MODEL SOLUTIONS

H

OCR
Oxford Cambridge and RSA

# GCSE (9–1) Mathematics

**J560/04** Paper 4 (Higher Tier)

## Thursday 24 May 2018 – Morning

**Time allowed: 1 hour 30 minutes**

*70471959825

**You may use:**
- a scientific or graphical calculator
- geometrical instruments
- tracing paper

*J560/04*

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| First name | | | | | | | | | |
| Last name | | | | | | | | | |
| Centre number | | | | | | Candidate number | | | | |

**INSTRUCTIONS**
- Use black ink. You may use an HB pencil for graphs and diagrams.
- Complete the boxes above with your name, centre number and candidate number.
- Answer **all** the questions.
- Read each question carefully before you start to write your answer.
- Where appropriate, your answers should be supported with working. Marks may be given for a correct method even if the answer is incorrect.
- Write your answer to each question in the space provided. Additional paper may be used if required but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the barcodes.

**INFORMATION**
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- Use the $\pi$ button on your calculator or take $\pi$ to be 3.142 unless the question says otherwise.
- This document consists of **20** pages.

Answer **all** the questions.

**1** **(a)** The ratio 2 centimetres to 5 metres can be written in the form 1 : *n*.

Find the value of *n*.

2cm : 500cm  1m = 100cm

1 : 250  Don't need to include units.

n = 250

**(a)** *n* = 250 **[2]**

**(b)** Jay, Sheila and Harry share £7200 in the ratio 1 : 2 : 5.

How much does Harry receive?

1 + 2 + 5 = 8 total parts

7200 ÷ 8 = 900  900 is the value of 1 part.

Harry gets 5 parts, so he gets 900 X 5 = £4500

**(b)** £ 4500 **[2]**

**2** Given that $y^{18} \div y^{6} = y^{k}$ , find the value of *k*.

$y^{18} \div y^{6} = y^{18-6}$

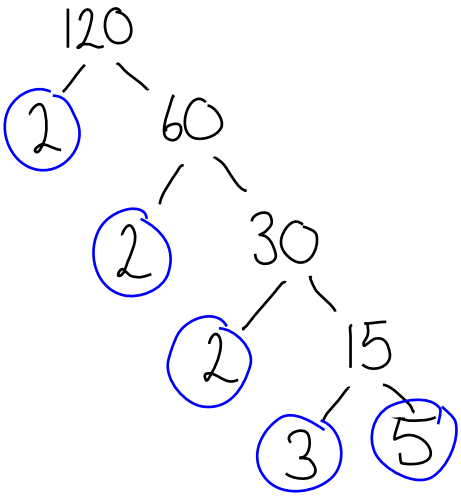
$= y^{12}$

K = 12

$x^{a} \div x^{b} = x^{a-b}$

*k* = 12 **[1]**

**3** **(a)** **(i)** Write 120 as a product of its prime factors.



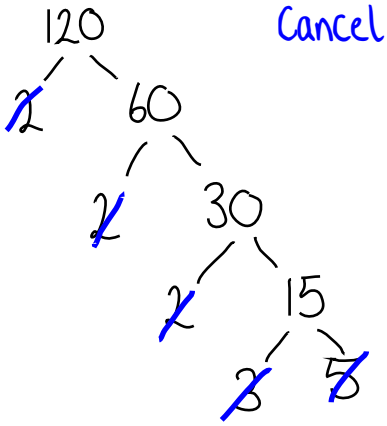

$120 = 2 \times 2 \times 2 \times 3 \times 5$

$= 2^3 \times 3 \times 5$

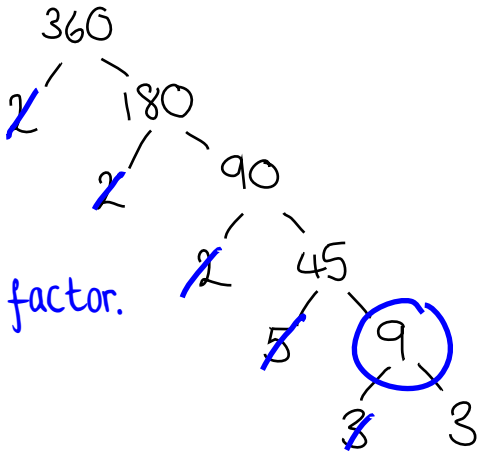
**(a)(i)** $2^3 \times 3 \times 5$ **[3]**

**(ii)** The lowest common multiple (LCM) of $x$ and 120 is 360.

Find the smallest possible value of $x$.




Cancel out common factors.




Choose highest remaining factor.

$9 > 3$ so $x = 9$

**(ii)** 9 **[2]**

**(b)** Two numbers, $A$ and $B$, are written as a product of prime factors.

$A = 2^4 \times 3^2 \times 7^2$ $B = 2^3 \times 3 \times 5 \times 7$

Find the highest common factor (HCF) of $A$ and $B$.

$A = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 7 \times 7$

$B = 2 \times 2 \times 2 \times 3 \times 5 \times 7$

Common factors are circled.

HCF = A AND B (the prime factors that are common.)

$= 2 \times 2 \times 2 \times 3 \times 7$

$= 168$

**(b)** 168 **[2]**

**4** Lee wishes to find out if there is a relationship between a person's age and the time it takes them to complete a puzzle.

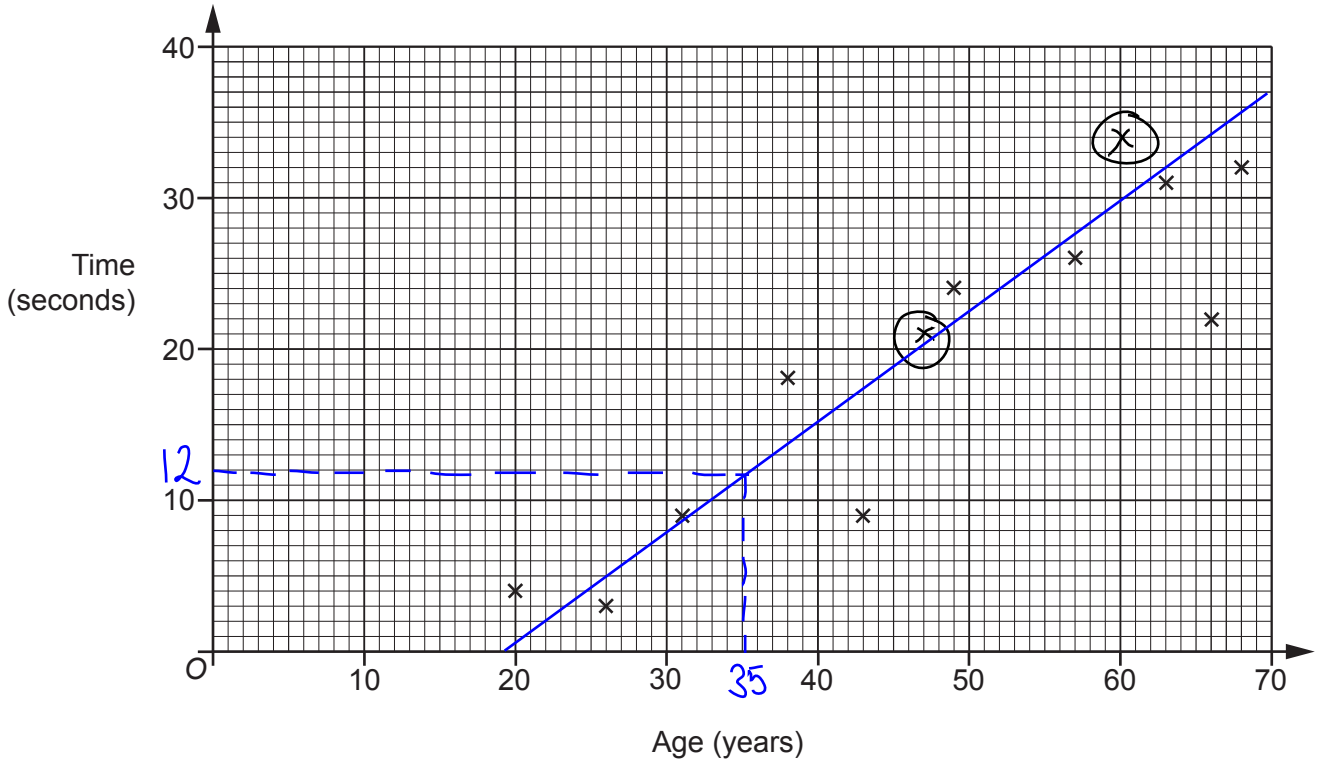
Lee decides to conduct an experiment.
She asks 12 people to complete the puzzle.
She records each person's age and the time taken to complete the puzzle.

**(a)** Make one criticism of Lee's method.

Her sample size is too small.

..... **[1]**

**(b)** This scatter diagram shows the results for ten of the people in Lee's experiment.




Here are the other two results.

| Age (years) | 47 | 60 |
|---|---|---|
| Time (seconds) | 21 | 34 |

Plot these results on the scatter diagram. **[2]**

**(c)** What type of correlation is shown in the scatter diagram?

As age increases, so does time, so correlation is positive.

**(c)** Positive **[1]**

**(d)** Estimate the time it would take a person aged 35 to complete the puzzle.

Show your working to justify your answer.

Draw a line of best fit. Find the time when age is 35 years.

**(d)** 12 seconds **[2]**

**(e)** Lee says that at least 80% of the 12 people completed the puzzle in under 30 seconds.

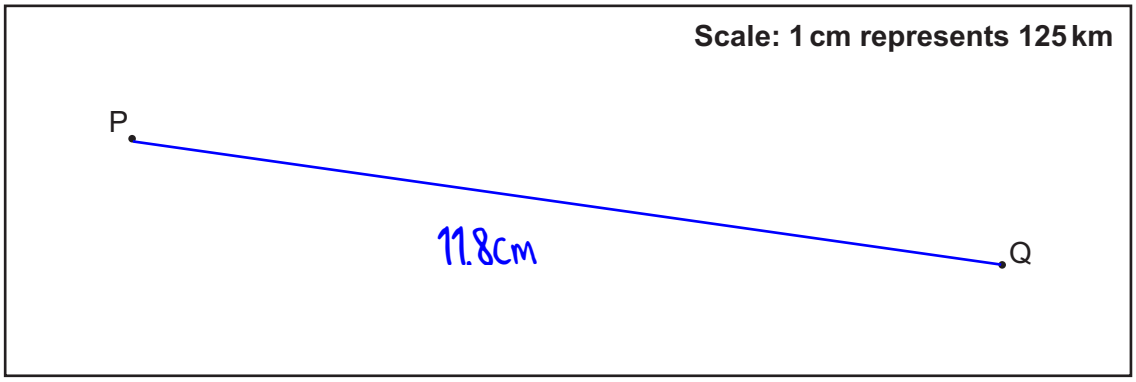
Is Lee correct?
Show working to support your answer.

$\frac{9}{12} \times 100 = 75\%$

$75\% < 80\%$ so Lee is incorrect.

**[3]**

**5** The scale diagram below shows two cities, P and Q.




A plane departs from P at 09 47 and arrives at Q at 12 07.

**(a)** Work out the average speed, in kilometres per hour, of the plane.

11.8cm = 11.8 X 125km

= 1475km

09:47 to 12:07 is 2 hours and 20 minutes

20 minutes = $\frac{20}{60} = \frac{1}{3}$ hours

speed = $\frac{\text{distance}}{\text{time}}$

= $\frac{1475}{23}$

= 632km/hour

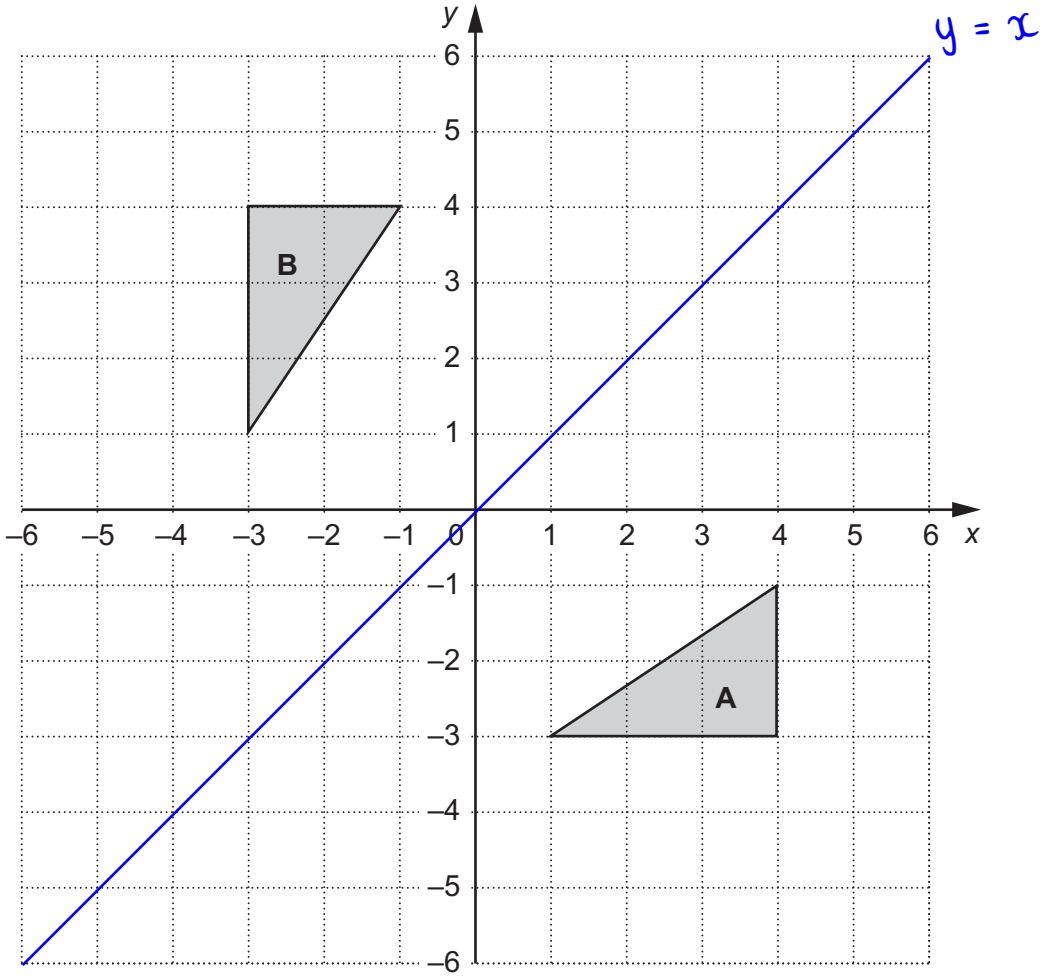
**(a)** 632 km/h **[5]**

**(b)** Give one reason why your answer may be inaccurate.

The plane may not have flown in a straight line.

**[1]**

**6** Triangles **A** and **B** are drawn on a coordinate grid.




**(a)** Describe fully the **single** transformation that maps triangle **A** onto triangle **B**.

Reflection in y = x.

..... **[2]**

**(b)** Triangle **A** can also be mapped onto triangle **B** using a combination of two transformations:

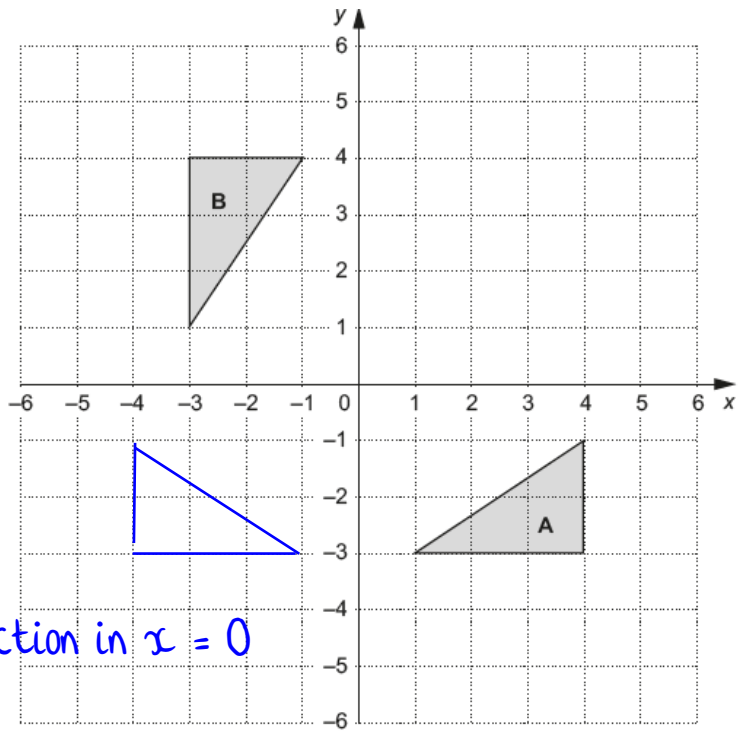
- a transformation T, followed by
- a reflection in the line $x = 0$.

Describe fully transformation T.

T: rotation about (0,0), 90° anti-clockwise.

.....

.....



**7** The scale diagram below shows towns, A, B and C.
Line AB represents the road from A to B and line AC represents the road from A to C.

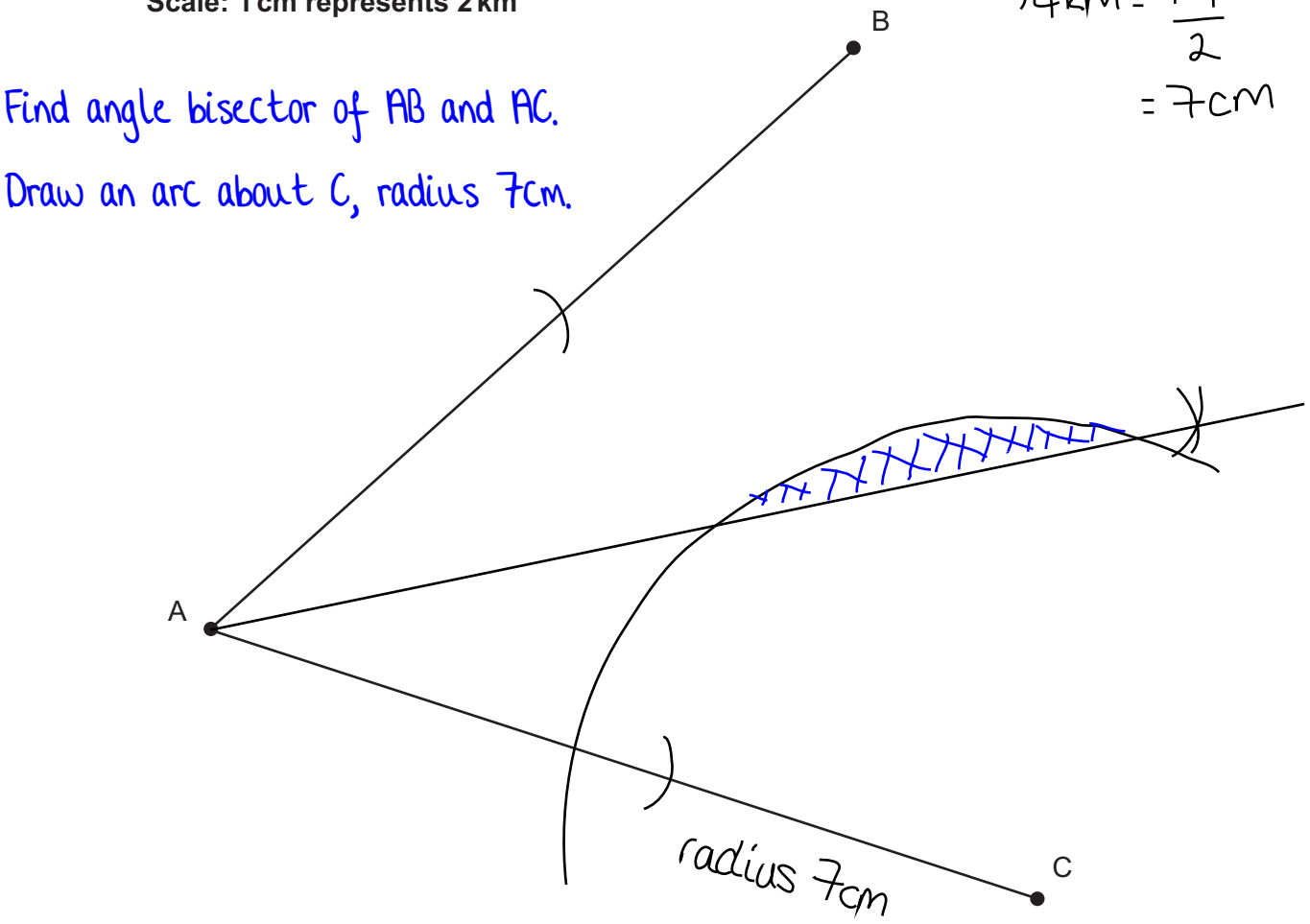
A shopping centre is to be built so that it is

- nearer to the road from A to B than the road from A to C,
- less than 14 km from town C.

**(a)** Using construction, shade the region where the shopping centre could be built.
Show all your construction lines.

**Scale: 1 cm represents 2 km**

14km = $\frac{14}{2}$ = 7cm



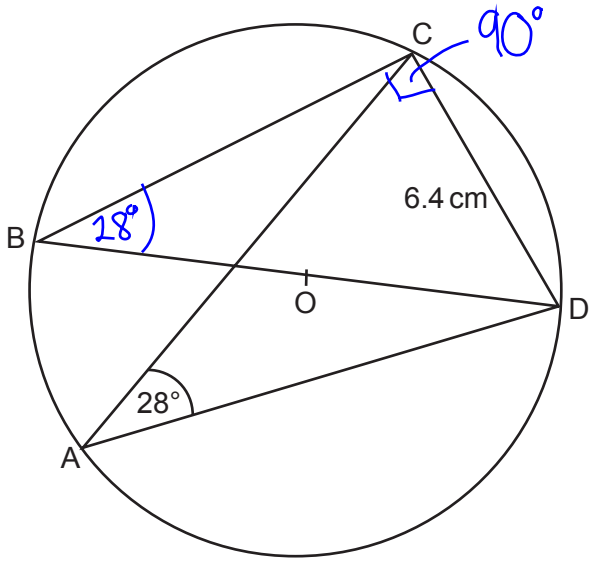

**[5]**

**(b)** Explain why the region found in part **(a)** may not be an appropriate site for the shopping centre.

.....

The roads may not be straight. **[1]**

**8** A, B, C and D are points on the circumference of a circle, centre O.




**Not to scale**

Angle CAD = 28° and CD = 6.4 cm.
BD is a diameter of the circle.

Calculate the area of the circle.

$C\hat{B}D = 28°$ Angles in the same segment are equal.

$B\hat{C}D = 90°$ Angle subtended at circumference by a semi-circle is 90°.

△BCD is right-angled.

$\sin\theta = \frac{O}{H}$

$\sin 28 = \frac{6.4}{BD}$

$\frac{6.4}{\sin 28} = BD$

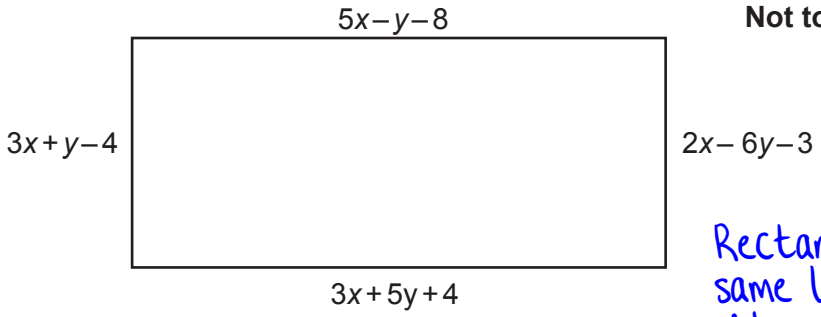
$BD = 13.6 = \text{diameter}$

$\text{radius} = \text{diameter} \div 2 = 13.6 \div 2 = 6.8$

$\text{area} = \pi r^2 = \pi \times 6.8^2 = 145.3\text{cm}^2$

145.3 ..... $cm^2$ **[5]**

**9** The dimensions, in centimetres, of this rectangle are shown as algebraic expressions.




**Not to scale**

Rectangle: vertical sides are same length and horizontal sides are same length.

Work out the length and width of the rectangle.

$5x - y - 8 = 3x + 5y + 4$

$-3x$: $2x - y - 8 = 5y + 4$

$-5y$: $2x - 6y - 8 = 4$

$+8$: $2x - 6y = 12$

$\div 2$: $x - 3y = 6$ (1)

$3x + y - 4 = 2x - 6y - 3$

$-2x$: $x + y - 4 = -6y - 3$

$+6y$: $x + 7y - 4 = -3$

$+4$: $x + 7y = 1$ (2)

(1) − (2): $x - 3y = 6$, $-x + 7y = 1$

$-10y = 5$

$\div(-10)$: $y = -\frac{1}{2}$

sub $y = -\frac{1}{2}$ into (1).

$x - 3(-\frac{1}{2}) = 6$

$x + \frac{3}{2} = 6$

$-\frac{3}{2}$: $x = \frac{9}{2}$

width $= 3 \times \frac{9}{2} + (-\frac{1}{2}) - 4 = 9\text{cm}$

length $= 5 \times \frac{9}{2} - (-\frac{1}{2}) - 8 = 15\text{cm}$

length = 15 ..... cm

width = 9 ..... cm

**[6]**

**10** 60% of the people in a town are males.
20% of the males are left-handed.
21.6% of all the people are left-handed.

Work out the percentage of the people who are not male who are left-handed.

| | Right-Handed | Left-Handed | Total |
|---|---|---|---|
| Male | | 12% | 60% |
| Not Male | | 24% | 40% |
| Total | | 21.6% | 100% |

100 - 21.6 = 78.4% right-handed, not male

0.6 X 0.2 = 0.12%

21.6 - 12 = 9.6%

9.6 ÷ 40 = 24%

24 ..... % **[5]**

**11** $y$ is directly proportional to the square of $x$.

Find the percentage increase in $y$ when $x$ is increased by 15%.

$y \propto x^2$

$y = Kx^2$ where K is a constant

$K(1.15x)^2 = 1.3225Kx$

$= 1.3225y$

32.25%

Increased by 15% is the same as 115% of x.

$115\% = \frac{115}{100} = 1.15$

32.25 % **[4]**

**12** The value of a car, £$V$, is given by

$$V = 16\,500 \times 0.82^n$$

where $n$ is the number of years after it is bought from new.

**(a)** Write down the value of the car when new.

$V = 16500 \times 0.82^0$

$= £16500$ $\quad x^0 = 1$

**(a)** £ 16500 **[1]**

**(b)** Write down the annual percentage decrease in the value of the car.

$100 - 82 = 18\%$

total $\quad 0.82 = \frac{82}{100} = 82\%$

**(b)** 18 % **[1]**

**(c)** Show that the value of the car after 4 years is less than half its value when new. **[2]**

$V = 16500 \times 0.82$

$= £7460.01$

$\frac{7460.01}{16500} = 0.452$

$0.452 < 0.5$

**13** A menu has

- 6 starters
- 10 main dishes
- 7 desserts.

**(a)** A three-course meal consists of a starter, a main dish and a dessert.

How many different three-course meals are possible?

$6 \times 10 \times 7 = 420$ Multiply to find number of combinations.

**(a)** 420 **[2]**

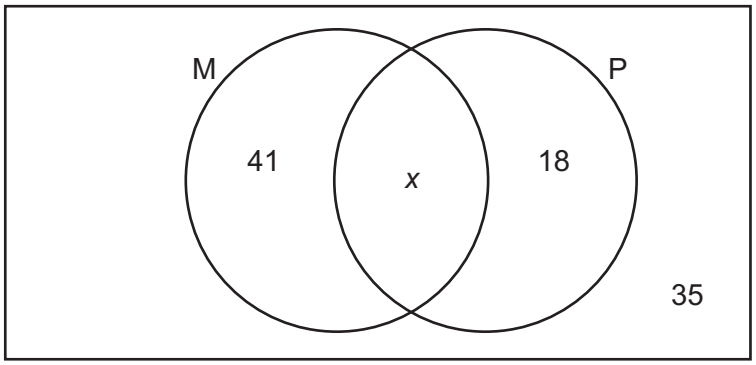
**(b)** A two-course meal consists either of a starter with a main dish, a starter with a dessert or a main dish with a dessert.

Show that there are 172 possible different two-course meals. **[3]**

(starter AND main) OR (starter AND dessert) OR (main AND desert)

$(6 \times 10) + (6 \times 7) + (10 \times 7) = 172$

**14** The Venn diagram shows the number of students studying Mathematics (M) and the number of students studying Physics (P) in a college.
35 students do not study either subject.




**(a)** The total number of students is 121.

Find the value of $x$.

$x + (41 + 18 + 35) = 121$ total number of students

$x = 121 - (94)$

$= 27$

**(a)** $x =$ 27 **[1]**

**(b)** One of the 121 students is selected at random.

Find the probability that this student studies Mathematics, given that they study Physics.

M+P

$\frac{27}{27+18} = \frac{3}{5}$

P

**(b)** $\frac{3}{5}$ **[2]**

**15** **(a)** Write $x^2 - 8x + 25$ in the form $(x - a)^2 + b$.

$= x^2 - 8x + 25$

$= (x - 4)^2 - 4^2 + 25$ halve the b term

$= (x - 4)^2 - 16 + 25$ remove the extra product $a^2$

$= (x - 4)^2 + 9$

**(a)** $(x - 4)^2 + 9$ **[3]**

**(b)** Write down the coordinates of the turning point of the graph of $y = x^2 - 8x + 25$.

$(x - 4)^2 + 9 = (x + q)^2 + p$

turning point $= (-p, q)$

$(4, 9)$

**(b)** ( 4 , 9 ) **[2]**

**(c)** Hence describe the single transformation which maps the graph of $y = x^2$ onto the graph of $y = x^2 - 8x + 25$.

Translated by the vector $\begin{pmatrix} 4 \\ 9 \end{pmatrix}$

4 units right, 9 units up.

The turning point of $y = x$ is (0,0) so the graph has moved 4 units across and 9 units up.

**[2]**

**16** Solve by factorisation.

$$3x^2 + 11x - 20 = 0$$

$3x^2 + 11x - 20 = 0$

$3x^2 + 15x - 4x - 20 = 0$

$3x(x + 5) - 4(x + 5) = 0$

$(3x - 4)(x + 5) = 0$

$3x - 4 = 0$ so $x = \frac{4}{3}$

$+4, \div 3$

$x + 5 = 0$ so $x = -5$

$-5$

Two numbers must multiply to make the c term (-20) and add to make the b term (11).

$15 + (-4) = 11$

$15 \times (-4) = -20$

3

Split the b-term.

Factorise each part.
Brackets must be the same.

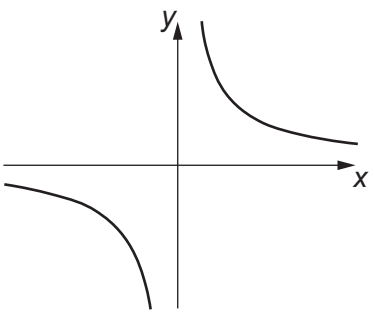
$x =$ $\frac{4}{3}$ or $x =$ $-5$ **[3]**

**17** For each graph below, select its possible equation from this list.

$y = \frac{1}{x}$ $y = \cos x$ $y = x^2$ $y = \left(\frac{1}{2}\right)^x$ $y = 2^x$

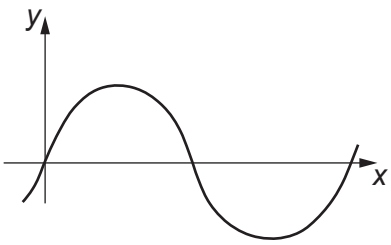
$y = \sin x$ $y = 2^{-x}$ $y = \tan x$ $y = x^3$ $y = \frac{1}{x^2}$

**(a)**



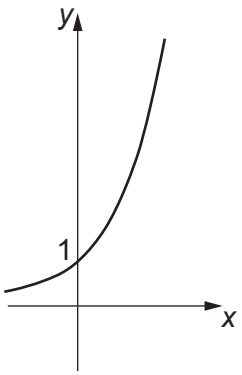

**(a)** $y =$ $\frac{1}{x}$ Reciprocal graph.

**(b)**




**(b)** $y =$ $\sin x$ Sine graph.

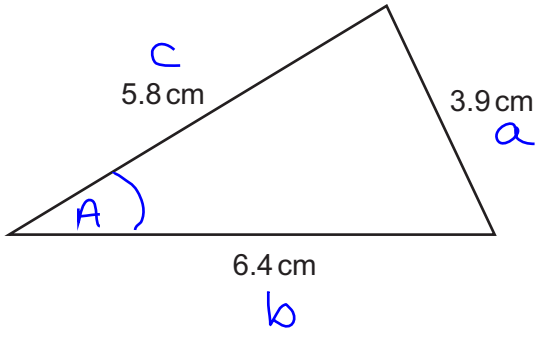
**(c)**




**(c)** $y =$ $2^x$ Exponential graph.

**[3]**

**18** Calculate the area of this triangle.




**Not to scale**

$a^2 = b^2 + c^2 - 2bc\cos A$

$3.9^2 = 6.4^2 + 5.8^2 - (2 \times 6.4 \times 5.8 \times \cos A)$

$15.21 = 74.6 - 74.24\cos A$ ) $-74.6$

$-59.39 = -74.24\cos A$ ) $\div -74.24$

$\cos A = -59.39 \div -74.24$

$\cos^{-1}\left(\frac{-59.39}{-74.24}\right) = A$ ) $\cos^{-1}$ both sides

$A = 36.9°$

$\text{area} = \frac{1}{2}ab\sin C$

$= \frac{1}{2} \times 5.8 \times 6.4 \times \sin 36.9$

$= 11.1 \text{cm}^2$

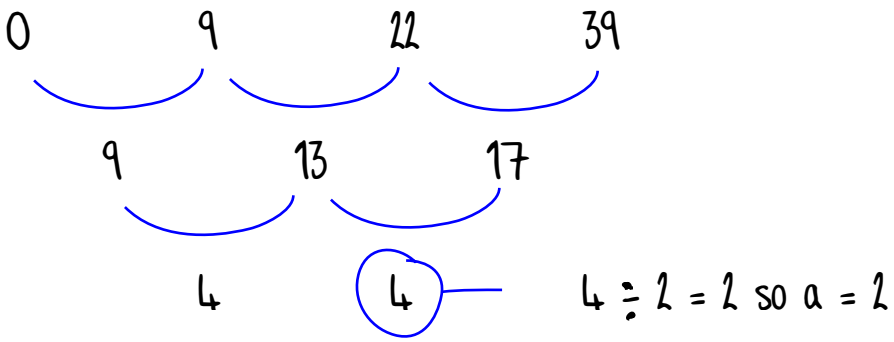
11.1 ..... $\text{cm}^2$ **[6]**

**19** Here are the first four terms of a quadratic sequence.

0 9 22 39

The $n$th term can be written as $an^2 + bn + c$.

Find the values of $a$, $b$ and $c$.




| nth term: | 0 | 9 | 22 | 39 | |
|---|---|---|---|---|---|
| $2n^2$: | 2 | 8 | 18 | 32 | |
| difference: | -2 | 1 | 4 | 7 | |
| | 3 | 3 | 3 | | so b = 3 |

| nth term: | 0 | 9 | 22 | 39 | |
|---|---|---|---|---|---|
| $2n^2 + 3n$: | 5 | 14 | 27 | 44 | |
| difference: | -5 | -5 | -5 | -5 | so c = -5 |

$2n^2 + 3n - 5$

$a$ = 2

$b$ = 3

$c$ = -5

**[4]**

**20** Solve this equation, giving your answers correct to 1 decimal place.

$$\frac{5}{x+2}+\frac{3}{x-3}=2$$

$$\frac{5}{x+2}+\frac{3}{x-3}=2$$

$$\frac{5(x-3)+3(x+2)}{(x+2)(x+3)}=2$$ ) multiply by denominator

$$5(x-3)+3(x+2)=2(x+2)(x-3)$$

$$5x-15+3x+6=2(x^2-x-6)$$

$-8x$ $+9$ (

$$8x-9=2x^2-2x-12$$

$$2x^2-10x-3=0 \quad (ax^2+bx+c=0)$$

$$x=\frac{-(-10)\pm\sqrt{(-10)^2-4\times2\times(-3)}}{2\times2}$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=5.3,\ -0.3$$

$x =$ 5.3 or $x =$ −0.3 **[6]**

**END OF QUESTION PAPER**