MODEL SOLUTIONS

H

OCR
Oxford Cambridge and RSA

GCSE (9–1) Mathematics

J560/05 Paper 5 (Higher Tier)

Thursday 7 June 2018 – Morning

Time allowed: 1 hour 30 minutes

You may use:
- geometrical instruments
- tracing paper

Do not use:
- a calculator

First name	
Last name	
Centre number	
Candidate number	

INSTRUCTIONS
- Use black ink. You may use an HB pencil for graphs and diagrams.
- Complete the boxes above with your name, centre number and candidate number.
- Answer **all** the questions.
- Read each question carefully before you start to write your answer.
- Where appropriate, your answers should be supported with working. Marks may be given for a correct method even if the answer is incorrect.
- Write your answer to each question in the space provided. If additional space is required, use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- Do **not** write in the barcodes.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document consists of **20** pages.

No calculator can be used for this paper

Answer **all** the questions.

1 **(a)** Calculate.

$$\frac{3}{5}+\frac{5}{8}$$

Give your answer as a mixed number in its simplest form.

$$\frac{3}{5}+\frac{5}{8} = \frac{3\times 8}{5\times 8}+\frac{5\times 5}{8\times 5}$$

Find the common denominator.

$$= \frac{24}{40}+\frac{25}{40}$$

$$= \frac{49}{40} = 1\frac{9}{40}$$

(a) ...... $1\frac{9}{40}$ ...... **[3]**

(b) Work out.

$$5\times 10^{4} - 1.6\times 10^{3}$$

Give your answer in standard form.

$$5 \times 10^{4} - 1.6 \times 10^{3} = 50 \times 10^{3} - 1.6 \times 10^{3}$$

Convert to same power of 10.

$$= (50 - 1.6) \times 10^{3}$$

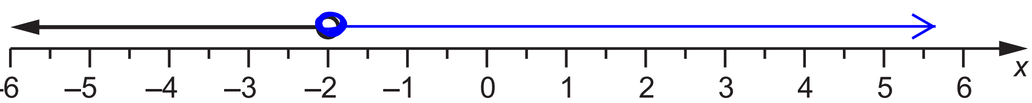
$$= 48.4 \times 10^{3}$$

Back into standard form.

$$= 4.84 \times 10^{4}$$

(b) ...... 4.84×10^{4} ...... **[3]**

2 Gemma's solution to the inequality $3x + 1 > -5$ is shown on the number line.



Correct solution in blue.

Is Gemma's solution correct?
Explain your reasoning.

$$3x + 1 > -5$$

$$3x > -6$$

$$x > -2$$

No, Gemma's solution is incorrect, as she has shown $x < -2$.

...... **[3]**

3 Work out.

(a) $\begin{pmatrix} -3 \\ 2 \end{pmatrix} + \begin{pmatrix} 5 \\ 7 \end{pmatrix}$

$= \begin{pmatrix} -3+5 \\ 2+7 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ 9 \end{pmatrix}$

(a) $\begin{pmatrix} 2 \\ 9 \end{pmatrix}$ **[1]**

(b) $\begin{pmatrix} 3 \\ 4 \end{pmatrix} - 2\begin{pmatrix} 1 \\ -3 \end{pmatrix}$

$= \begin{pmatrix} 3 \\ 4 \end{pmatrix} - \begin{pmatrix} 2 \times 1 \\ 2 \times (-3) \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} - \begin{pmatrix} 2 \\ -6 \end{pmatrix}$

$= \begin{pmatrix} 3-2 \\ 4-(-6) \end{pmatrix} = \begin{pmatrix} 1 \\ 10 \end{pmatrix}$

(b) $\begin{pmatrix} 1 \\ 10 \end{pmatrix}$ **[2]**

4 Here is the nutritional information for a 110 g serving of cereal.

Carbohydrates	99.4 g
Proteins	9.5 g
Fats	1.1 g

Emily says that more than 90% of this serving is carbohydrates.

Is she correct?
Explain your reasoning.

$10\% = 110 \div 10$
$= 11g$

$90\% = 9 \times 11$
$= 99g$

$\underbrace{99.4}_{\text{carbohydrates}} < 99$, so she is correct.

[3]

5 The table shows the relative frequencies of the results for a football team after a number of games.

Result of game	won	lost	drew
Relative frequency	0.2	0.45	0.35

(a) Complete the table. [2]

Probabilities must add to 1

$0.2 + 0.45 = 0.65$

$1 - 0.65 = 0.35$ games are drawn.

(b) The team lost 10 more games than they won.

How many games did the team play altogether?

w = won l = lost

$l = w + 10$

sub in values

$0.45 = 0.2 + 10$

-0.2

$0.25 = 10$ games

$0.25 = \frac{1}{4}$

$\times 4$

Multiply by 4 to make frequency equal to 1.

$10 \times 4 = 40$ games

(b) ....40 games........ [3]

6 Jack sent 15% more text messages in March than in February.
Jack sent 460 text messages in March.

How many more texts did Jack send in March than in February?

15% more = 115%

115% = 460 texts

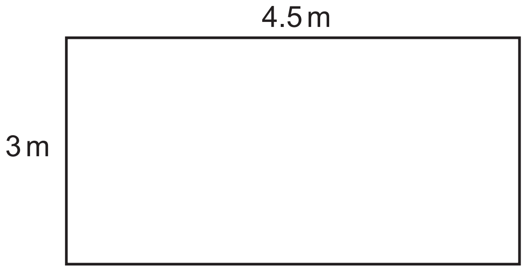
$460 \div 115 = 4$ texts so 1% = 4 texts

$100\% = 4 \times 100 = 400$ texts

$460 - 400 = 60$ more texts in March than in February

........60........ [4]

7 Here is the floor plan of a rectangular room.



Not to scale

Tim buys carpet tiles for this room.

Each tile is a square measuring 50 cm by 50 cm.
The tiles are only sold in packs of ten.
Each pack costs £20.
Tim pays for fitting at a rate of £7.50 per square metre, with any fraction of a square metre rounded up.

Work out the **total** cost of the tiles and fitting.

3m = 300cm   1 tile = 50cm   300 ÷ 50 = 6 so 6 tiles fit along the width

4.5m = 400cm   1 tile = 50cm   450 ÷ 50 = 9 so 9 tiles fit along the length

6 X 9 = 54 tiles in the room

54 ÷ 10 = 5.4 packs   round up to 6 packs

£20 X 6 = £120 to buy the tiles

3m X 4.5m = 13.5m^2 area = width x height

13.5 X £7.50 = £105 to fit the tiles

£120 + £105 = £225 total cost

£ ....225........................ **[6]**

8 Hannah wants to display all the possible outcomes when rolling two fair 6-sided dice.

(a) Give a reason why a tree diagram is not the best method to use.

There will be too many outcomes (or branches). **[1]**

(b) **(i)** Draw a sample space to display all the possible outcomes. **[2]**

	1	2	3	4	5	6
1	1, 1	2, 1	3, 1	4, 1	5, 1	6, 1
2	1, 2	2, 2	3, 2	4, 2	5, 2	6, 2
3	1, 3	2, 3	3, 3	4, 3	5, 3	6, 3
4	1, 4	2, 4	3, 4	4, 4	5, 4	6, 4
5	1, 5	2, 5	3, 5	4, 5	5, 5	6, 5
6	1, 6	2, 6	3, 6	4, 6	5, 6	6, 6

(ii) Show that the probability of the scores on the two dice adding to 11 is $\frac{1}{18}$.

Two possible combinations: $6 + 5 = 11$
$5 + 6 = 11$

2 out of 36 combinations $= \frac{2}{36} \overset{\div 2}{=} \frac{1}{18}$

$\div 2$

[2]

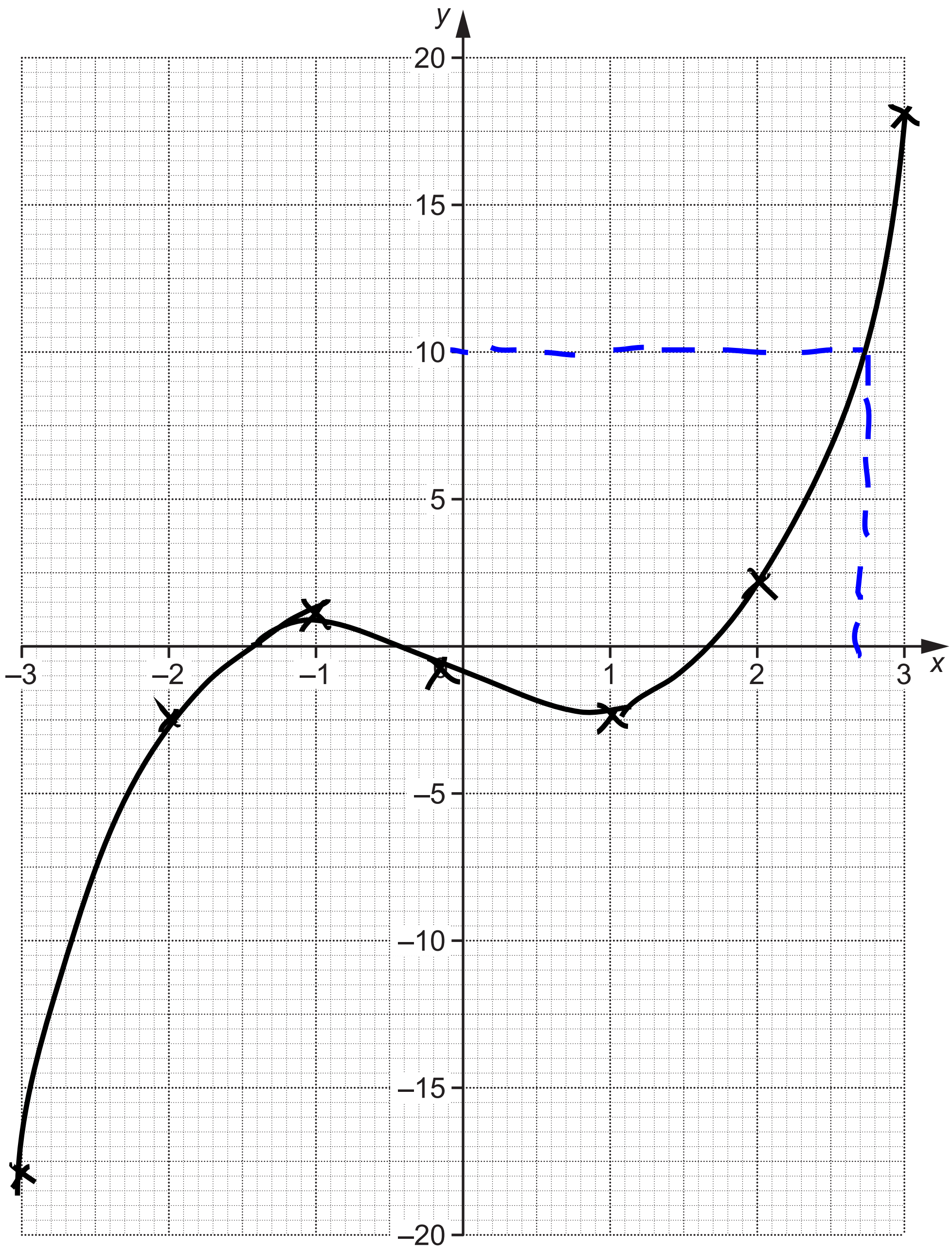
9 **(a)** Complete the table for $y = x^3 - 3x$.

$y = (-1) - 3(-1) = (-1) - (-3)$
$= -1 + 3$
$= 2$

x	-3	-2	-1	0	1	2	3
y	-18	-2	2	0	-2	2	18

[1]

(b) Draw the graph of $y = x^3 - 3x$ for $-3 \leqslant x \leqslant 3$.



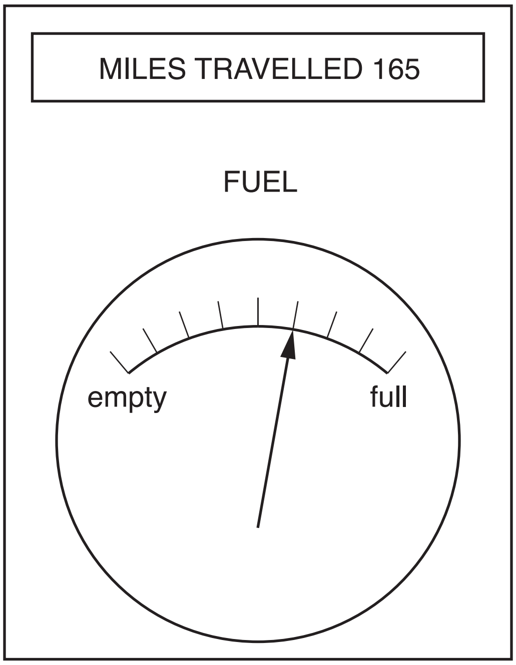
[3]

(c) Use your graph to solve $x^3 - 3x = 10$.

Find x when $y = 10$

(c) $x = $ ....2.6.......... **[1]**

10 Ifsaw noticed this information on her car's dashboard at the end of her journey.
She started her journey with a full tank of fuel and her miles travelled set to zero.



(a) Work out how far Ifsaw's car can travel on a full tank of fuel.

$\frac{3}{8}$ of a tank = 165 miles

165 ÷ 3 = 55

$\frac{1}{8}$ of a tank = 55 miles

55 X 8 = 440 miles with a full tank.

$$3 \overline{)165} = 055$$

$$55 \times 8 = 440$$

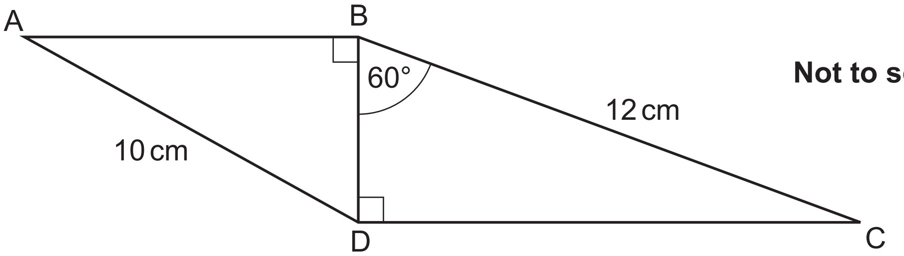
(a) 440 miles **[3]**

(b) What assumption have you made when answering part **(a)**?

The rate of fuel consumption remains constant.

[1]

11 The diagram shows two right-angled triangles ABD and BCD, sharing a common side BD.
AD = 10 cm, BC = 12 cm and angle DBC = 60°.



Work out the length of AB.

$\cos\theta = \frac{A}{H}$

$\cos 60 = \frac{BD}{12}$

$\times 12 \quad \times 12$

$12\cos 60 = BD$

$12 \times \frac{1}{6} = 6 \quad 6 = BD$

Pythagoras Theorem

$a^2 + b^2 = c^2$

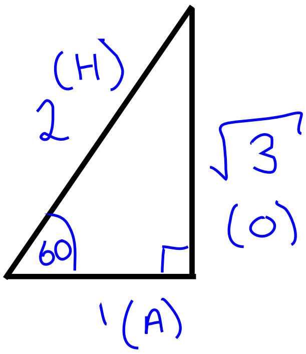
$-b \quad -b$

$a^2 = c^2 - b^2$

$a = \sqrt{c^2 - b^2}$

$AB = \sqrt{10^2 - 6^2}$

$= 8cm$



$\cos = \frac{A}{H}$

$\cos 60 = \frac{1}{2}$

8 cm **[6]**

12 Carol says that $64^{-\frac{1}{2}} = \frac{1}{32}$.

Explain her error and give the correct value of $64^{-\frac{1}{2}}$ in the form $\frac{p}{q}$.

$64^{-\frac{1}{2}} = \frac{1}{64^{\frac{1}{2}}} = \frac{1}{\sqrt{64}} = \frac{1}{8}$

Carol has done $64 \div 2$, instead of $\sqrt{64}$ **[3]**

13 **(a)** Write $\frac{5}{12}$ as a recurring decimal.

$$12 \overline{\big) 5.{}^{5}0{}^{2}0{}^{5}0{}^{8}0{}^{8}0\ldots}$$ quotient $0.41666\ldots$

so $\frac{5}{12} = 0.14\dot{6}$

(a) $0.14\dot{6}$ **[2]**

(b) Convert $0.\dot{7}\dot{6}$ to a fraction.

$x = 0.767676\ldots$

$100x = 76.7676\ldots$

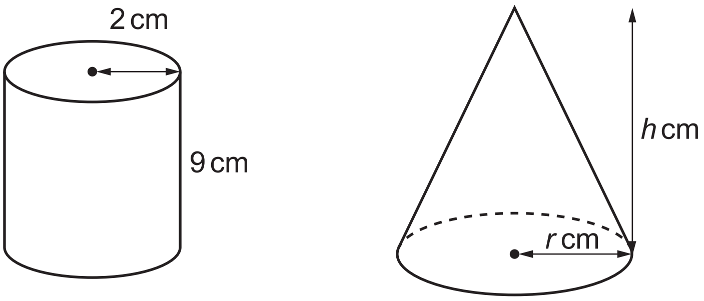
$100x - x = 76.7676\ldots - 0.767676\ldots$

$99x = 76$

$x = \frac{76}{99}$ $\div 99$

(b) $\frac{76}{99}$ **[2]**

14 The diagram shows a cylinder and a cone.



The cylinder has radius 2 cm and height 9 cm.
The cone has radius *r* cm and height *h* cm.

The ratio *r* : *h* is 1 : 4.
The volume of the cone is **equal to** the volume of the cylinder.

Work out the value of ***r***.

[The volume V of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.]

$\frac{r}{h} = \frac{1}{4}$) $\times h$

$r = \frac{1}{4}h$) $\times 4$

$4r = h$

Volume of Cylinder:

$V = \pi r^2 h$

$V = \pi \times 2^2 \times 9$

$V = 36\pi$

Volume of Cone:

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi r^2 4r$) $\frac{1}{3} \times 4 \times r^2 \times r \times \pi$

$V = \frac{4}{3}\pi r^3$

Volume of Cylinder = Volume of Cone

$36\pi = \frac{4}{3}\pi r^3$) $\times 3$

$108 = 4r^3$) $\div 4$

$r^3 = 27$) $\sqrt[3]{\ }$

$r = 3$

.......................3cm....................... **[5]**

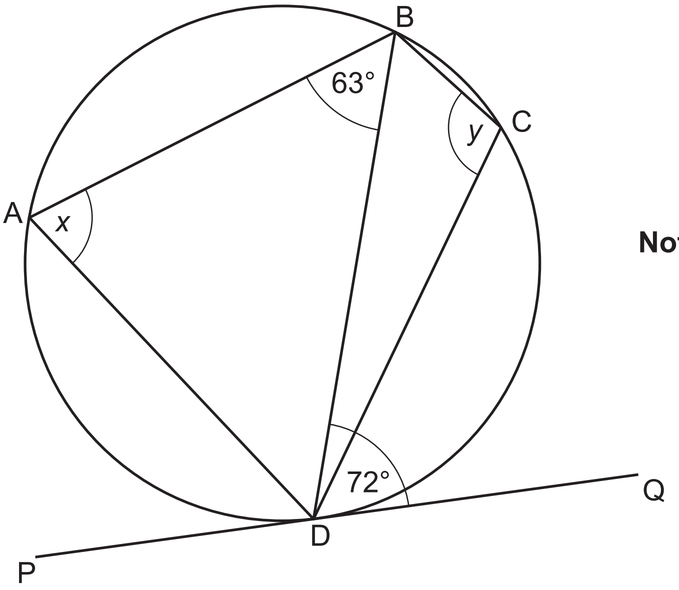
15 n is a positive integer.

Prove that $13n + 3 + (3n - 5)(2n + 3)$ is a multiple of 6. **[4]**

$$13n + 3 + (3n - 5)(2n + 3) = 13n + 3 + (6n^2 + 9n - 10n - 15)$$
$$= 6n^2 + 13n + 9n - 10n - 15$$
$$= 6n^2 + 12n - 12$$
$$= 6(n^2 + 2n + 2)$$

Therefore $13n + 3 + (3n - 5)(2n + 3)$ is a multiple of 6.

16 A, B, C and D are points on the circumference of a circle.



Not to scale

PQ is a tangent to the circle at D.
Angle BDQ = 72° and angle ABD = 63°.

(a) Work out angle *x*.
Give a reason for your answer.

Angle *x* = 72° because Alternate angles are equal (alternate angle theorem) **[2]**

(b) Work out angle *y*.
Give a reason for your answer.

180 - 72 = 108°

Angle *y* = 108° because opposite angles in a cyclic quadrilateral add up to 180° **[2]**

17 $(x+a)(x+3)(2x+1) = bx^3 + cx^2 + dx - 12$

Find the value of a, b, c and d.

$$(x + a)(x + 3)(2x + 1) = bx^3 + cx^2 + dx + 12$$

$$= (x^2 + ax + 3x + 3a)(2x + 1)$$

$$= 2x^3 + 2ax^2 + 6x^2 + 6ax + x^2 + ax + 3x + 3a$$

$$= \underbrace{2}_{b}x^3 + \underbrace{(2a + 7)}_{c}x^2 + \underbrace{(3 + 7a)}_{d}x + 3a$$

$3a = -12$
$a = -4$

$b = 2$

$c = 2(-4) + 7$
$= -8 + 7$
$= -1$

$d = 3 + 7(-4)$
$= 3 + (-28)$
$= -25$

$a =$ .... -4 ....

$b =$ .... 2 ....

$c =$ .... -1 ....

$d =$ .... -25 .... **[4]**

18 **(a)** A straight line passes through the point (0, 6) and is perpendicular to $y = 4x - 5$.

Find the equation of this line, giving your answer in the form $y = mx + c$.

gradient

y-intercept

$y = mx + c$

$m_1 \times m_2 = -1$

$4 \times m_2 = -1$

$m_2 = -\frac{1}{4}$

$y = -\frac{1}{4}x + c$

$6 = -\frac{1}{4}(0) = c$

$6 = c$

$y = -\frac{1}{4}x + 6$

(a) $y = -\frac{1}{4}x + 6$ **[3]**

(b) Work out the coordinates of the intersection of the graphs of $y = 4x - 5$ and $y = x^2 - 17$.

$4x - 5 = y$ and $y = x^2 - 17$

As both equations are equal to y, set them equal to each other to solve simultaneously.

$4x - 5 = x^2 - 17$

$-4x$

$-5 = x^2 - 4x - 17$

$+5$

$x^2 - 4x - 12 = 0$

quadratic equation

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 1 \times (-12)}}{2 \times 1}$

$= \frac{4 \pm \sqrt{16 - (-48)}}{2}$

$= \frac{4 \pm \sqrt{64}}{2}$

$= \frac{4 \pm 8}{2}$

$x = \frac{4 + 8}{2} = \frac{12}{2} = 6$

$x = \frac{4 - 8}{2} = \frac{-4}{2} = -2$

$y = 4x - 5$

$y = 4(6) - 5 = 24 - 5 = 19$

$(6, 19)$

$y = 4(-2) - 5 = -8 - 5 = -13$

$(-2, -13)$

(b) (6 , 19)

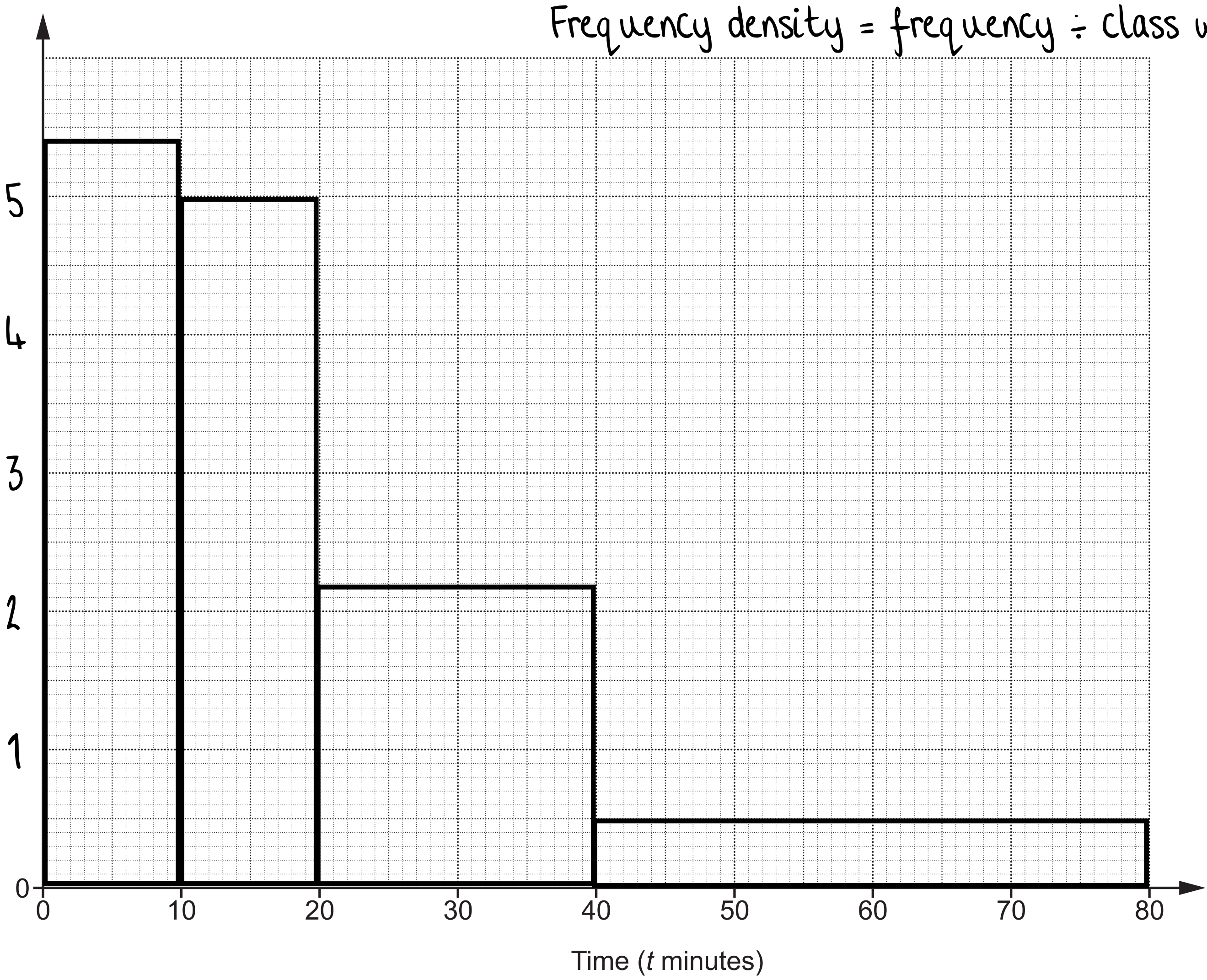
(-2 , -13) **[6]**

19 Ceri records the time taken, t minutes, to travel to school for a sample of 168 students at her Academy.

Time taken (t minutes)	Frequency	Class Width	Frequency Density
$0 < t \leqslant 10$	54	10	$54 \div 10 = 5.4$
$10 < t \leqslant 20$	50	10	$50 \div 10 = 5$
$20 < t \leqslant 40$	44	20	$44 \div 20 = 2.2$
$40 < t \leqslant 80$	20	40	$20 \div 40 = 0.5$

(a) Draw a histogram to represent this information.

Frequency density = frequency ÷ class width



[4]

(b) Ceri says

The longest time that any of these students took to travel to school was 80 minutes.

Is she correct?
Give a reason for your answer.

Ceri is incorrect because exact data is not given - it is given in classes instead.

[1]

(c) Ceri also claims that 25% of all of the students at this Academy took more than 30 minutes to travel to school.

(i) Show how Ceri might have worked out her claim. **[2]**

Using the midpoints of the classes:

$44 \div 2 = 22$ divide by 2 as 30 is the midpoint of the class.

$22 + 20 = 42$ add 20 for the final bar, as these values are greater than 30

$\frac{42}{168} = \frac{1}{4}$ 168 is the total number

$\frac{1}{4} \times 100 = 25\%$

(ii) State one assumption that Ceri has made in making her claim.

Data is distributed equally across each class.

[1]

20 In the following equation, n is an integer greater than 1.

$$(\sqrt{2})^n = k\sqrt{2}$$

(a) (i) Find k when $n = 7$.

$(\sqrt{2})^7 = K\sqrt{2}$

$\underbrace{\sqrt{2} \times \sqrt{2}} \times \underbrace{\sqrt{2} \times \sqrt{2}} \times \underbrace{\sqrt{2} \times \sqrt{2}} \times \sqrt{2}$

$= 2 \times 2 \times 2 \times \sqrt{2}$

$= 8\sqrt{2}$

(a)(i) $k =$ .... 8 .... **[2]**

(ii) Find n when $k = 64$.

$(\sqrt{2})^n = 64\sqrt{2}$

$64 = 2^6$ so $(\sqrt{2})^{12} = 64$

$x^a \times x^b = x^{a+b}$

$(\sqrt{2})^n = (\sqrt{2})^{12} \times \sqrt{2}^{1}$

$= (\sqrt{2})^{13\,(12+1)}$ so $n = 13$

(ii) $n =$ .... 13 .... **[2]**

(b) Show that $\dfrac{14}{3-\sqrt{2}}$ can be written in the form $a + b\sqrt{2}$. **[5]**

$$\frac{14}{3-\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}} = \frac{14(3+\sqrt{2})}{(3-\sqrt{2})(3+\sqrt{2})}$$

rationalise denominator

$$= \frac{42 + 14\sqrt{2}}{9-2}$$

$$= \frac{42 + 14\sqrt{2}}{7}$$

$$= 6 + 2\sqrt{2}$$

where $a = 6$, $b = 2$

END OF QUESTION PAPER

ADDITIONAL ANSWER SPACE

If additional space is required, you should use the following lined page(s). The question number(s) must be clearly shown in the margin(s).