Pearson Edexcel

Mark Scheme (Results)

Summer 2021

Pearson Edexcel International GCSE

In Further Pure Mathematics (4PM1)

Paper 01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the last candidate in exactly the same way as they mark the first.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme - not according to their perception of where the grade boundaries may lie.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification/indicative content will not be exhaustive.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, a senior examiner must be consulted before a mark is given.
- Crossed out work should be marked **unless** the candidate has replaced it with an alternative response.

- **Types of mark**
  - M marks: method marks
  - A marks: accuracy marks
  - B marks: unconditional accuracy marks (independent of M marks)

- **Abbreviations**
  - cao – correct answer only
  - ft – follow through
  - isw – ignore subsequent working
  - SC - special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - awrt – answer which rounds to
  - eeoo – each error or omission

- **No working**
  If no working is shown then correct answers normally score full marks
  If no working is shown then incorrect (even though nearly correct) answers score no marks.

- **With working**
  If the final answer is wrong, always check the working in the body of the script (and on any diagrams), and award any marks appropriate from the mark scheme.
  If it is clear from the working that the “correct” answer has been obtained from incorrect working, award 0 marks.
  If a candidate misreads a number from the question. Eg. Uses 252 instead of 255; method marks may be awarded provided the question has not been simplified. Examiners should send any instance of a suspected misread to review.
  If there is a choice of methods shown, then award the lowest mark, unless the answer on the answer line makes clear the method that has been used.
  If there is no answer achieved then check the working for any marks appropriate from the mark scheme.

- **Ignoring subsequent work**
  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. Incorrect cancelling of a fraction that would otherwise be correct.
  It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect eg algebra.
  Transcription errors occur when candidates present a correct answer in working, and write it incorrectly on the answer line; mark the correct answer.

- **Parts of questions**
  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded to another.

General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:

$$(x^2+bx+c)=(x+p)(x+q), \text{ where } |pq|=|c| \quad \text{leading to } x=....$$

$$(ax^2+bx+c)=(mx+p)(nx+q) \text{ where } |pq|=|c| \text{ and } |mn|=|a| \quad \text{leading to } x=....$$

2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for *a*, *b* and *c*, leading to $x=....$

3. Completing the square:

$$x^2+bx+c=0: \ \left(x\pm\tfrac{b}{2}\right)^2 \pm q \pm c = 0, \quad q\neq 0 \qquad \text{leading to } x=....$$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $(x^n \to x^{n-1})$

2. Integration:

Power of at least one term increased by 1. $(x^n \to x^{n+1})$

Use of a formula:

Generally, the method mark is gained by **either** quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

or, where the formula is not quoted, the method mark can be gained by implication

from the substitution of correct values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers may be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show...."

Exact answers:

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)

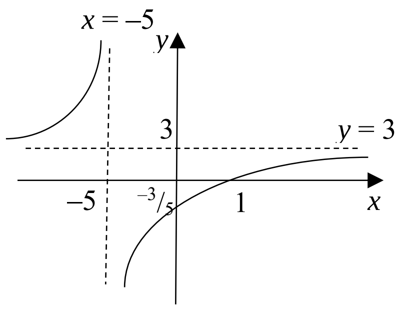
Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark

Paper 1		
Question number	Scheme	Marks
1	$b^2-4ac\geq 0$	
	$(k+5)^2-4k(3k+6)\geq 0$	M1
	$k^2+10k+25-12k^2-24k\geq 0$	M1
	$11k^2+14k-25\leq 0$	A1
	$(11k+25)(k-1)\leq 0$	M1
	[Critical values are $-\frac{25}{11}$ and 1]	
	$-\frac{25}{11}\leq k\leq 1$ oe	M1A1
		Total 6 marks

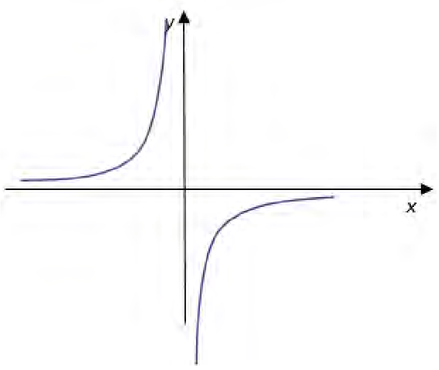
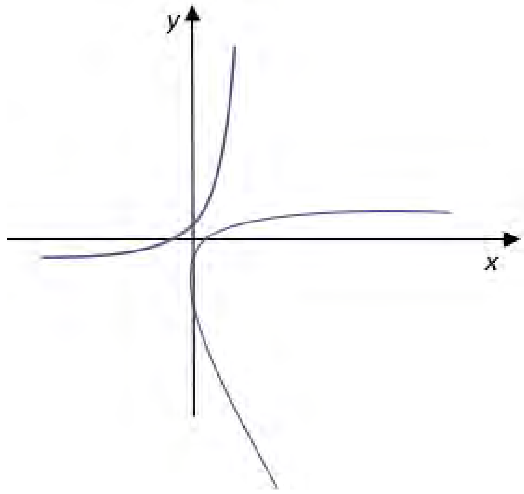
Mark	Notes
M1	Uses b^2-4ac on the given quadratic equation with correct *a*, *b* and *c*; $a=3(k+2)$ $b=k+5$ $c=k$ **and** a correct substitution to obtain $(k+5)^2-4\times 3\times(k+2)(k)$ **Note:** Accept for this mark any inequality, equals sign and even b^2-4ac used on its own.
M1	For attempting to expand the brackets and form a 3TQ in terms of *k*. Allow as a minimum at least one term correct. $k^2+10k+25-12k^2-24k\Rightarrow(-11k^2-14k+25)$ M0M1 is possible here.
A1	For the correct 3TQ with the correct inequality. **Note:** Allow > or < in place of $\geqslant$ and $\leqslant$ for this mark $-11k^2-14k+25\geqslant 0$ or $11k^2+14k-25\leqslant 0$
M1	For an attempt to solve their 3TQ, (provided it is a 3TQ) in terms of *k* by any acceptable method. See General Guidance for the definition of an attempt by factorisation, formula or completing the square. Use of calculators: if their 3TQ is incorrect, do not award this mark if working is not seen. $(11k+25)(k-1)=0\Rightarrow k=1,\ -\frac{25}{11}$
M1	For forming the correct inequality with **their** critical values, provided they have been obtained from a 3TQ, must be a closed region. $\left(-\frac{25}{11}\leqslant k\leqslant 1\right)$ ft their values from their $-11k^2+14k-25\geqslant 0$ or $11k^2-14k+25\leqslant 0$
A1	For the correct inequality. $-\frac{25}{11}\leqslant k\leqslant 1$

Question number	Scheme	Marks
2 (a)(i)	$(\tan\alpha =)\dfrac{4}{3}$	B1
(a)(ii)	$(\tan\beta =)-\dfrac{1}{\sqrt{3}}\quad\left(=-\dfrac{\sqrt{3}}{3}\right)$	B1 B1 (3)
(b)	$\tan(\alpha+\beta)=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$	
	$=\dfrac{\frac{4}{3}+\left(-\frac{1}{\sqrt{3}}\right)}{1-\left(\frac{4}{3}\right)\left(-\frac{1}{\sqrt{3}}\right)}$	M1
	$=\dfrac{\frac{4\sqrt{3}\pm 3}{3\sqrt{3}}}{\frac{3\sqrt{3}\pm 4}{3\sqrt{3}}}$ or $\dfrac{\frac{12+\left(-3\sqrt{3}\right)}{9}}{\frac{9+4\sqrt{3}}{9}}$	dM1
	$=\dfrac{4\sqrt{3}-3}{3\sqrt{3}+4}$	A1 cso (3)
		Total 6 marks

Part	Mark	Notes
(a)(i)	B1	For $(\tan\alpha =)\frac{4}{3}$ accept $1.\dot{3}$ or 1.3^r or 1.3 recurring
(a)(ii)	B1	For $(\tan\beta =)\pm\frac{1}{\sqrt{3}}$ (i.e. 1st B mark allow + or – sign with the exact value shown oe)
	B2	$(\tan\beta =)-\frac{1}{\sqrt{3}}$ or $-\frac{\sqrt{3}}{3}$ oe (i.e. 2 B marks correct sign with correct exact value)
	B marks are awarded independent of method, so award for the exact values stated oe	
(b)	M1	For using the correct formula for $\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$ **Note:** The formula is given on page 2 of this paper and there must be **correct** substitution for **their exact** values obtained in part (a) $\tan(\alpha+\beta)=\frac{\frac{4}{3}+\left(-\frac{1}{\sqrt{3}}\right)}{1-\left(\frac{4}{3}\right)\left(-\frac{1}{\sqrt{3}}\right)}$.
	dM1	For attempting to simplify their expression for $\tan(\alpha+\beta)$ as far as $\frac{a\pm b\sqrt{c}}{d\sqrt{c}\pm e}$ where *a*, *b*, *c*, *d* and *e* are integers. Ft the correct substitution of their values and check the common denominators are correct for their values in both the numerator and denominator of $\tan(\alpha+\beta)$ The numerator and denominator must be of the form $p\pm q$ where *q* contains a surd. If they use $\tan\beta=\pm\frac{1}{\sqrt{3}}$ they will get to: $\tan(\alpha+\beta)=\frac{\frac{4}{3}+\left(\pm\frac{1}{\sqrt{3}}\right)}{1-\left(\frac{4}{3}\right)\left(\pm\frac{1}{\sqrt{3}}\right)}=\frac{\frac{4\sqrt{3}\pm3}{3\sqrt{3}}}{\frac{3\sqrt{3}\mp4}{3\sqrt{3}}}=\frac{4\sqrt{3}\pm3}{3\sqrt{3}\mp4}$ If they use $\tan\beta=-\frac{\sqrt{3}}{3}$ they will get to: $\tan(\alpha+\beta)=\frac{\frac{4}{3}+\left(\pm\frac{\sqrt{3}}{3}\right)}{1-\left(\frac{4}{3}\right)\left(\pm\frac{\sqrt{3}}{3}\right)}=\frac{\frac{4+(\pm\sqrt{3})}{3}}{\frac{9\mp4\sqrt{3}}{9}}$ or $\frac{\frac{12+(\pm3\sqrt{3})}{9}}{\frac{9\mp4\sqrt{3}}{9}}=\frac{12+(\pm3\sqrt{3})}{9\mp4\sqrt{3}}$ **Note:** This mark is dependent on the previous M mark.
	A1	For simplifying to the correct final answer with no errors seen. $\tan(\alpha+\beta)=\frac{4\sqrt{3}-3}{3\sqrt{3}+4}$ o.e. for example $\frac{12\sqrt{3}-9}{9\sqrt{3}+12}$ Must be in the form $\frac{m\sqrt{3}-n}{n\sqrt{3}+m}$

Question number	Scheme	Marks
3 (a)	$\frac{dy}{dx} = \frac{a(x+5)-(ax-3)}{(x+5)^2}$	M1
	When $x = 2$ $\frac{dy}{dx} = \frac{7a-2a+3}{49} = \frac{18}{49} \Rightarrow a = ...$	M1
	$a = 3$ *	A1cso (3)
3 (b)(i)	$y = 3$	B1
(b)(ii)	$x = -5$	B1 (2)
(c)(i)	$(1,0)$	B1
(c)(ii)	$\left(0, \frac{-3}{5}\right)$	B1 (2)
(d)	 Curve drawn	B1
	Asymptotes drawn and labelled	B1ft
	$^{-3}/_5$, 1 labelled on axes	B1ft (3)
		[10]
		Total 10 marks

Part	Mark	Notes
(a)	**M1**	For an attempt at Quotient rule. The definition of an attempt is that there must be a correct attempt to differentiate both terms and the denominator must be squared. Allow the terms in the numerator to be the wrong way around, but the terms must be subtracted. $ax-3 \Rightarrow a$ and $x+5 \Rightarrow 1$ must be correct. $\frac{dy}{dx} = \frac{a(x+5)-(ax-3)}{(x+5)^2}$ $\left[\text{allow } \frac{dy}{dx} = \frac{(ax-3)-a(x+5)}{(x+5)^2}\right]$
	M1	For substituting $x = 2$ into their differentiated expression, setting it equal to $\frac{18}{49}$ and attempting to solve the linear equation leading to a value for a $\frac{dy}{dx} = \frac{7a-2a+3}{49} = \frac{18}{49} \Rightarrow 5a+3=18 \Rightarrow a = ...$ Allow one slip in their method.
	A1 cso	For $a = 3$ * No errors in working.

(b)(i)	**B1**	For $y = 3$ This must be an equation of a line. Do not award for just 3.
(ii)	**B1**	For $x = -5$ This must be the equation of a line. Do not award for just -5
(c)(i)	**B1**	For (1, 0) or clearly listing $x = 1$, $y = 0$ as a pair.
(ii)	**B1**	For $\left(0, -\frac{3}{5}\right)$ oe. or clearly listing $x = 0$, $y = -\frac{3}{5}$ as a pair.
(d)	**B1**	For the curve drawn with two branches anywhere on the grid provided it is a negative reciprocal curve. The ends of the curves must be asymptotic and must not turn back on themselves. Do not allow any obvious overlap across the ends of the curve with evidence of the presence of asymptotes. For example, accept: Negative reciprocal curve in incorrect position  Do not accept: • Overlap of asymptotes • Ends turning back on themselves 
	B1ft	For their asymptotes correctly drawn and clearly labelled with their equation. At least one branch of the curve is required. It must be a negative reciprocal and it must be in the correct position for their asymptotes. The follow through is available for their answers in part b. **If correct asymptotes appear on the sketch, do not award marks retrospective marks in part b.**
	B1ft	The curve must be drawn going through their two points of intersection. It must be a negative reciprocal, in the correct position for their intersections and clearly marked on the axes. The follow through is available for their answers in part c. Allow the other branch to even be missing. **If correct coordinates appear on the sketch, do not award marks retrospective marks in part c.**

Question number	Scheme	Marks
4	$u_1 = (1+1)\ln 4 = 2\ln 4$ and $d = \ln 4$	M1
	$S_n = \frac{n}{2}\left(2\times'2\ln 4'+(n-1)'\ln 4'\right)$ or $S_n = \frac{n}{2}\left('2\ln 4'+(n+1)'\ln 4'\right)$	M1
	ln 4 to either 2ln 2 or ln 2^2 at any stage.	M1
	$S_n = \frac{n}{2}(2n+6)\ln 2$ or $S_n = \frac{n}{2}(n+3)\ln 4$ or $S_n = \frac{n}{2}\left(\ln 2^6 + \ln 2^{2n}\right)$ or $S_n = \frac{n}{2}\left(\ln 4^3 + \ln 4^n\right)$	M1
	$S_n = \ln 2^{n^2+3n}$	A1 cso
		Total 5 marks

Note: You may see the use of $\ln 4\sum_1^n (r+1)$

<u>Solution</u>

$$S_n = \ln 4\sum_1^n (r+1)$$
$$= \ln 4\left(\sum_1^n r + \sum_1^n 1\right)$$
$$= \ln 4\left(\frac{n}{2}(n+1)+n\right)$$
$$= 2\ln 2\left(\frac{n^2}{2}+\frac{3n}{2}\right)$$
$$= \left(n^2+3n\right)\ln 2$$
$$S_n = \ln 2^{n^2+3n}$$

If this is a full and correct solution (no errors) as shown – award full marks – otherwise, please send to Review.

<table>
<tr><th>Mark</th><th>Notes</th></tr>
<tr><td>M1</td><td>Finds the first term and the common difference.
$u_1 = (1+1)\ln 4 = 2\ln 4$ and $d = \ln 4$
Both must be correct for this mark.</td></tr>
<tr><td colspan="2">The general principle of marking this question is as follows:
Note: Their a and d must be in terms of ln 4 or 2ln 2
• Second M1 for a correct substitution of their a and their d into either $\frac{n}{2}(2a+(n-1)d)$ or $\frac{n}{2}(a+L)$
• Third M1 is for dealing correctly with all terms in ln 4 at any stage $\left(\ln 4 = 2\ln 2 \text{ or } \ln 2^2\right)$ seen anywhere in the solution.
• Fourth M1 for attempting to simplify the sum to the required form using their a and d
• Final mark is for obtaining the given answer with no errors seen.</td></tr>
<tr><td>M1</td><td>Uses either form of the summation formula for an arithmetic series with their a and d provided both are in terms of ln 4 or 2ln 2
There must be no errors in the use of and substitution of their values into the formula for this question – it is given on page 2.
$S_n = \frac{n}{2}\left(2\times'2\ln 4'+(n-1)'\ln 4'\right)$ or $S_n = \frac{n}{2}\left('2\ln 4'+(n+1)'\ln 4'\right)$</td></tr>
<tr><td>M1</td><td>For correctly changing all terms in ln 4 to either 2ln 2 or $\ln 2^2$ at any stage.

You may see this step at the end of the solution.</td></tr>
<tr><td>M1</td><td>Simplifies their expression in either ln 2 or ln 4 to obtain one of the following.
$S_n = \frac{n}{2}(2n+6)\ln 2$
or
$S_n = \frac{n}{2}(n+3)\ln 4$
or
$S_n = \frac{n}{2}\left(\ln 2^6 + \ln 2^{2n}\right)$
or
$S_n = \frac{n}{2}\left(\ln 4^3 + \ln 4^n\right)$</td></tr>
<tr><td>A1cso*</td><td>For obtaining the given answer in full with no errors.
$S_n = \ln 2^{n^2+3n}$</td></tr>
</table>

Question number	Scheme	Marks
5 (a)	$1+anx+\frac{n(n-1)}{2}a^2x^2+\frac{n(n-1)(n-2)}{3!}a^3x^3$	M1 A1 (2)
(b)	$an=15$	B1
	$\frac{n(n-1)}{2}a^2=\frac{n(n-1)(n-2)}{6}a^3 \Rightarrow (3=(n-2)a)$	M1A1
	Solving simultaneously leading to $a=\ldots$ or $n=\ldots$	M1
	$a=6$ and $n=2.5$	A1 A1 (6)
(c)	$\frac{'2.5'\times 1.5\times 0.5\times '6'^3}{6}=67.5$	M1 A1 (2)
		Total 10 marks

Part	Mark	Notes
(a)	**M1**	For an attempt to expand the given expression up to the term in x^3 The definition of an attempt is as follows: • The first two terms must be correct $[1+anx]$ • The powers of *x* must be correct • The denominators must be correct • Simplification not required e.g accept $(ax)^2$ or $(ax)^3$ $(1+ax)^n=1+anx+\frac{n(n-1)}{2}a^2x^2+\frac{n(n-1)(n-2)}{3!}a^3x^3$
	A1	For a fully correct expansion. Simplification not required.
(b)	**B1**	For setting $an=15$
	M1	For setting their coefficient of x^2 equal to their coefficient of x^3 $\frac{n(n-1)}{2}a^2=\frac{n(n-1)(n-2)}{6}a^3$ Do not condone the presence of either x^2 or x^3 for this mark unless there is later recovery.
	A1	For the fully correct equation simplified or unsimplified. $\frac{n(n-1)}{2}a^2=\frac{n(n-1)(n-2)}{6}a^3 \Rightarrow (3=(n-2)a)$
	M1	For simplifying the correct equation above to an equation where the powers of *a* and *n* are 1 and attempting to solve the two equations simultaneously leading to $a=\ldots$ or $n=\ldots$ Condone one arithmetical slip at any point.
	A1	For either $a=6$ or $n=2.5$
	A1	For both $a=6$ and $n=2.5$
(c)	**M1**	For substituting their values of *a* and *n* into their coefficient of x^3 $\frac{'2.5'\times 1.5\times 0.5\times '6'^3}{6}=67.5$ where $a,n\neq 1$ or 0
	A1	For 67.5

Question number	Scheme	Marks
6 (a)	$(\alpha-\beta)^2=\alpha^2-2\alpha\beta+\beta^2$	M1
	$(\alpha-\beta)^2=(\alpha+\beta)^2-2\alpha\beta-2\alpha\beta$	M1
	$(\alpha-\beta)^2=(\alpha+\beta)^2-4\alpha\beta$ *	A1 cso (3)
(b)	$\alpha+\beta=7k$ and $\alpha\beta=k^2$	B1
	$\alpha-\beta=\sqrt{49k^2-4k^2}$	M1
	$=\sqrt{45}k=3k\sqrt{5}$ *	A1 cso (3)
(c)	Sum = $\alpha+\beta\ (=7k)$	B1ft
	Product $(\alpha+1)(\beta-1)=\alpha\beta-(\alpha-\beta)-1\Rightarrow(\alpha+1)(\beta-1)=k^2-3k\sqrt{5}-1$	M1
	So $x^2-7kx+k^2-3k\sqrt{5}-1=0$	M1A1 (4)
		Total 10 marks

Note: You may see a method based on the difference of two squares for part (a)
i.e. $(\alpha+\beta)^2-(\alpha-\beta)^2=4\alpha\beta$

Solution

$$(\alpha-\beta)^2=(\alpha+\beta)^2-4\alpha\beta\Rightarrow(\alpha+\beta)^2-(\alpha-\beta)^2=4\alpha\beta$$

$$(\alpha+\beta)^2-(\alpha-\beta)^2=([\alpha+\beta]+[\alpha-\beta])([\alpha+\beta]-[\alpha-\beta])$$

$$=(2\alpha)(2\beta)$$

$$=4\alpha\beta$$

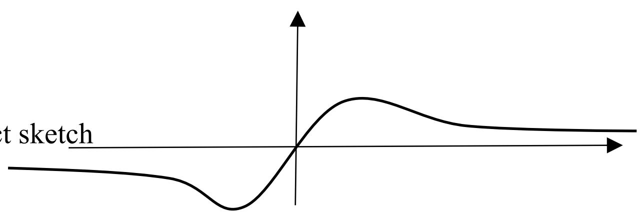
LHS = RHS (hence shown)

If this is a full and correct solution as shown (no errors) – award full marks – otherwise, please send to Review.

Part	Mark	Notes
(a)	M1	For expanding $\left(\alpha-\beta\right)^2=\alpha^2-2\alpha\beta+\beta^2 \Rightarrow \left(\alpha^2+\beta^2-2\alpha\beta\right)$ or expanding $(\alpha+\beta)^2=\alpha^2+\beta^2+2\alpha\beta$ This must be correct for this mark.
	M1	Replaces $\alpha^2+\beta^2$ with $(\alpha+\beta)^2-2\alpha\beta$ in their expansion of $(\alpha-\beta)^2$. And attempts to collect terms $\left(\alpha-\beta\right)^2=(\alpha+\beta)^2-2\alpha\beta-2\alpha\beta$
	A1 cso	For the correct given expression **with no errors seen.** $(\alpha-\beta)^2=(\alpha+\beta)^2-4\alpha\beta$ *
	ALT	
	M1	For expanding $\left(\alpha-\beta\right)^2=\alpha^2-2\alpha\beta+\beta^2 \Rightarrow \left(\alpha^2+\beta^2-2\alpha\beta\right)$ or expanding $(\alpha+\beta)^2=\alpha^2+\beta^2+2\alpha\beta$ This must be correct for this mark.
	M1	For expanding the RHS and equates to their expansion of $\left(\alpha-\beta\right)^2$ and attempting to simplify. $\alpha^2+\beta^2-2\alpha\beta=\left(\alpha^2+\beta^2+2\alpha\beta\right)-4\alpha\beta$
	A1 cso	Both sides of the equivalence are shown to be equal **with no errors seen.** $(\alpha-\beta)^2=(\alpha+\beta)^2-4\alpha\beta$ *
(b)	B1	For both $\alpha+\beta=7k$ and $\alpha\beta=k^2$ This may be implied in later work e.g. by use of $49k^2$ and $4k^2$
	M1	For substituting their values for the sum and product into the **given** expression for $\left(\alpha-\beta\right)^2$, simplifying and square rooting both sides. $(\alpha-\beta)^2=(7k)^2-4k^2 \Rightarrow \alpha-\beta=\sqrt{(7k)^2-4k^2}=\sqrt{45k^2}$ Condone $\pm\sqrt{45k^2}$ for this mark.
	A1 cso	For the correct value of $\alpha-\beta=3k\sqrt{5}$ *
(c)	B1ft	For the sum $\left(\alpha+1+\beta-1\right)=\alpha+\beta=7k$ Ft their $\alpha+\beta$
	M1	For the product in terms of *k*. Correctly multiplying out $(\alpha+1)(\beta-1)$ and substituting in their value $\alpha\beta$ and the correct value $\alpha-\beta=3k\sqrt{5}$ $(\alpha+1)(\beta-1)=\alpha\beta-(\alpha-\beta)-1 \Rightarrow (\alpha+1)(\beta-1)=k^2-3k\sqrt{5}-1$
	M1	For correctly forming an equation with their sum and product $x^2-'7k'x+'k^2-3k\sqrt{5}-1'(=0)$ Condone the absence of = 0 for this mark
	A1	For the correct equation $x^2-7kx+k^2-3k\sqrt{5}-1=0$ Allow $p=-7k$ $q=k^2-3k\sqrt{5}-1$

Question number	Scheme	Marks
7 (a)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(x^2+4)-x(2x)}{\left(x^2+4\right)^2}$	M1
	$=\frac{4-x^2}{\left(x^2+4\right)^2}$	A1
	$\frac{\mathrm{d}y}{\mathrm{d}x}=0\Rightarrow 4-x^2=0$	M1
	So $\left(2,\frac{1}{4}\right)$ and $\left(-2,-\frac{1}{4}\right)$	A1 A1 (5)
(b)	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{-2x(x^2+4)^2-4x(4-x^2)(x^2+4)}{(x^2+4)^4}$	M1
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{-2x^3-8x-16x+4x^3}{(x^2+4)^3}$	M1
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{2x^3-24x}{(x^2+4)^3}$	M1
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{2x(x^2-12)}{(x^2+4)^3}$ *	A1 cso (4)
(c)	When $x=2$ $\left[\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-\frac{1}{16}\right]$ When $x=-2$ $\left[\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{1}{16}\right]$	
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}<0$ so maximum $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}>0$ so minimum	M1 A1ft (2)
		Total 11 marks

Part	Mark	Notes
(a)	**M1**	For an attempt at Quotient rule. The definition of an attempt is that there must be a minimal attempt to differentiate both terms and the denominator must be $\left(x^2+4\right)^2$. Allow the terms in the numerator to be the wrong way around, but the terms must be subtracted. [See General Guidance for an attempt at differentiation]. $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(x^2+4)-x(2x)}{\left(x^2+4\right)^2}$
	A1	For the correct $\frac{\mathrm{d}y}{\mathrm{d}x}$ simplified or unsimplified. Award the mark for a correct $\frac{\mathrm{d}y}{\mathrm{d}x}$ seen even if there are later errors in simplification.
	M1	For setting their $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ which must be a quadratic equation solving to find x: $4-x^2=0\Rightarrow x=\pm 2$

	A1	For the correct coordinates of either $\left(2, \frac{1}{4}\right)$ or $\left(-2, -\frac{1}{4}\right)$
	A1	For both correct coordinates $\left(2, \frac{1}{4}\right)$ and $\left(-2, -\frac{1}{4}\right)$
(b)	**M1**	For an attempt at Quotient rule on their $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(x^2+4)-x(2x)}{\left(x^2+4\right)^2}$ which must be as a minimum: $\frac{ax^2+bx+c}{\left(x^2+4\right)^2}$ where *a*, *b* and *c* are constants and $a, c \neq 0$ The definition of an attempt is that there must be a minimal attempt to differentiate the numerator and denominator and the correct formula applied. $\left(x^2+4\right)^2$ must differentiate to $ax(x^2+4)$, the denominator must be $\left(x^2+4\right)^4$ Allow the terms in the numerator to be the wrong way around, but the terms must be subtracted. Apply General Guidance for an attempt at differentiation on ax^2+bx+c. $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{-2x(x^2+4)^2 - 4x(4-x^2)(x^2+4)}{(x^2+4)^4}$
	M1	For cancelling through by (x^2+4) $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{-2x(x^2+4)-4x(4-x^2)}{(x^2+4)^3}$
	M1	For simplifying the numerator to achieve as a minimum $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{ax^3+bx}{(x^2+4)^3}$ where *a* and *b* are constants
	A1 cso	For obtaining the answer as given with no errors. $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{2x(x^2-12)}{(x^2+4)^3}$ *
(c)	**M1**	Substitutes either their 2 or their – 2 into their $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ Note: When $x = 2$ $\left[\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = -\frac{1}{16}\right]$ and when $x = -2$ $\left[\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{1}{16}\right]$
	A1ft	For the conclusion $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} < 0$ so maximum $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} > 0$ so minimum
	ALT – tests gradient or sight of a sketch	
	M1	Tests gradient on either side of one the turning points (their 2 or their – 2) using their $\frac{\mathrm{d}y}{\mathrm{d}x}$ or a correct sketch 
	A1ft	For the correct conclusion

Question number	Scheme	Marks
8 (a)	$\log_a n = \log_a 3(2n-1)$	M1
	$n = 3(2n-1)$	M1
	$n = \dfrac{3}{5}$	A1 (3)
(b)(i)	$x = p^3$	B1 (1)
(b)(ii)	$\log_p y - \log_p 2^3 = 4 \Rightarrow \log_p\left(\dfrac{y}{2^3}\right) = 4$ or $\log_p\left(\dfrac{y}{8}\right) = 4$	M1
	$\dfrac{y}{2^3} = p^4 \Rightarrow \left(y = 2^3 p^4 \text{ or } 8p^4\right)$	M1
	$xy = 8p^7$	M1A1 (4)
	ALT (b)(ii)	
	$\log_p x + \log_p y - 3\log_p 2 = 4 + 3 \Rightarrow \log_p\left(\dfrac{xy}{2^3}\right) = 7$	{M1}
	$\dfrac{xy}{2^3} = p^7$	{M1}
	$xy = 8p^7$	{M1A1} (4)
		Total 8 marks

Part	Mark	Notes
(a)	M1	Uses the addition law of logs correctly $\log_a n = \log_a 3 + \log_a(2n-1) \Rightarrow \log_a n = \log_a 3(2n-1)$ Accept also $\log_a n = \log_a 3 + \log_a(2n-1) \Rightarrow 0 = \log_a\left(\frac{n}{3(2n-1)}\right) = (\log_a 1)$
	M1	For obtaining a linear equation from their log equation and attempting to find a value for *n*. $n = 3(2n-1)$ leading to a numerical value for n
	A1	For $n = \frac{3}{5}$
(b)(i)	B1	For $x = p^3$
(b)(ii)	M1	For stating that $3\log_p 2 = \log_p 8$ or $\log_p 2^3$ and for using the addition law correctly to combine the LHS: $\log_p y - \log_p 2^3 = 4 \Rightarrow \log_p\left(\frac{y}{2^3}\right) = 4$ or $\log_p\left(\frac{y}{8}\right) = 4$
	M1	Correctly removes logs on both sides to obtain: $\frac{y}{2^3} = p^4 \Rightarrow \left(y = 2^3 p^4 \text{ or } 8p^4\right)$
	M1	For correctly finding the product of their *x* and their *y*: $xy = 'p^3' \times '8p^4'$
	A1	For the correct answer of $xy = 8p^7$
	ALT	
	M1	For stating that $3\log_p 2 = \log_p 8$ or $\log_p 2^3$ and states $\log_p x + \log_p y - 3\log_p 2 = 3 + 4$ Uses the addition law correctly to combine the LHS $\log_p x + \log_p y - 3\log_p 2 = 4 + 3 \Rightarrow \log_p\left(\frac{xy}{2^3}\right) = 7$
	M1	Correctly remove logs on both sides to obtain: $\frac{xy}{2^3} = p^7$
	M1	Correctly rearrange their expression to make *xy* the subject
	A1	For the correct answer of $xy = 8p^7$

Question number	Scheme	Marks
9	$\frac{dy}{dx} = -(x^3 - 2x)e^{1-x} + (3x^2 - 2)e^{1-x}$	M1 A1
	When $x = 1$ $\frac{dy}{dx} = 2$ $\Rightarrow$ Gradient of **normal** = $-\frac{1}{"2"}$	M1
	$(y+1) = -\frac{1}{2}(x-1)$ oe and isw once seen	M1 A1 (5)
		Total 5 marks

Mark	Notes
M1	For the use of product rule. This is not given on page 2 so please mark as follows: • There must be an acceptable attempt to differentiate both terms. For this question $x^3 - 2x \to ax^2 + b \quad a, b \neq 0$ $e^{1-x} \to \pm e^{1-x}$ • Allow their $u\frac{dv}{dx} \pm v\frac{du}{dx}$ (as long as it fulfils these minimum conditions) $\frac{dy}{dx} = -(x^3 - 2x)e^{1-x} + (3x^2 - 2)e^{1-x}$
A1	For the correct simplified or unsimplified $\frac{dy}{dx}$ as shown above.
M1	For substituting $x = 1$ correctly into their $\frac{dy}{dx}$ to obtain a value for the gradient of the **normal**. When $x = 1$ $\frac{dy}{dx} = "2" \Rightarrow m_n = -\frac{1}{"2"}$ (must come from their $\frac{dy}{dx}$)
M1	For correctly forming an equation using the given coordinates with their gradient of the normal which is the negative reciprocal of their value of $\frac{dy}{dx}$ $(y+1) = -\frac{1}{2}(x-1)$ If $y = mx + c$ is used, then they must find a value for c and find an equation. $c = -\frac{1}{2}$ so $y = -\frac{x}{2} - \frac{1}{2}$ oe
A1	For the correct equation as shown above in any form.

Question number	Scheme	Marks
10 (a)		
	$\left(\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}\right) = 5\mathbf{p} + 3\mathbf{q}$	M1
	$\overrightarrow{DC} = \left(\overrightarrow{DO} + \overrightarrow{OC}\right) = \overrightarrow{DO} + \frac{3}{2}\overrightarrow{OB}$	M1
	$\overrightarrow{DC} = \frac{9}{2}(\mathbf{p} + \mathbf{q}) = \frac{9\mathbf{p}}{2} + \frac{9\mathbf{q}}{2}$	A1 (3)
(b)	$\overrightarrow{OF} = \overrightarrow{OD} + \lambda\overrightarrow{DC}$	M1
	$= 3\mathbf{p} + \frac{9}{2}\lambda(\mathbf{p} + \mathbf{q})$	A1
	$\overrightarrow{OF} = \overrightarrow{OA} + \mu\overrightarrow{AB}$	M1
	$= 5\mathbf{p} + 3\mu\mathbf{q}$	A1
	$3 + \frac{9}{2}\lambda = 5$	M1
	$\lambda = \frac{4}{9}$ $\left(\frac{9}{2}\lambda = 3\mu\right)$ or $\mu = \frac{2}{3}$	A1
	$\overrightarrow{OF} = 5\mathbf{p} + 2\mathbf{q}$	A1 (7)
(c)	$\overrightarrow{OG} = \alpha(5\mathbf{p} + 3\mathbf{q})$	M1
	$\overrightarrow{OG} = 3\mathbf{p} + 2(\mathbf{p} + \mathbf{q}) + 5\beta\mathbf{p} = 5\mathbf{p} + 5\beta\mathbf{p} + 2\mathbf{q}$ ft their $\overrightarrow{OF}$	M1
	$3\alpha = 2 \Rightarrow \alpha = \frac{2}{3}$ $\left(\beta = -\frac{1}{3}\right)$	M1
	$\overrightarrow{OG} = \frac{10}{3}\mathbf{p} + 2\mathbf{q}$	A1 (4)
	ALT (c)	
	$\overrightarrow{OG} = \alpha(5\mathbf{p} + 3\mathbf{q})$	{M1}
	$\overrightarrow{FG} = -5\mathbf{p} - 2\mathbf{q} + \alpha\ 5\mathbf{p} + 3\mathbf{q}$ ft their $-\left(\overrightarrow{OF}\right)$	{M1}
	As FG is parallel to AO there can be no **q** component in $\overrightarrow{FG}$	
	$3\alpha = 2 \Rightarrow \alpha = \frac{2}{3}$	{M1}
	$\overrightarrow{OG} = \frac{10}{3}\mathbf{p} + 2\mathbf{q}$	{A1} (4)
		Total 14 marks

Part	Mark	Notes
(a)	M1	For finding the vector $\overrightarrow{OB}$: $\left(\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}\right) = 5\mathbf{p} + 3\mathbf{q}$ **This must be correct.**
	M1	For the correct vector statement for $\overrightarrow{DC}$: $\overrightarrow{DC} = \left(\overrightarrow{DO} + \overrightarrow{OC}\right) = \overrightarrow{DO} + \frac{3}{2}\overrightarrow{OB}$ This mark can be implied by a correct (unsimplified) vector.
	A1	For the correct simplified $\overrightarrow{DC}$ in terms of a single **p** and a single **q** only $\overrightarrow{DC} = -3\mathbf{p} + \frac{3}{2}(5\mathbf{p} + 3\mathbf{q}) = \frac{9}{2}(\mathbf{p} + \mathbf{q})$ accept $\overrightarrow{DC} = \frac{9\mathbf{p}}{2} + \frac{9\mathbf{q}}{2}$ oe
(b)	M1	For stating $\overrightarrow{OF} = \overrightarrow{OD} + \lambda\overrightarrow{DC}$ (or any other variable in place of λ)
	A1	For the vector $\overrightarrow{OF} = 3\mathbf{p} + \frac{9}{2}\lambda(\mathbf{p} + \mathbf{q})$
	M1	For stating $\overrightarrow{OF} = \overrightarrow{OA} + \mu\overrightarrow{AB}$ (or any other variable in place of μ)
	A1	For the vector $\overrightarrow{OF} = 5\mathbf{p} + 3\mu\mathbf{q}$
	M1	Collects like terms correctly, equates coefficients correctly and attempts to solve their two equations in λ and μ They must achieve a value for μ or λ for this mark $3\mathbf{p} + \frac{9}{2}\lambda(\mathbf{p} + \mathbf{q}) = 5\mathbf{p} + 3\mu\mathbf{q} \Rightarrow \mathbf{p}\left(3 + \frac{9}{2}\lambda\right) + \frac{9}{2}\lambda\mathbf{q} = 5\mathbf{p} + 3\mu\mathbf{q}$ $3 + \frac{9}{2}\lambda = 5$ and $\frac{9}{2}\lambda = 3\mu$ $\Rightarrow \mu = \frac{2}{3}$ or $\lambda = \frac{4}{9}$
	A1	For the correct $\mu = \frac{2}{3}$ or $\lambda = \frac{4}{9}$
	A1	For the correct vector $\overrightarrow{OF} = 5\mathbf{p} + 2\mathbf{q}$
		Part (b) can be solved in different ways. These are the general principles of marking this part question. **M1** – for any correct vector statement leading to $\overrightarrow{OF}$ which introduces a parameter. **A1** – for one correct vector $\overrightarrow{OF}$ **M1** - For any correct second vector statement for $\overrightarrow{OF}$ which can be used with the first vector and introduces a second parameter. **A1** – For a second correct vector $\overrightarrow{OF}$ **M1A1A1** – as main scheme above
(c)	M1	For one vector for $\overrightarrow{OG} = \alpha(5\mathbf{p} + 3\mathbf{q})$

	M1	For a second vector for $\overrightarrow{OG} = \left(\overrightarrow{OF} + \overrightarrow{FG}\right) = 3\mathbf{p} + 2(\mathbf{p}+\mathbf{q}) + 5\beta\mathbf{p} = \left(5\mathbf{p} + 5\beta\mathbf{p} + 2\mathbf{q}\right)$ ft their $\overrightarrow{OF}$ Accept simplified or unsimplified.
	M1	Collects like terms correctly, equates coefficients correctly to find the value of α $\alpha(5\mathbf{p}+3\mathbf{q}) = 5\mathbf{p} + 5\beta\mathbf{p} + 2\mathbf{q} \Rightarrow 5\mathbf{p}\alpha + 3\mathbf{q}\alpha = (5+5\beta)\mathbf{p} + 2\mathbf{q}$ $\Rightarrow 3\alpha = 2 \Rightarrow \alpha = \dfrac{2}{3}$
	A1	For the correct vector $\overrightarrow{OG} = \dfrac{10}{3}\mathbf{p} + 2\mathbf{q}$
	ALT	
	M1	For the vector for $\overrightarrow{OG} = \alpha(5\mathbf{p}+3\mathbf{q})$
	M1	For the vector $\overrightarrow{FG} = \left(\overrightarrow{FO} + \overrightarrow{OG}\right) = -5\mathbf{p} - 2\mathbf{q} + \alpha\ 5\mathbf{p} + 3\mathbf{q}$ **Note:** ft their $-\left(\overrightarrow{OF}\right)$ from part (b)
	M1	As FG is parallel to OA there is no $\mathbf{q}$ component in $\overrightarrow{FG} \Rightarrow 3\alpha = 2 \Rightarrow \alpha = \dfrac{2}{3}$
	A1	For the correct vector $\overrightarrow{OG} = \dfrac{10}{3}\mathbf{p} + 2\mathbf{q}$

Question number	Scheme	Marks
11 (a)	$\cos 2x = \cos(x+x)$	
	$= \cos^2 x - \sin^2 x = 1 - \sin^2 x - \sin^2 x$	M1 M1
	$= 1 - 2\sin^2 x$ *	A1 cso
		(3)
(b)	$\sin x + 2\sin^2 x - 1 = 0 \Rightarrow (2\sin x - 1)(\sin x + 1) = 0$	M1dM1
	When $\sin x = \frac{1}{2}$ When $\sin x = -1$	
	$x = \frac{\pi}{6}, \frac{5\pi}{6}$ $x = \frac{3\pi}{2}$	A1
	$\left(\frac{\pi}{6}, \frac{5}{2}\right), \left(\frac{5\pi}{6}, \frac{5}{2}\right), \left(\frac{3\pi}{2}, 1\right)$	A1
		(4)
(c)	$\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (\sin x + 2)\,dx - \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (\cos 2x + 2)\,dx$	M1
	$[-\cos x + 2x]_{\frac{\pi}{6}}^{\frac{5\pi}{6}} - \left[\frac{\sin 2x}{2} + 2x\right]_{\frac{\pi}{6}}^{\frac{5\pi}{6}}$	A1
	$\left[\left(-\cos\frac{5\pi}{6} + 2\left(\frac{5\pi}{6}\right)\right) - \left(-\cos\frac{\pi}{6} + 2\left(\frac{\pi}{6}\right)\right)\right]$ $-\left[\left(\frac{\sin 2\left(\frac{5\pi}{6}\right)}{2} + 2\left(\frac{5\pi}{6}\right)\right) - \left(\frac{\sin 2\left(\frac{\pi}{6}\right)}{2} + 2\left(\frac{\pi}{6}\right)\right)\right]$	M1
	$= \frac{3\sqrt{3}}{2}$	A1
	$\int_{\frac{5\pi}{6}}^{\frac{3\pi}{2}} (\cos 2x + 2)\,dx - \int_{\frac{5\pi}{6}}^{\frac{3\pi}{2}} (\sin x + 2)\,dx$	M1
	$\left[\frac{\sin 2x}{2} + 2x\right]_{\frac{5\pi}{6}}^{\frac{3\pi}{2}} - [-\cos x + 2x]_{\frac{5\pi}{6}}^{\frac{3\pi}{2}}$	
	$\left[\left(\frac{\sin 2\left(\frac{3\pi}{2}\right)}{2} + 2\left(\frac{3\pi}{2}\right)\right) - \left(\frac{\sin 2\left(\frac{5\pi}{6}\right)}{2} + 2\left(\frac{5\pi}{6}\right)\right)\right]$ $-\left[\left(-\cos\left(\frac{3\pi}{2}\right) + 2\left(\frac{3\pi}{2}\right)\right) - \left(-\cos\left(\frac{5\pi}{6}\right) + 2\left(\frac{5\pi}{6}\right)\right)\right]$	M1
	$\frac{3\sqrt{3}}{4}$	A1
	$R_1 : R_2 = 2 : 1$	B1ft
	See Special Case at end of notes	(8)
		Total 15 marks

Part	Mark	Notes
(a)	M1	For using the addition formula on page 2 and reaching the result $\cos 2x = \cos^2 x - \sin^2 x$ This **must** be correct for this mark.
	M1	For using **and** applying $\sin^2 x + \cos^2 x = 1$ in their $\cos 2x$ $\cos 2x = 1 - \sin^2 x - \sin^2 x$
	A1 cso	For the correct identity as shown in full. $\cos 2x = 1 - 2\sin^2 x$ * **Note: This is a show question, there must be no errors seen.**
(b)	M1	Equates the two given equations together. $\sin x + 2 = \cos 2x + 2$ and uses the given identity in part (a) to attempt to form a 3TQ in sin *x*: $\sin x + 2\sin^2 x - 1 = 0$ or $2\sin^2 x + \sin x - 1 = 0$ At least either $2\sin^2 x$ or $\sin x$ must be correct.
	dM1	Solves their 3TQ by any method to reach two values for $\sin x$ -see General Guidance $(2\sin x - 1)(\sin x + 1) = 0 \Rightarrow \sin x = \left(-1, \frac{1}{2}\right)$ This mark is dependent on the previous M mark.
	A1	For finding all three values of *x:* $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$ Allow *x* = $30^\circ, 150^\circ, 270^\circ$ for this mark – condone missing degree signs
	A1	For all three sets of correct coordinates: $\left(\frac{\pi}{6}, \frac{5}{2}\right), \left(\frac{5\pi}{6}, \frac{5}{2}\right), \left(\frac{3\pi}{2}, 1\right)$ Allow $\left(30^\circ, \frac{5}{2}\right), \left(150^\circ, \frac{5}{2}\right), \left(270^\circ, 1\right)$ – condone missing degree signs
(c)	**Area R_1**	
	M1	For stating a correct method to find the area R_1 using their limits from part (b) of $\frac{\pi}{6}$ and $\frac{5\pi}{6}$ applied the correct way. Condone angles in degrees. $\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (\sin x + 2)\,dx - \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (\cos 2x + 2)\,dx$ (The + 2 – 2 may have been simplified).
	M1	For an attempt to integrate their expression for R_1 which must involve $\sin x$ and $\cos 2x$ A minimally acceptable integral of $\sin x \to -\cos x$ **or** $\cos 2x \to \pm\frac{\sin 2x}{2}$ and $2 \to 2x$ (The $2 \to 2x$ may not be present if they've simplified first). At least one trig term must be acceptable **with** $2 \to 2x$ (if present) correct. $\left[-\cos x + 2x\right]_{\frac{\pi}{6}}^{\frac{5\pi}{6}} - \left[\frac{\sin 2x}{2} + 2x\right]_{\frac{\pi}{6}}^{\frac{5\pi}{6}}$ Ignore incorrect, angles in degrees or even absent limits for this mark.

	M1	Substitutes their limits correctly into their integrated expression. Must involve a changed expression. $\left[\left(-\cos\frac{5\pi}{6}+2\left(\frac{5\pi}{6}\right)\right)-\left(-\cos\frac{\pi}{6}+2\left(\frac{\pi}{6}\right)\right)\right]-$ $\left[\left(\frac{\sin 2\left(\frac{5\pi}{6}\right)}{2}+2\left(\frac{5\pi}{6}\right)\right)-\left(\frac{\sin 2\left(\frac{\pi}{6}\right)}{2}+2\left(\frac{\pi}{6}\right)\right)\right]$ **Do not accept a substitution in degrees – it must be in radians.** **If the work for the first two method marks to find the area of R_1 is fully correct, this mark can be implied by $\frac{3\sqrt{3}}{2}$. If the student does not get** $\left[-\cos x+2x\right]_{\frac{\pi}{6}}^{\frac{5\pi}{6}}-\left[\frac{\sin 2x}{2}+2x\right]_{\frac{\pi}{6}}^{\frac{5\pi}{6}}$ **from the first two methods marks, they must show the clear substitution of their limits to be awarded this mark.**
	A1	For the correct area.
Area R_2		
	M1	For an attempt to integrate their expression for R_2 which must involve $\int(\cos 2x+2)\,dx-\int(\sin x+2)\,dx$ or $\int \cos 2x-\sin x\,dx$ if they have simplified the +2 – 2. Ignore incorrect, angles in degrees or even absent limits for this mark. A minimally acceptable integral of $\sin x\to-\cos x$ or $\cos 2x\to\pm\frac{\sin 2x}{2}$ and $2\to 2x$ (The $2\to 2x$ may not be present if they've simplified first). At least one trig term must be acceptable **with** $2\to 2x$ (if present) correct. $\left[\frac{\sin 2x}{2}+2x\right]_{\frac{5\pi}{6}}^{\frac{3\pi}{2}}-\left[-\cos x+2x\right]_{\frac{5\pi}{6}}^{\frac{3\pi}{2}}$
	M1	For substituting their limits correctly into their integrated expression. Must involve a changed expression. $\left[\left(\frac{\sin 2\left(\frac{3\pi}{2}\right)}{2}+2\left(\frac{3\pi}{2}\right)\right)-\left(\frac{\sin 2\left(\frac{5\pi}{6}\right)}{2}+2\left(\frac{5\pi}{6}\right)\right)\right]$ $-\left[\left(-\cos\left(\frac{3\pi}{2}\right)+2\left(\frac{3\pi}{2}\right)\right)-\left(-\cos\left(\frac{5\pi}{6}\right)+2\left(\frac{5\pi}{6}\right)\right)\right]$ **Do not accept a substitution in degrees – it must be in radians.** **If the student's work for the first method mark to find the area of R_2 is fully correct, this M mark can be implied by $\frac{3\sqrt{3}}{4}$. If the student does not get**

		$\left[\frac{\sin 2x}{2}+2x\right]_{\frac{5\pi}{6}}^{\frac{3\pi}{2}}-\left[-\cos x+2x\right]_{\frac{5\pi}{6}}^{\frac{3\pi}{2}}$ **from the first method mark, they must show the clear substitution of their limits to be awarded this mark.**
	A1	For the correct area.
Ratio area of R_1: area of R_2		
	B1ft	For a ratio given in its simplest form *a* : *b* where *a, b* are positive integers, ft their values for the area of R_1 *and* R_2 Correct ratio is $R_1{:}R_2 = 2 : 1$
	SC	**Special case: For the use of degrees in part (c).** **<u>If the area of R_1 and R_2 are both correct</u> when limits in degrees have been substituted** (possible, for example, when the candidate has simplified before integrating): Penalise the first substitution in degrees for the area R_1 by awarding M1 M1 M0 A0. For the area R_2 allow M1 M1 A1 if the substitution (in degrees) and the area of R_2 is correct. B1ft for the correct ratio.

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