Model Solutions

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel International GCSE

Centre Number | Candidate Number

Tuesday 21 May 2019

Morning (Time: 2 hours) | Paper Reference **4MA1/1H**

Mathematics A
Level 1/2
Paper 1H
Higher Tier

You must have:
Ruler graduated in centimetres and millimetres, protractor, compasses, pen, HB pencil, eraser, calculator. Tracing paper may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- **Calculators may be used.**
- You must **NOT** write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over

P58365A

1/1/1/1/

Pearson

International GCSE Mathematics

Formulae sheet – Higher Tier

Arithmetic series

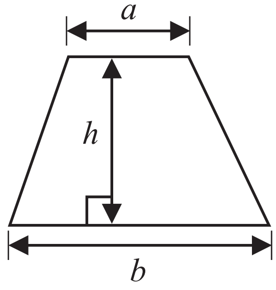
Sum to n terms, $S_n = \frac{n}{2}\,[2a + (n - 1)d]$

The quadratic equation

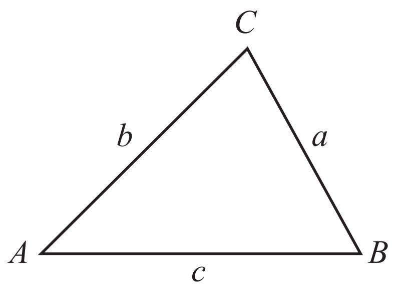
The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$ are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Area of trapezium $= \frac{1}{2}(a + b)h$



Trigonometry



In any triangle *ABC*

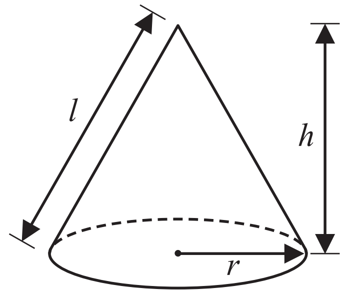
Sine Rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine Rule $a^2 = b^2 + c^2 - 2bc\cos A$

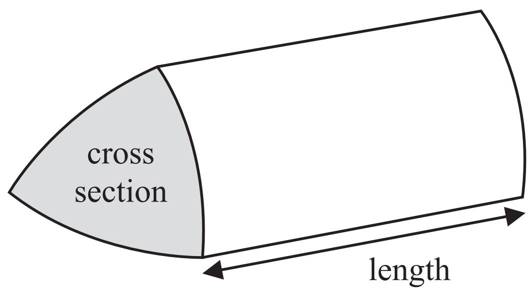
Area of triangle $= \frac{1}{2}ab\sin C$

Volume of cone $= \frac{1}{3}\pi r^2 h$

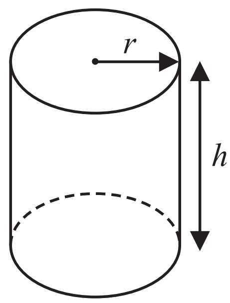
Curved surface area of cone $= \pi rl$



Volume of prism
= area of cross section × length

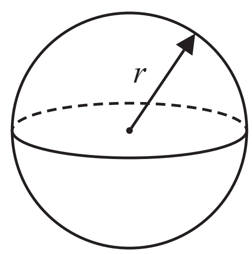


Volume of cylinder $= \pi r^2 h$
Curved surface area of cylinder $= 2\pi rh$



Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$



DO NOT WRITE IN THIS AREA

Answer ALL TWENTY FOUR questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 Show that $4\frac{2}{3} \div 1\frac{1}{9} = 4\frac{1}{5}$

$4\frac{2}{3} = \frac{14}{3}$

$1\,{}^1/_9 = \frac{10}{9}$

$\frac{14}{3} \div \frac{10}{9}$

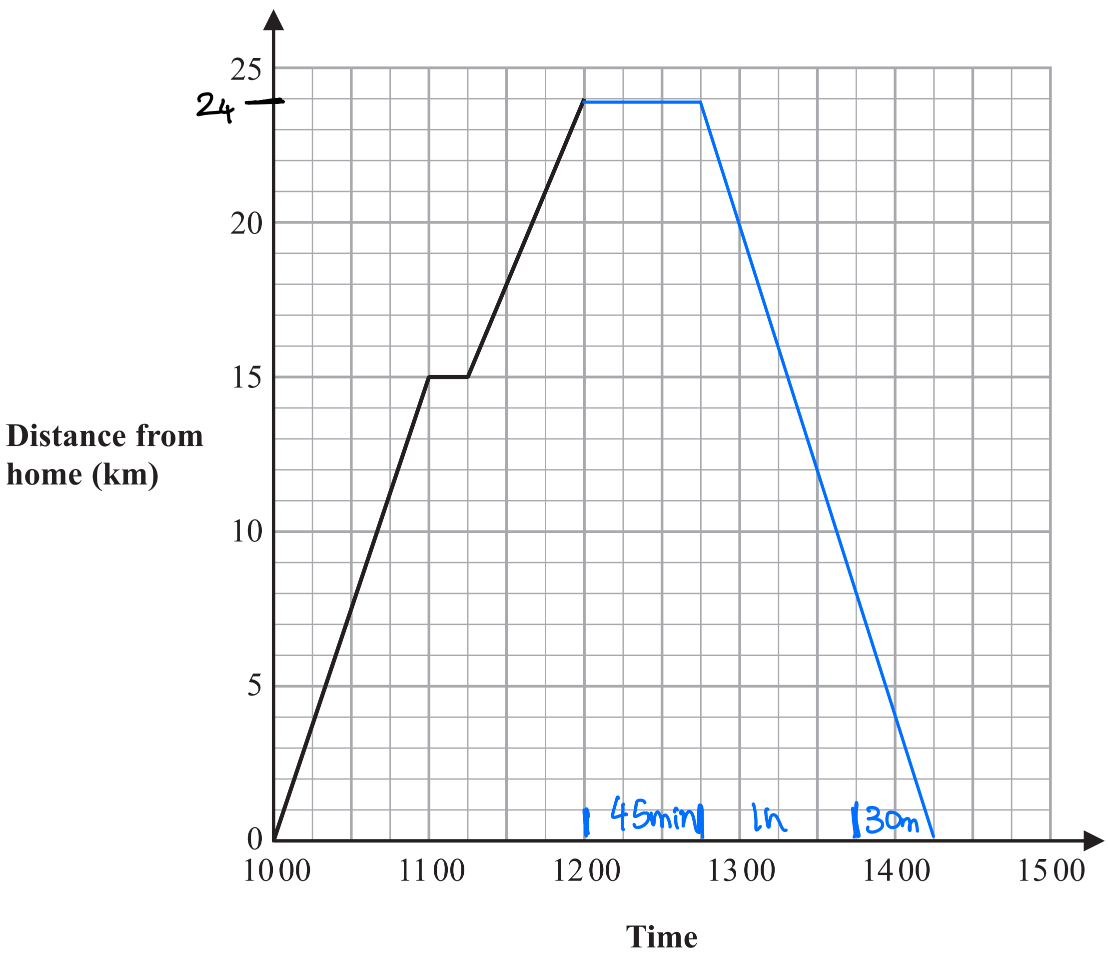
$= \frac{14}{3} \times \frac{9}{10} = \frac{126}{30}$

$\frac{126}{30} \underset{\div 6}{\overset{\div 6}{=}} \frac{21}{5}$

$\frac{21}{5} = 4\,{}^1/_5$

(Total for Question 1 is 3 marks)

2 Jalina left her home at 10 00 to cycle to a park.
On her way to the park, she stopped at a friend's house and then continued her journey to the park.
Here is the distance-time graph for her journey to the park.



(a) On her journey to the park, did Jalina cycle at a faster speed before or after she stopped at her friend's house?
Give a reason for your answer.

Before the line is steeper before the stop, therefore the speed was greater.

(1)

Jalina stayed at the park for 45 minutes.
She then cycled, without stopping, at a constant speed of 16 km/h from the park back to her home.

(b) Show all this information on the distance-time graph.

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

Distance = 24

$$\text{Time} = \frac{24\text{ km}}{16\text{ km/h}} = \frac{3}{2}\text{h}$$

(2)

(c) Work out Jalina's average cycling speed, in kilometres per hour, for the complete journey to the park and back.
Do **not** include the times when she was not cycling in your calculation.
Give your answer correct to 1 decimal place.

Total Distance : 24 + 24 = 48km

Total Time : 90min + 60min + 45min = 195min

$\div 60$

= 3.25hour

Average Speed : $\dfrac{48}{3.25} = 14.769$

round up

14.8 km/h

(3)

(Total for Question 2 is 6 marks)

3 (a) Simplify $e^9 \div e^5$

e^{9-5}

e^4

(1)

(b) Simplify $(y^2)^8$

$y^{2\times 8}$

y^{16}

(1)

(c) Expand and simplify $(x + 9)(x - 2)$

$x^2 - 2x + 9x - 18$

$x^2 + 7x - 18$

(2)

(d) Factorise fully $16c^4p^2 + 20cp^3$

$\text{HCF} = 4cp^2$

$4cp^2(4c^3 + 5p)$

$\frac{16c^4p^2}{4cp^2}$ $\frac{20cp^3}{4cp^2}$

$4cp^2(4c^3+5p)$

(2)

(Total for Question 3 is 6 marks)

DO NOT WRITE IN THIS AREA

4 (a) Complete the table of values for $y = x^2 - 3x - 1$

type in calculator

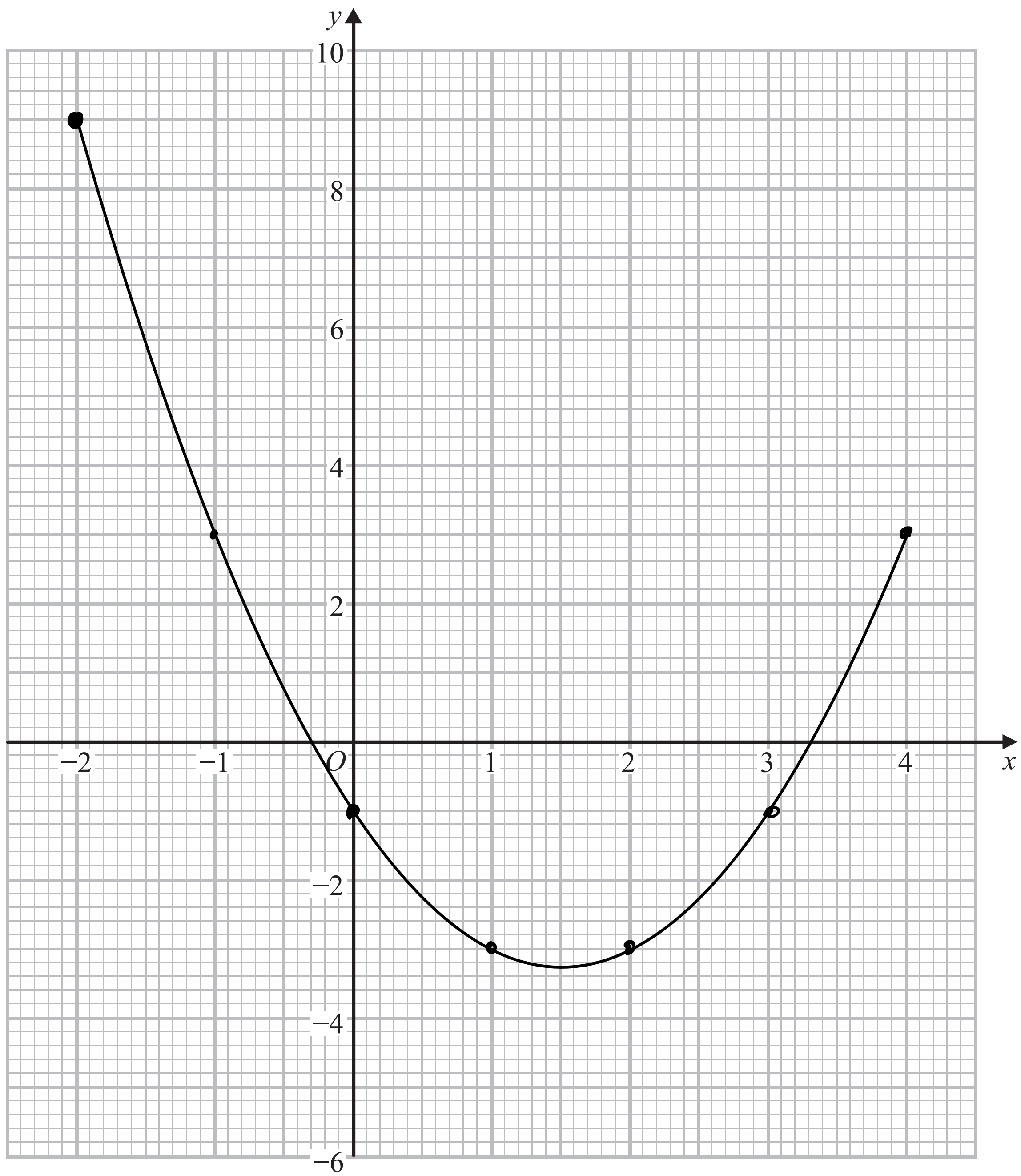
$y = (-2)^2 - 3(-2) - 1 = 9$

$y = (-1)^2 - 3(-1) - 1 = 3$

x	−2	−1	0	1	2	3	4
y	9	3	−1	−3	−3	−1	3

(2)

(b) On the grid, draw the graph of $y = x^2 - 3x - 1$ for all values of x from −2 to 4



(2)

(Total for Question 4 is 4 marks)

5 Becky has a biased 6-sided dice.

The table gives information about the probability that, when the dice is thrown, it will land on each number.

Number	1	2	3	4	5	6
Probability	$2x$	0.18	$2x$	$3x$	0.26	x

Becky is going to throw the dice 200 times.

Work out an estimate for the number of times that the dice will land on an even number.

Probability adds to 1:

$2x + 0.18 + 2x + 3x + 0.26 + x = 1$

$8x + 0.44 = 1$

$8x = 0.56$

$x = 0.07$

$P(\text{even}) = P(2) + P(4) + P(6)$

$= 0.18 + 3(0.07) + 0.07$

$P(\text{even}) = 0.46$

number of times rolling dice

$0.46 \times 200 = 92$

Probability

..................92..................

(Total for Question 5 is 4 marks)

DO NOT WRITE IN THIS AREA

6 The diagram shows a solid cuboid made from wood.

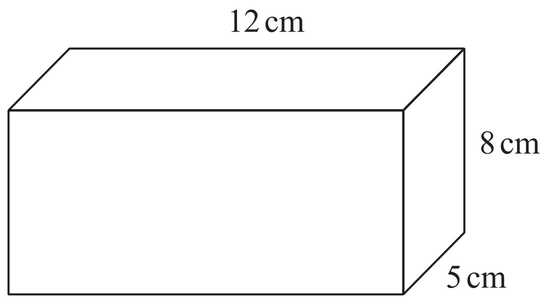


Diagram **NOT** accurately drawn

The wood has density 0.7 g/cm³

Work out the mass of the cuboid.

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

Volume : $12 \times 8 \times 5 = 480 \text{cm}^3$

Mass $= 0.7 \times 480$
$= 336\text{g}$

336 grams

(Total for Question 6 is 3 marks)

7 (a) Write 5.7×10^6 as an ordinary number.

$5.7000000\,0\,0$

1 2 3 4 5 6

5,700,000

(1)

(b) Write 0.004 in standard form.

123

between 1 and 10

4×10^{-3}

(1)

(c) Work out $\dfrac{2 \times 10^4 + 3 \times 10^5}{6.4 \times 10^{-2}}$

$= \dfrac{20000 + 300000}{0.064} = \dfrac{320,000}{0.064}$

5000 000

(2)

(Total for Question 7 is 4 marks)

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8 On 1st January 2016 Li bought a boat for \$170 000
The value of the boat depreciates by 8% per year.

Work out the value of the boat on 1st January 2019
Give your answer correct to the nearest dollar.

$100\% - 8\% = 92\% \xrightarrow{\div 100} = \times 0.92$

depreciate

multiplier

$2019 - 2016 = 3$ years

3 years

$170,000 \times 0.92^3 = \$132376.96$

Starting value

multiplier

round up

\$ 132 377

(Total for Question 8 is 3 marks)

9 The diagram shows a shape made from a right-angled triangle and a semicircle.

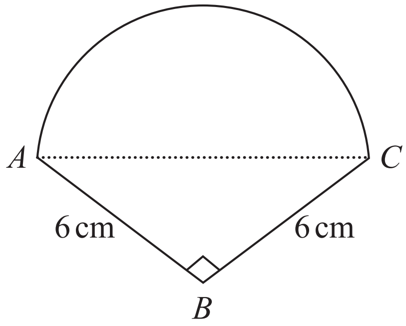

Diagram **NOT** accurately drawn

AC is the diameter of the semicircle.
$BA = BC = 6$ cm
Angle $ABC = 90°$

Work out the area of the shape.
Give your answer correct to 1 decimal place.

Area of triangle ABC $= \frac{1}{2} \times 6 \times 6 = 18 \text{cm}^2$

Pythogoras: $a^2 + b^2 = c^2$

$6^2 + 6^2 = AC^2$

$AC^2 = 72$

$AC = 6\sqrt{2}$

AC = diameter $= 6\sqrt{2}$

radius $= 3\sqrt{2}$

Area of semicircle : $\frac{1}{2}\pi r^2$

$\frac{1}{2}\pi(3\sqrt{2})^2 = 9\pi$

Total Area $= 9\pi + 18$

$= 46.27\ldots$

round up

46.3 cm²

(Total for Question 9 is 5 marks)

10 $A = 2^n \times 3 \times 5^m$

Write $8A$ as a product of powers of its prime factors.

index laws – add powers

$8A = 2^3 \times 2^n \times 3 \times 5^m$

$8 = 2^3$

$= 2^{3+n} \times 3 \times 5^m$

$2^{n+3} \times 3 + 5^m$

(Total for Question 10 is 2 marks)

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11 $C = b - a$

$a = \ \ 6$ correct to the nearest integer
$b = 15$ correct to the nearest 5

Work out the upper bound for the value of C
Show your working clearly.

all values round to 6

Bounds: $5.5 \leqslant a < 6.5$

$12.5 \leqslant b < 17.5$ – all values round to 15

UB for C = UB – LB

$= 17.5 - 5.5 = 12$

12

(Total for Question 11 is 3 marks)

DO NOT WRITE IN THIS AREA

12 (a) Factorise $2x^2 - 7x + 6$

2 numbers that multiply to $2 \times 6 = 12$

and add to -7 - -3 and -4

factorise each side

split

$2x^2 - 4x \mid -3x + 6$

$2x(x-2) \mid -3(x-2)$

brackets should be equal

$(2x-3)(x-2)$

(2)

(b) Solve $\dfrac{4m+9}{3} = 7 - 2m$

Show clear algebraic working.

$\dfrac{4m+9}{3} = 7 - 2m$ $\times 3$ $\times 3$

$4m + 9 = 21 - 6m$

$+6m$

$10m + 9 = 21$

-9

$10m = 12$

$\div 10$

$m = 1.2$

$m =$ 1.2

(4)

(c) Write $\dfrac{\sqrt[4]{y}}{y}$ in the form y^b where b is a fraction.

$\sqrt[4]{y} = y^{\frac{1}{4}}$

$y^{\frac{1}{4}} \div y^1 = y^{1/4 - 1} = y^{-3/4}$

$y^{-3/4}$

(2)

(Total for Question 12 is 8 marks)

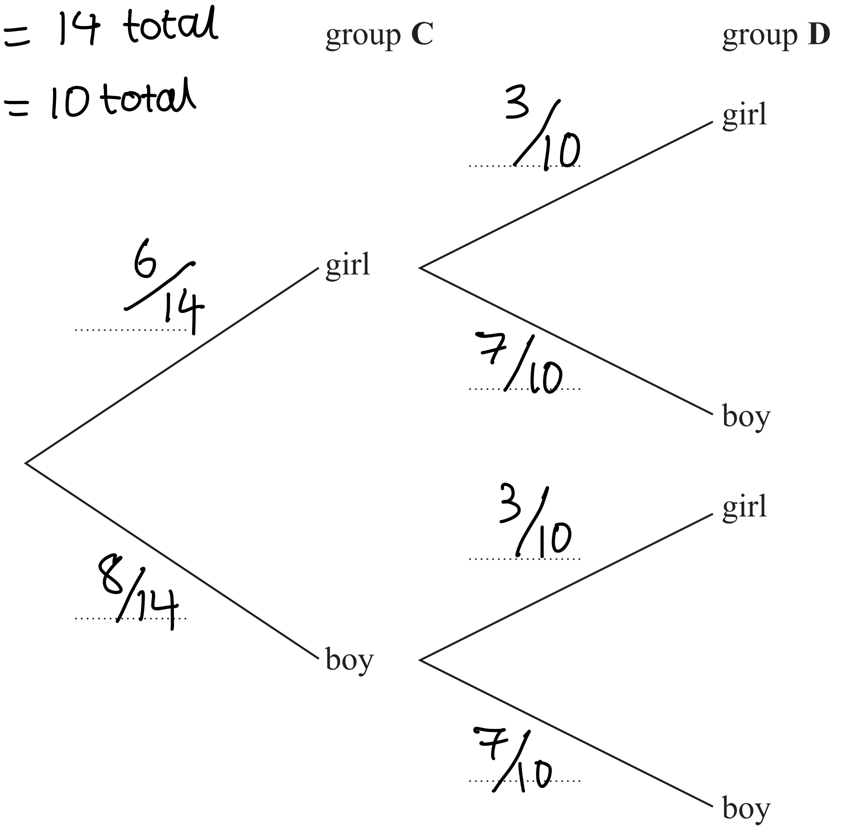
13 In group **C**, there are 6 girls and 8 boys.
In group **D**, there are 3 girls and 7 boys.

A team is made by picking at random one child from group **C** and one child from group **D**.

(a) Complete the probability tree diagram.

C: $6 + 8 = 14$ total

D: $3 + 7 = 10$ total



(2)

(b) Work out the probability that there are two boys in the team.

P(boy <u>AND</u> boy)

multiply

$$= \frac{8}{14} \times \frac{7}{10} = \frac{56}{140} =$$

2/5

(2)

DO NOT WRITE IN THIS AREA

After the first team has been picked, a second team is picked.
One child is picked at random from the children left in group **C** and one child is picked at random from the children left in group **D**.

(c) Work out the probability that there are two boys in each of the two teams.

2boys picked before

'After the first team is picked' - 1 less person in each group

P(2boys picked in 2nd team)

$$\frac{8-1}{14-1} \times \frac{7-1}{10-1} = \frac{7}{13} \times \frac{6}{9} = \frac{42}{117} = \frac{14}{39}$$

P(2boys and 2boys)

$$= \frac{14}{39} \times \frac{2}{5} = \frac{28}{195}$$

$\frac{28}{195}$

(3)

(Total for Question 13 is 7 marks)

14 $\mathscr{E}$ = {positive integers less than 20}
$A = \{x : x < 12\}$
$B = \{x : 7 \leqslant x < 16\}$

(a) List the members of $A \cap B$

$A \cap B$ = Numbers in set A and B
↳ Any integer less than 12 and more than or equal to 7

7, 8, 9, 10, 11

(2)

C is a set such that $C \subset A$ and $n(C) = 3$

Given that all members of C are even numbers,

(b) list the members of one possible set C.

C is a subset of A.
3 even numbers in C, less than 7

2, 4, 6

(1)

(Total for Question 14 is 3 marks)

15 Use algebra to show that the recurring decimal $0.2\dot{5}\dot{4} = \frac{14}{55}$

$x = 0.2\dot{5}\dot{4} = 0.25454\ldots$

$10x = 2.\dot{5}\dot{4} = 2.54545\ldots$

$1000x = 254.\dot{5}\dot{4} = 254.54545\ldots$

both align

$$\begin{aligned} 1000x &= 254.54\ldots \\ - \quad 10x &= 2.54\ldots \end{aligned}$$

cancels out as they all align

$990x = 252$

$\div 990$

$x = \frac{252}{990} = \frac{14}{55}$

$\div 18$

$\div 18$

(Total for Question 15 is 2 marks)

16 Here are the first five terms of an arithmetic sequence.

7 10 13 16 19

Find the sum of the first 100 terms of this sequence.

$S_n = \frac{n}{2}(2a + (n-1)d)$

First term $= 7 = a$

difference $= 3 = d$

last term $= 100 = n$

Sum: $\frac{100}{2}(2 \times 7 + (100-1) \times 3)$

$= 50 \times 311$

$= 15\,550$

15,550

(Total for Question 16 is 2 marks)

DO NOT WRITE IN THIS AREA

17 **A** and **B** are two similar vases.

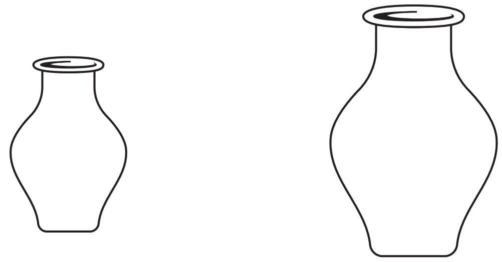


Diagram **NOT** accurately drawn

Vase **A** has height 24 cm.
Vase **B** has height 36 cm.

Vase **A** has a surface area of 960 cm^2

(a) Work out the surface area of vase **B**.

Linear scale : $36 \div 24 = 3/2$

Area scale : $(3/2)^2 = (9/4)$

Surface area of B : $960 \times \frac{9}{4} =$

2160 cm^2

(2)

Vase **B** has a volume of V cm^3

(b) Find in terms of V, an expression for the volume, in cm^3, of vase **A**.

inverse linear scale (B to A) : $2/3$

volume scale : $\left(\frac{2}{3}\right)^3 = \frac{8}{27}$

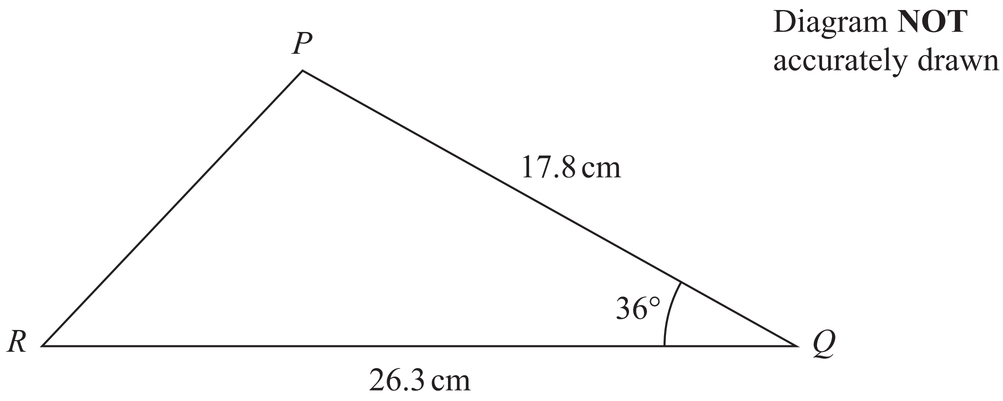
Volume of A : $V \times \frac{8}{27}$

$\frac{8}{27} V$ cm^3

(2)

(Total for Question 17 is 4 marks)

18 The diagram shows triangle PQR.



Calculate the length of PR.
Give your answer correct to 3 significant figures.

$a^2 = b^2 + c^2 - 2bc \cos A$

$(PR)^2 = 17.8^2 + 26.3^2 - 2 \times 17.8 \times 26.3 \times \cos 36$

$= 1008.53 - 936.28 \cos 36$

$(PR)^2 = 251.06356...$

$PR = 15.84498...$ cm

round down

3 sf

15.8 cm

(Total for Question 18 is 3 marks)

DO NOT WRITE IN THIS AREA

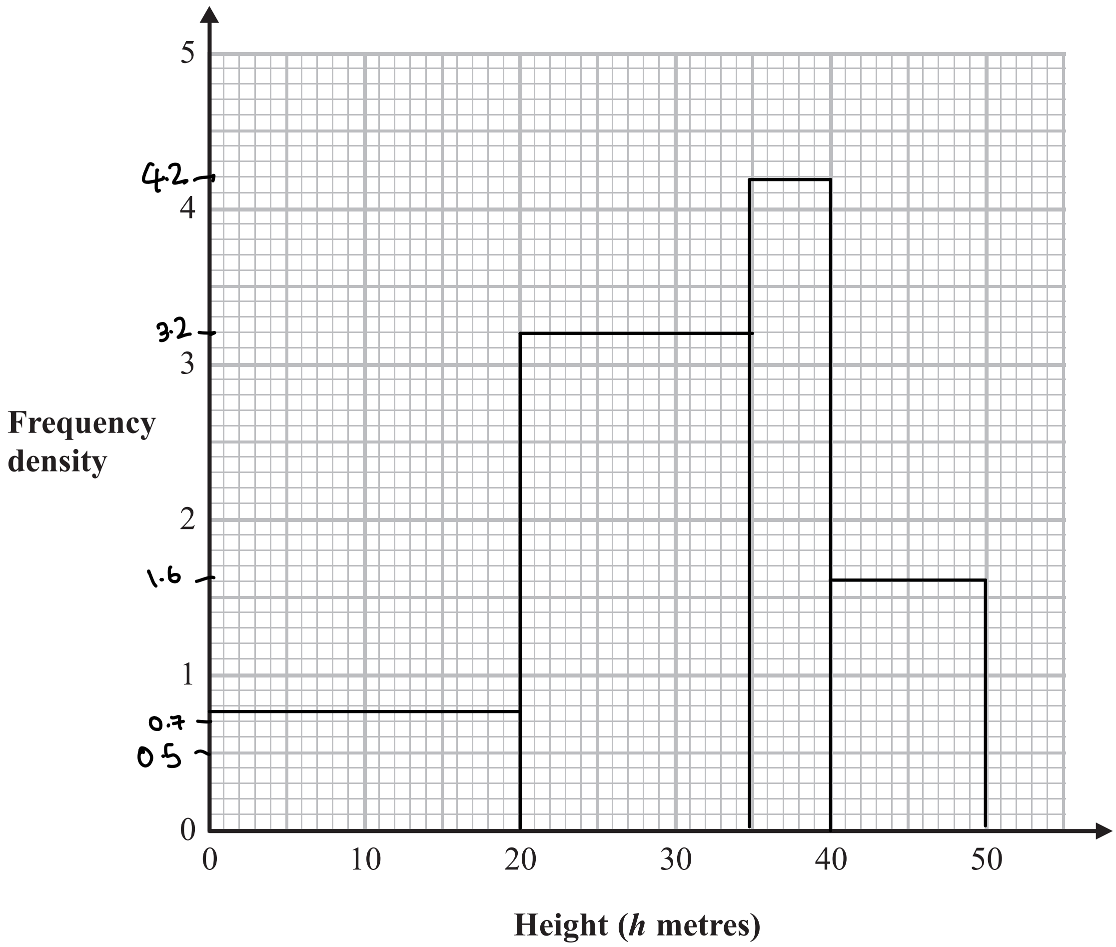
19 The table gives information about the heights of some trees.

Freq Density = Freq / class width

Height (*h* metres)	Frequency	FD
$0 < h \leqslant 20$ (20)	15	15 ÷ 20 = 0·75
$20 < h \leqslant 35$ (15)	48	3·2
$35 < h \leqslant 40$ (5)	21	4·2
$40 < h \leqslant 50$ (10)	16	1·6

→ class width

On the grid, draw a histogram for this information.



(Total for Question 19 is 3 marks)

20

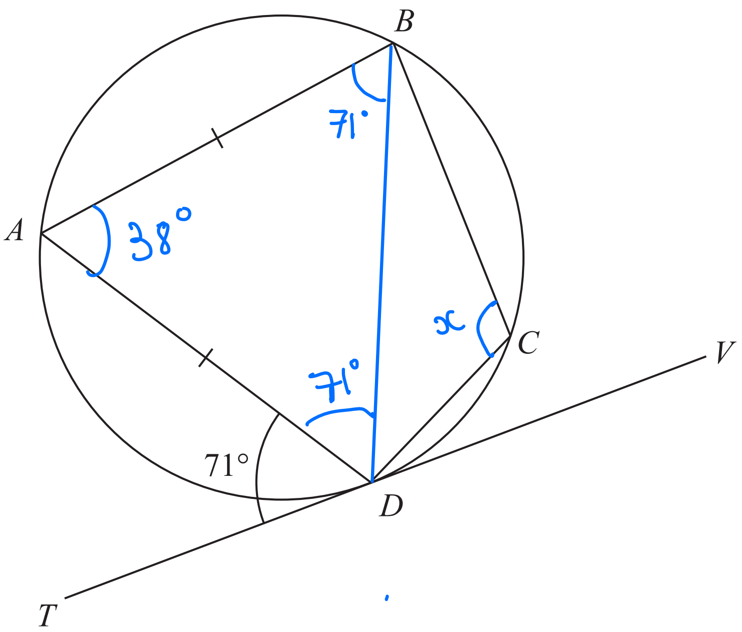


Diagram **NOT** accurately drawn

A, *B*, *C* and *D* are points on a circle.
TDV is the tangent to the circle at *D*.

$AB = AD$
Angle $ADT = 71°$

Work out the size of angle *BCD*.
Give a reason for each stage of your working.

Angle ABD = 71° Alternate angle theorm

Angle ADB = 71° Base angles in an isosceles triangle are equal

Angle BAD = 180 − 71 − 71 = 38° Angles in a triangle add to 180°

Angle BCD = 180 − 38 = 142° Opposite angles in cyclic quadrilateral add to 180°

142 °

(Total for Question 20 is 5 marks)

DO NOT WRITE IN THIS AREA

21 A solid is made from a hemisphere and a cylinder.
The plane face of the hemisphere coincides with the upper plane face of the cylinder.

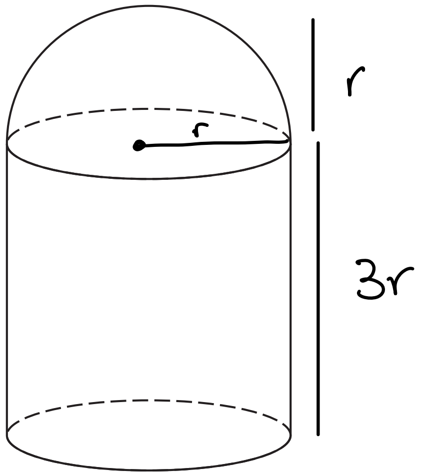


Diagram **NOT** accurately drawn

The hemisphere and the cylinder have the same radius.

The ratio of the radius of the cylinder to the height of the cylinder is 1 : 3

Given that the solid has volume $792\pi\,\text{cm}^3$
work out the height of the solid.

Volume of hemisphere: $\frac{2}{3}\pi r^3$

Volume of cylinder : $\pi r^2 h$

Volume of hemisphere: $\frac{2}{3}\pi r^3$

Volume of cylinder: $\pi r^2 \times 3r = 3r^3\pi$

Total Volume $= \frac{2}{3}\cancel{\pi} r^3 + 3r^3\cancel{\pi} = 792\cancel{\pi}$

$\frac{2}{3}+3 \quad \frac{11}{3} r^3 = 792$

$\div\frac{11}{3}$

$r^3 = 216$

$\sqrt[3]{\ }$

$r = 6$

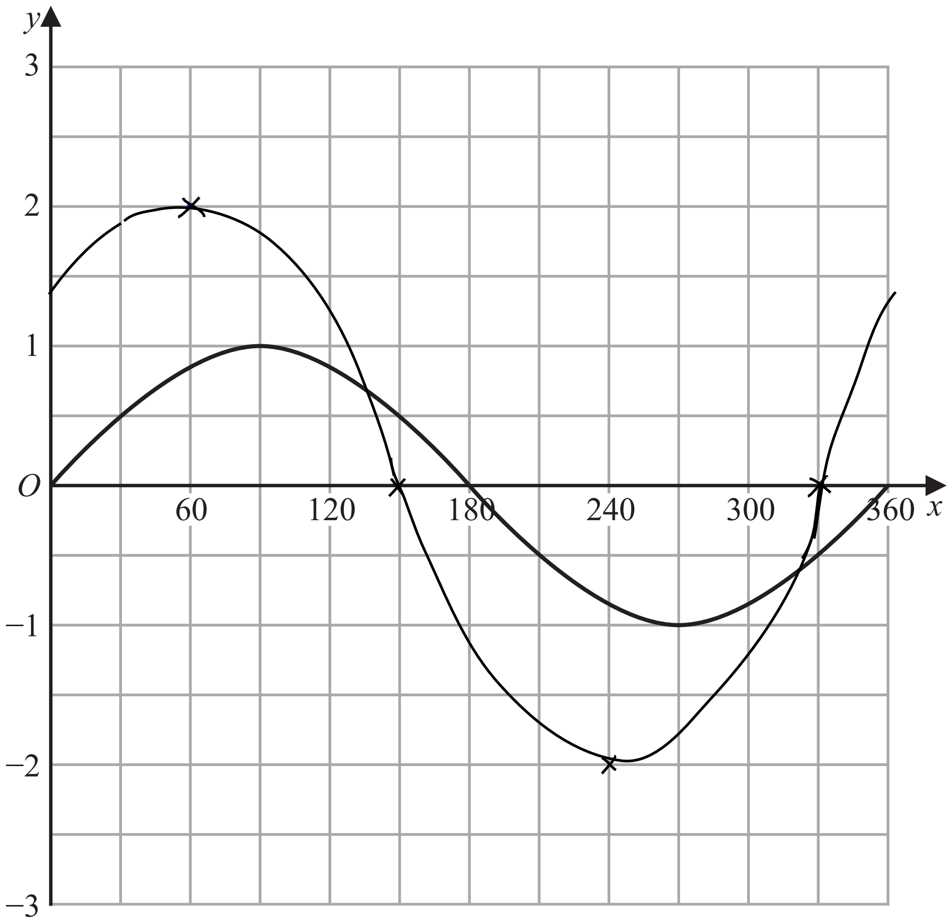
Cylinder height $= 3 \times 6 = 18$

Solid height : $18+6$

24 cm

(Total for Question 21 is 5 marks)

22 The graph of $y = \sin x°$ for $0 \leqslant x \leqslant 360$ is drawn on the grid.



(a) On the grid, draw the graph of $y = 2\sin(x + 30)°$ for $0 \leqslant x \leqslant 360$

Stretch s.f. 2

$\begin{pmatrix} -30 \\ 0 \end{pmatrix}$

(2)

(b) (i) Write $x^2 - 6x + 10$ in the form $(x - a)^2 + b$ where a and b are integers.

$$(x-3)^2 - 9 + 10 = (x-3)^2 + 1$$

$\frac{6x}{2x}$ $\quad -(-3)^2$

$(x-3)^2 + 1$

(2)

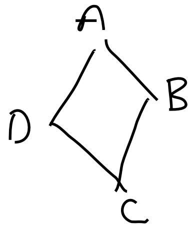
(ii) Hence, describe fully the single transformation that maps the curve with equation $y = x^2$ onto the curve with equation $y = x^2 - 6x + 10$

Translation of $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$.

(2)

(Total for Question 22 is 6 marks)

23 *ABCD* is a kite with $AB = AD$ and $CB = CD$.



B is the point with coordinates (10, 19)
D is the point with coordinates (2, 7)

Find an equation of the line *AC*.
Give your answer in the form $py + qx = r$ where p, q and r are integers.

midpoint: $\left(\frac{10+2}{2}, \frac{7+19}{2}\right) = (6, 13)$

the line AC passes through the midpoint of BD

gradient BD: $\frac{19-7}{10-2} = \frac{12}{8} = 1.5$

gradient AC: $m \times 1.5 = -1$

$m = -\frac{2}{3}$

$y = mx + c$

$13 = -\frac{2}{3}(6) + c$

$13 = -4 + c$

$c = 17$

$y = -\frac{2}{3}x + 17$

$3y = -2x + 51$

$2x + 3y = 51$

$2x + 3y = 51$

(Total for Question 23 is 5 marks)

24 A particle P is moving along a straight line that passes through the fixed point O.
The displacement, s metres, of P from O at time t seconds is given by

$$s = t^3 - 6t^2 + 5t - 4$$

Find the value of t for which the acceleration of P is 3 m/s²

$V = \frac{ds}{dt}$ differentiate $ax^n \rightarrow anx^{n-1}$

$= 3t^2 - 12t + 5$

$a = \frac{dv}{dt}$ differentiate $ax^n \rightarrow anx^{n-1}$

$= 6t - 12$

$3 = 6t - 12$

$15 = 6t$

$t = 2.5$

$t =$ 2.5

(Total for Question 24 is 4 marks)

TOTAL FOR PAPER IS 100 MARKS