OCR
Oxford Cambridge and RSA

H

GCSE (9–1) Mathematics

J560/05 Paper 5 (Higher Tier)

Monday 6 November 2017 – Morning

Time allowed: 1 hour 30 minutes

You may use:
- Geometrical instruments
- Tracing paper

Do not use:
- A calculator

Model Solutions

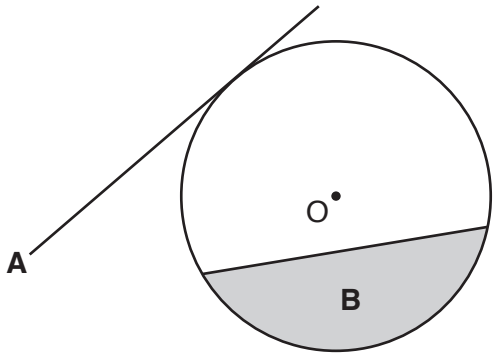
First name									
Last name									
Centre number						Candidate number				

INSTRUCTIONS
- Use black ink. You may use an HB pencil for graphs and diagrams.
- Complete the boxes above with your name, centre number and candidate number.
- Answer **all** the questions.
- Read each question carefully before you start to write your answer.
- Where appropriate, your answers should be supported with working. Marks may be given for a correct method even if the answer is incorrect.
- Write your answer to each question in the space provided.
- If additional space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- Do **not** write in the barcodes.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document consists of **16** pages.

Answer **all** the questions.

1 The diagram shows a circle, centre O.



Write down the mathematical name of

(a) line **A**,

(a) tangent [1]

(b) shaded region **B**.

(b) segment [1]

2 **(a)** Write the next term in each of these sequences.

(i) 1 1 2 3 5 8

fibonacci sequence

(a)(i) 13 [1]

(ii) 2 4 8 16 32 64

geometric progression

(ii) 128 [1]

(b) Write an expression for the *n*th term of the sequence below.

15 12 9 6

−3 −3 −3 → −3n

n^{th} term = −3n + k

0th term = 15 + 3 = 18

nth term = −3n + 18

= 18 − 3n

(b) 18 − 3n [2]

3 Andrew is thinking of a number.

- It is between 1 and 150.
- It is one more than a square number.
- It is three less than a cube number.
- It is not a prime number.

What is Andrew's number?
You must show all your reasoning.

Square numbers : 1 4 9 16 25 36 49 64 81 100 121 144
+1 : 2 5 10 17 26 37 50 65 82 101 122 145

Cube numbers : 1 8 27 64 125
−3 : −2 5 24 61 122

5 is a prime number → Andrew's number can't be 5

122 ..... **[4]**

4 **(a)** Factorise.

$x^2 - 43^2$

difference of two squares:

$(x+43)(x-43)$

(a) $(x+43)(x-43)$ **[1]**

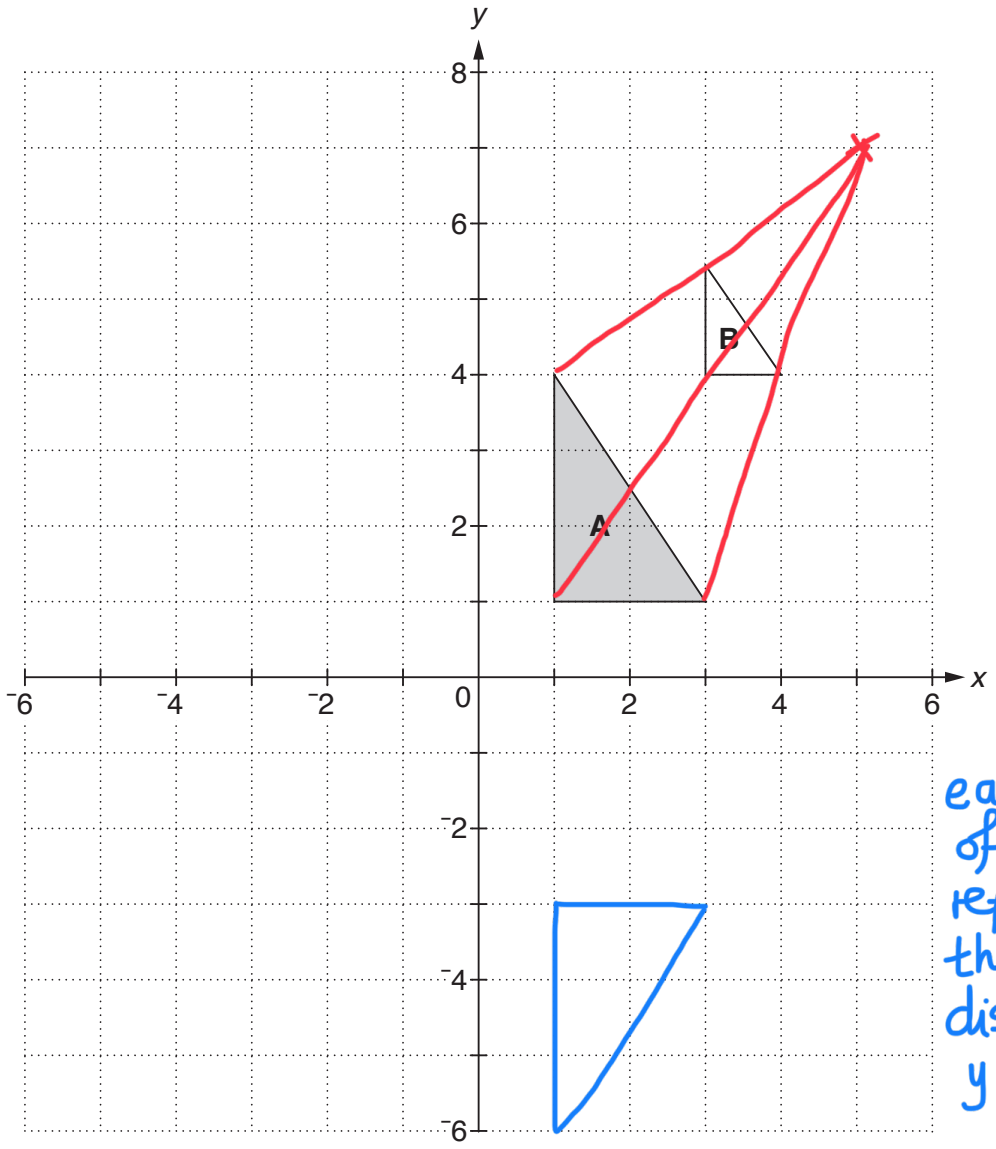
(b) Calculate.

$57^2 - 43^2$

$(57+43)(57-43)$
$= 100 \times 14$
$= 1400$

(b) 1400 **[2]**

5 Here is a coordinate grid.



each vertex of A and its reflection are the same distance from $y = 1$.

(a) Draw the image of triangle **A** after a reflection in the line $y = -1$. **[2]**

(b) Describe fully the **single** transformation that maps triangle **A** onto triangle **B**.

enlargement - scale factor $\frac{1}{2}$

centre (5, 7)

[3]

(c) Complete this statement.

A rotation of 180° around (0, 0) has the same effect as an enlargement by

scale factor ...−1... with centre of enlargement (...0..., ...0...). **[2]**

6 This rectangle has length $(4x - 5)$ cm and width $(x + 3)$ cm.

Not to scale

The perimeter of the rectangle is 46 cm.

Calculate the area of the rectangle.

Perimeter $= (x+3) + (4x-5) + (x+3) + (4x-5)$

$= 10x + 6 - 10$

$= 10x - 4$

$10x - 4 = 46$

$10x = 50$

$x = \frac{50}{10} = 5$

length: $4x - 5$

$= 4(5) - 5$

$= 15$ cm

width: $x + 3$

$= 5 + 3$

$= 8$ cm

area = length × width = $15 \times 8 = 120$ cm^2

120 cm^2 **[5]**

7 Naomi is given a 10% pay decrease.
Her new wage is £252 per week.

What would be her weekly wage if, instead, she had received a 10% pay increase?

10% pay decrease → 100 − 10 = 90% of original wage

let original wage = £N

0.9N = 252

0.1N = 28

N = 280

10% pay increase → 100 + 10 = 110% of original wage

110% of N = N + 0.1N = 1.1N

= 280 + 28

= 308

£ 308 **[5]**

8 The angles in a triangle are in the ratio 1 : 2 : 3.

(a) Show that the triangle is a right-angled triangle. **[2]**

1 : 2 : 3 → total number of parts of ratio = 1 + 2 + 3 = 6

6 parts = 180°

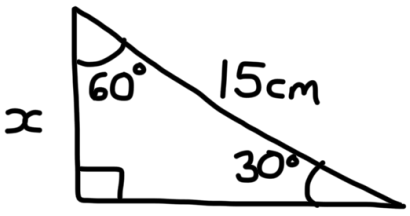
180 ÷ 6 = 30

1 : 2 : 3

30 : 60 : 90 → 90° angle so triangle is right-angled.

(b) The hypotenuse of the triangle is 15 cm long.

Calculate the length of the shortest side in the triangle.



$\sin 30 = \frac{x}{15}$

$15 \sin 30 = x$

$x = 7.5\text{cm}$

(b) 7.5 cm **[4]**

9 There is a total of 250 men, women and children on a train.
The ratio of men to women is 4 : 5.
The ratio of women to children is 10 : 7.

How many men are on the train?

men : women Women : children

4 : 5 10 : 7

×2 ↓ ×2

8 : 10

men : women : children

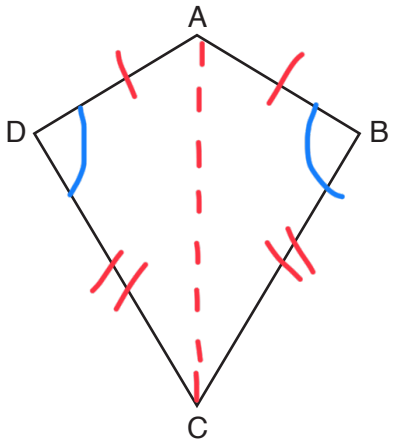
8 : 10 : 7

250 ÷ 25 parts = 10

8 × 10 = 80

80 ..... [4]

10 ABCD is a quadrilateral.
AD = AB and CD = CB.



Not to scale

Prove that angle ADC is equal to angle ABC.

AB = AD

CB = CD

AC is common to both triangles

3 pairs of equal sides → SSS

angle ADC = angle ABC

..... [4]

11 Amelia buys a new car.
The expected future value of this car, £V, is given by

$$V = 16000 \times 0.75^t$$

where t is the age of the car in complete years.

(a) **(i)** Write down the value of the car when new.

$a^0 = 1$

new → $t = 0$
value $V = 16000 \times 0.75^0$
$= 16000$

(i) £ 16000 **[1]**

(ii) Write down the annual percentage decrease in the expected value of the car.

$V = 16000 \times 0.75^t$

$0.75 = 75\%$

multiplier = $100\% - x\% = 75\%$ $x = 25\%$

(ii) 25 % **[1]**

(iii) Show that the expected value of the car when 2 years old is £9000. **[2]**

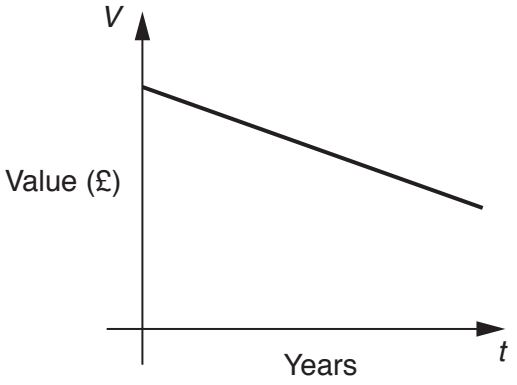
$V = 16000 \times 0.75^2$

$0.75 \rightarrow \frac{3}{4}$

$V = 16000 \times \frac{9}{16}$

$= £9000$

(b) Amelia sketches a graph to show the expected value of her car as it gets older.



Explain how you know that Amelia's graph is incorrect.

the equation does not give a straight line. The value of the car does not decrease by the same number of £5 each year. **[1]**

(c) Amelia assumes that her car will have no value at all after 20 years.

Explain why her assumption is mathematically incorrect.

If you calculate the value for a 20 year old car, the value is greater than 0. **[1]**

$v = 16000 \times 0.75^{20} = 50.74$

12 **(a)** Write $\frac{5}{6}$ as a recurring decimal.

$\frac{5}{6} = 5 \div 6$

$= 0.8333\ldots$

$= 0.8\dot{3}$

(a) $0.8\dot{3}$ **[2]**

(b) Convert $0.12\dot{6}$ to a fraction.
Give your answer in its lowest terms.

let $x = 0.12\dot{6}$

$1000x = 126.\dot{6}$

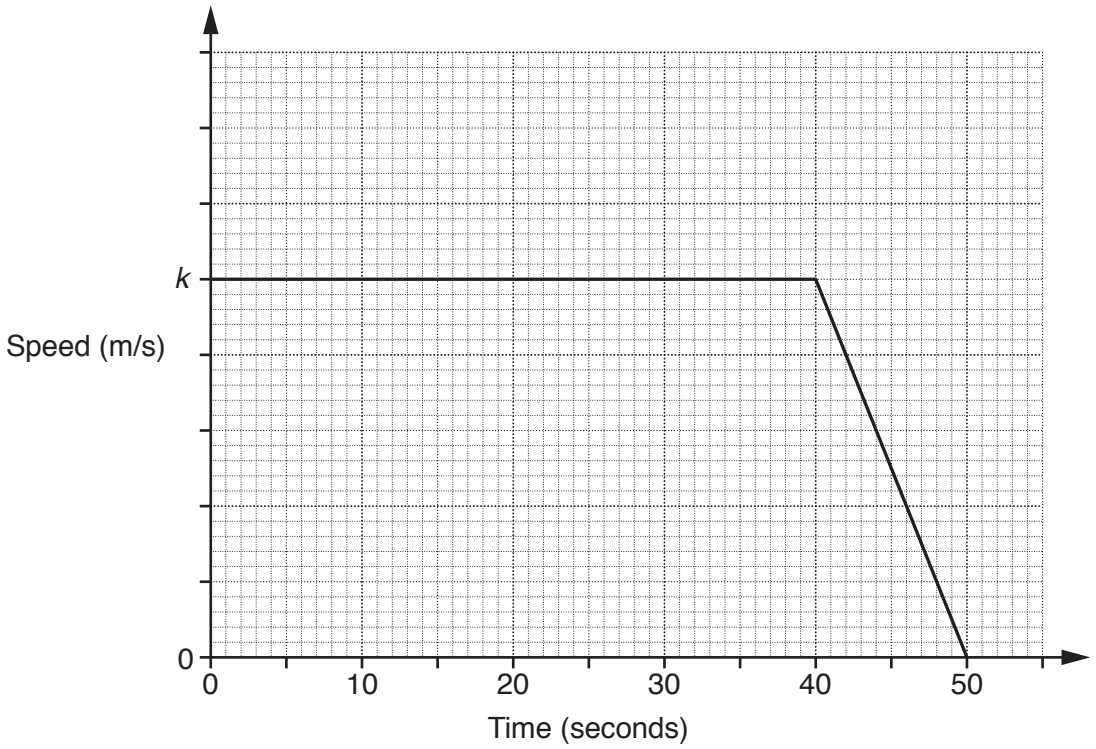
$-\ 100x = 12.\dot{6}$

$900x = 114$

$0.12\dot{6} = \frac{19}{150}$

(b) $\frac{19}{150}$ **[3]**

13 The graph shows information about the speed of a vehicle during the final 50 seconds of a journey.
At the start of the 50 seconds the speed is *k* metres per second.
The distance travelled during the 50 seconds is 1.35 kilometres.



(a) Work out the average speed of the vehicle during the 50 seconds.
Give your answer in metres per second.

average speed = $\frac{\text{total distance travelled}}{\text{total time taken}} = \frac{1350}{50}$

1.35 km = 1350 m

(a) ..... 27 ..... m/s **[2]**

(b) Work out the value of *k*.

distance travelled = area under speed-time graph

$a_1 = 40 \times k = 40k$

$a_2 = \frac{1}{2} \times 10 \times k = 5k$

total area = $40k + 5k = 45k$

$45k = 1350$

$k = 30$

(b) *k* = ... 30 ..... **[5]**

(c) **(i)** Calculate the gradient of the graph in the final 10 seconds of the journey.

gradient $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-30}{50-40} = \frac{-30}{10} = -3$

-3

(c)(i) ..... **[1]**

(ii) Describe what this gradient represents.

the deceleration of the vehicle is m/s^2

..... **[2]**

14 Adam has 10 sweets in a bag.
5 are cherry sweets, 4 are lemon sweets and 1 is an orange sweet.
C L O
Adam chooses a sweet at random from the bag and eats it.
He then takes another sweet at random from the bag and eats it.

(a) Adam says

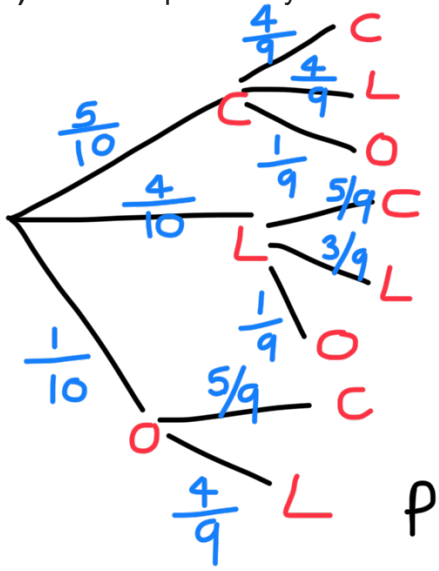
The probability that I choose two cherry sweets is $\frac{25}{100}$.

He is incorrect. Explain his error.

the probability of the second sweet being cherry is $\frac{4}{9}$ as Adam eats the first sweet. $P(2^{nd} \text{ cherry}) = \frac{5}{10} \times \frac{4}{9} = \frac{20}{90}$

..... **[2]**

(b) Find the probability that the two sweets he chooses have different flavours.



$P(C,C) = \frac{5}{10} \times \frac{4}{9} = \frac{20}{90}$

$P(L,L) = \frac{4}{10} \times \frac{3}{9} = \frac{12}{90}$

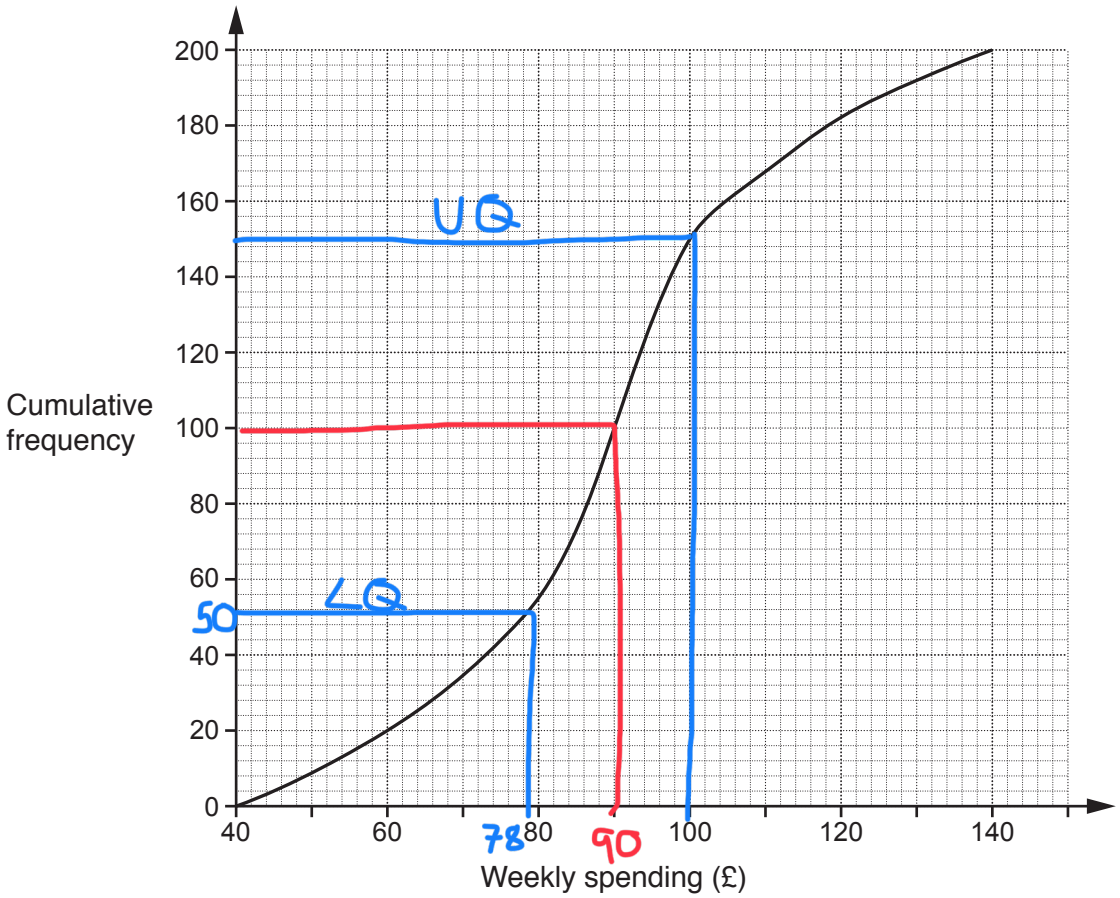
$P(\text{two sweets the same}) = \frac{20}{90} + \frac{12}{90} = \frac{32}{90}$

$P(\text{two different flavours}) = 1 - \frac{32}{90} = \frac{58}{90}$

$\frac{58}{90}$

(b) ..... **[4]**

15 Iqrah carries out a survey of 200 families in the **north** of England on their weekly spending on food.
The cumulative frequency diagram summarises the results.



(a) Find

(i) the median,

(a)(i) £ 90 **[1]**

(ii) the interquartile range.

£100 − £78
= £22

(ii) £ 22 **[2]**

(b) Iqrah says

15% of these families spent over £120.

Is her statement correct?
State the evidence you have used in making your decision.

182 families spent £120 or less
200 − 182 = 18 families spent over £120
$\frac{18}{200} \times 100\% = 9\%$ therefore, No.

[2]

(c) In a survey of 200 families in the **south** of England, the median weekly amount spent on food was £84 and the interquartile range was £28.

Make two comparisons between the weekly amounts spent on food in the north of England and the south of England.
State the evidence you have used in making your comparisons.

1 Families in the South spent less on average than families in the North as their median was lower. **[2]**

2 The spending of families in the South has a greater spread as their interquartile range was higher. **[2]**

16 **(a)** Write $\sqrt{12} + \sqrt{75}$ in the form $k\sqrt{3}$.

$\sqrt{12} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$
$\sqrt{75} = \sqrt{25} \times \sqrt{3} = 5\sqrt{3}$
$\sqrt{12} + \sqrt{75} = 7\sqrt{3}$

$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

(a) $7\sqrt{3}$ **[3]**

(b) Work out.

$16^{-\frac{3}{4}}$

$\left(16^{\frac{1}{4}}\right)^{-3}$
$= \left(\sqrt[4]{16}\right)^{-3}$
$= 2^{-3}$
$= \frac{1}{2^3} = \frac{1}{8}$

(b) $\frac{1}{8}$ **[3]**

17 Solve the inequality.

$$x^2 - 5x - 6 \leqslant 0$$

$x^2 - 5x - 6 = 0$

$(x-6)(x+1) = 0$

$-1 \leq x \leq 6$

$-1 \leq x \leq 6$ ..... **[4]**

18 Prove that the difference between two consecutive square numbers is always odd. **[4]**

two consecutive squares: n^2 and $(n+1)^2$

$(n+1)^2 = (n+1)(n+1) = n^2 + 2n + 1$

difference $= (n+1)^2 - n^2$

$= (n^2 + 2n + 1) - n^2$

$= 2n + 1$

19 Solve these simultaneous equations algebraically.

$y = 2x^2 - 7x + 4$
$y = 4x - 1$

① $y = 2x^2 - 7x + 4$

② $y = 4x - 1$

$2x^2 - 7x + 4 = 4x - 1$

$2x^2 - 11x + 5 = 0$

$(2x - 1)(x - 5) = 0$

$x = \frac{1}{2}$

$x = 5$

$y = 4x - 1$

$y = 1$

$y = 19$

$x =$ ½ $y =$ 1

$x =$ 5 $y =$ 19 **[6]**

END OF QUESTION PAPER

ADDITIONAL ANSWER SPACE

If additional space is required, you should use the following lined page(s). The question number(s) must be clearly shown in the margin(s).