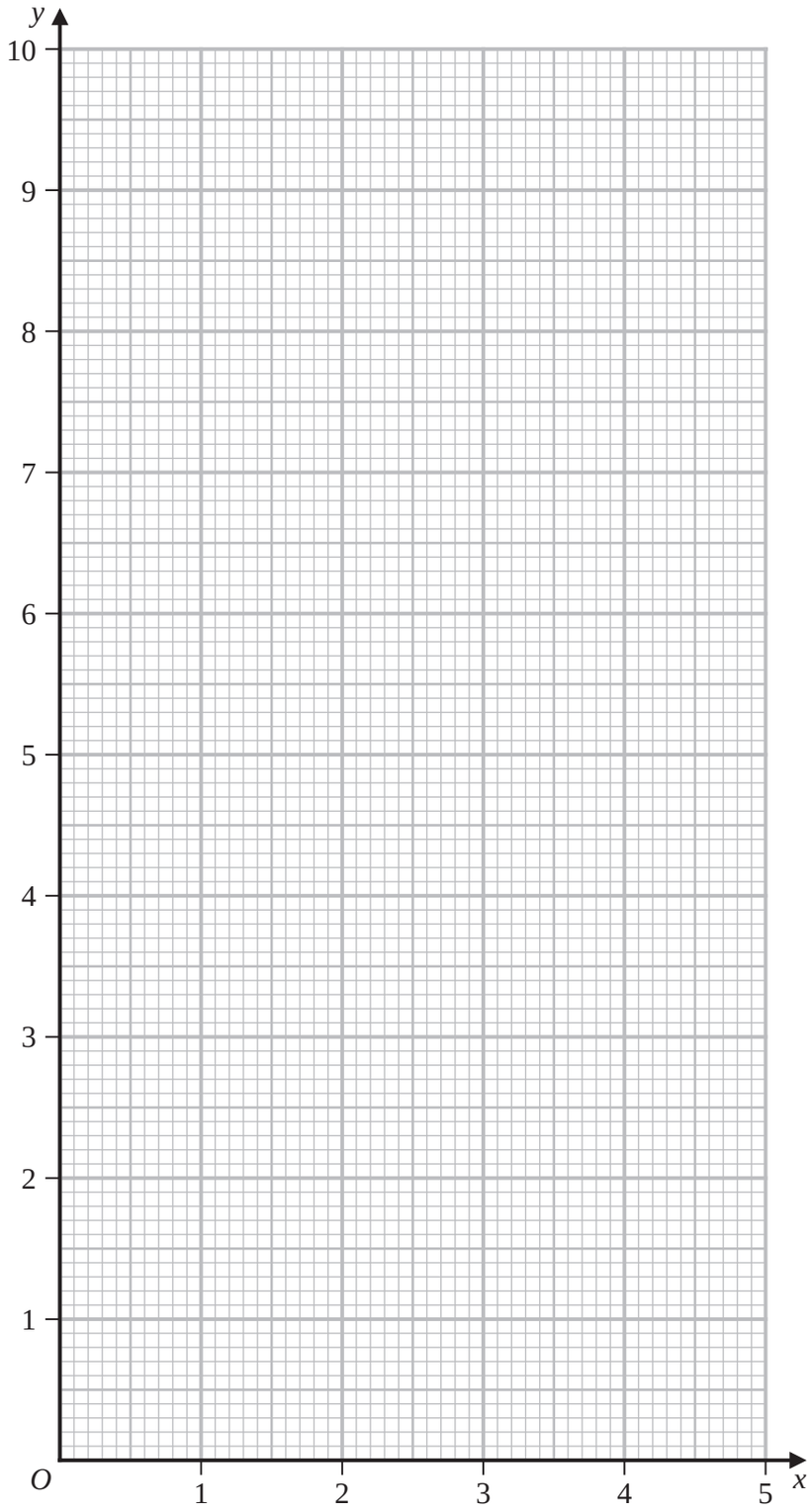


Turn over for a spare grid if you need to redraw your graph.

Question 8 continued

Question 8 continued

Only use this grid if you need to redraw your graph.



(Total for Question 8 is 9 marks)

9

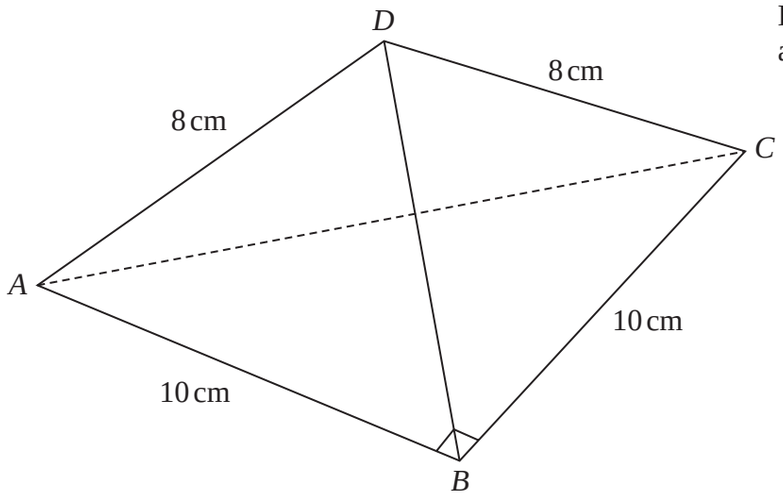


Diagram **NOT** accurately drawn

Figure 2

Figure 2 shows a triangular pyramid *ABCD* with base *ABC*

$AB = BC = 10$ cm $AD = CD = 8$ cm $\angle ABC = 90°$

(a) Find the exact length of *AC*

Give your answer in the form $p\sqrt{q}$ cm where *p* is an integer and *q* is a prime number.

(2)

The point *M* is the midpoint of *AC*

(b) Find the exact length of *BM*

Give your answer in the form $m\sqrt{n}$ cm where both *m* and *n* are prime numbers.

(2)

Given that *BD* = 6 cm,

(c) find, in degrees to one decimal place, the size of the acute angle between the plane *ACD* and the plane *ABC*

(4)

The base *ABC* of the pyramid is placed on a horizontal plane.

(d) Find, in cm to 3 significant figures, the vertical height of *D* above the base.

(2)

Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 10 marks)

10 (a) Show that $\dfrac{3}{\sqrt{9-3x}} = \left(1-\dfrac{x}{3}\right)^{-\frac{1}{2}}$

(2)

(b) Hence expand $\dfrac{3}{\sqrt{9-3x}}$ in ascending powers of x up to and including the term in x^3 expressing each coefficient as an exact fraction in its lowest terms.

(3)

$$f(x) = \frac{1+2x}{\sqrt{9-3x}}$$

(c) Find the expansion of $3f(x)$ in ascending powers of x up to and including the term in x^3 expressing each coefficient as an exact fraction in its lowest terms.

(4)

(d) Hence, using algebraic integration, obtain an approximation to 6 significant figures for

$$\int_{0.1}^{0.2} \frac{1+2x}{\sqrt{9-3x}}\,dx$$

(4)

Question 10 continued

Question 10 continued

Question 10 continued

(Total for Question 10 is 13 marks)

11 The points A and B have coordinates $(-3, -5)$ and $(7, 5)$ respectively.

(a) Find an equation for the line AB

(2)

The point C has coordinates $(p, 1)$ where $p < 0$

Given that AC and BC are perpendicular,

(b) prove that $p = -5$

(7)

The point D, where BCD is a straight line, is such that C divides BD in the ratio $4:3$

(c) Find the coordinates of D

(2)

(d) (i) Find the exact length of AC

(ii) Hence, or otherwise, find the area of triangle ABD

(4)

Question 11 continued

Question 11 continued

(Total for Question 11 is 15 marks)

TOTAL FOR PAPER IS 100 MARKS