AQA Model Solutions.

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

GCSE MATHEMATICS

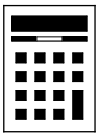
H

Higher Tier Paper 3 Calculator

Tuesday 12 June 2018 Morning Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:
- a calculator
- mathematical instruments.



Instructions
- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

Advice
- In all calculations, show clearly how you work out your answer.

For Examiner's Use	
Pages	Mark
2–3	
4–5	
6–7	
8–9	
10–11	
12–13	
14–15	
16–17	
18–19	
20–21	
22–23	
24–25	
TOTAL	

Answer **all** questions in the spaces provided

Do not write outside the box

1 Circle the decimal that is closest in value to $\frac{11}{20}$

[1 mark]

(0.56) 0.6 0.525 0.5

$\frac{11}{20} = 0.55$, so 0.56 is closest.

2 Circle the list of **all** the integers that satisfy $-2 < x \leqslant 4$

[1 mark]

–2, –1, 0, 1, 2, 3 –1, 0, 1, 2, 3

–2, –1, 0, 1, 2, 3, 4 (–1, 0, 1, 2, 3, 4)

3 Circle the largest number.

[1 mark]

($3.2\dot{7}$) 3.27 3.277 $3.20\dot{7}$

$3.2\dot{7} = 3.27777...$ ← Largest
3.27000
3.27700
$3.20\dot{7} = 3.20777..$

Do not write outside the box

4 What is the size of an exterior angle of a regular decagon?

Circle your answer.

[1 mark]

18° 36° 144° 162°

$\frac{360}{10} = 36°$ ← Decagon has 10 sides (all exterior angles of a regular polygon add to 360°)

5 a is a common factor of 72 and 120

b is a common multiple of 6 and 9

Work out the highest possible value of $\frac{a}{b}$

[4 marks]

72

1	72
2	36
3	24
4	18
6	12
8	9

120

1	120
2	60
3	40
4	30
5	24
6	20
8	15
10	12

a: factors in common:

1, 2, 3, 4, 6, 8, 12, 24

b: common multiples of 6, 9:

18, 36, 54, ...

largest value of $\frac{a}{b}$ = $\frac{\text{largest } a}{\text{smallest } b}$ ∴ $\frac{a}{b} = \frac{24}{18} = \frac{4}{3}$

Answer 4/3

Turn over for the next question

8

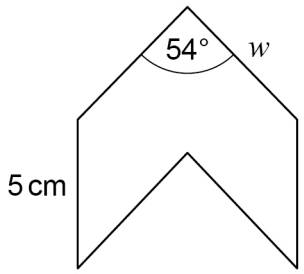
Do not write outside the box

6 A and B are similar shapes.

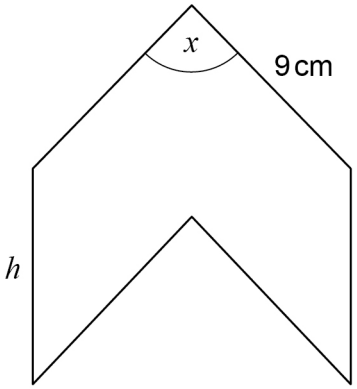
B is an enlargement of A with scale factor 1.5

Not drawn accurately

A



B



Work out the values of x, h and w.

[3 marks]

$x = 54^\circ$ (Angle stays the same when enlarged)

$9 = w \times 1.5 \rightarrow w = \frac{9}{1.5} = 6\text{cm}$

$h = 5 \times 1.5 = 7.5\text{cm}$

$x =$ 54 degrees

$h =$ 7.5 cm

$w =$ 6 cm

Do not write outside the box

7 Investment A Save £150 per month for 2 years.
2.5% interest is added to the total amount saved.

Investment B Invest £3500
Compound interest is added at 3% per year.

After 2 years, how much **more** is investment B worth than investment A?

[4 marks]

Investment A:

Saved ⇒ £150 × 12 × 2 = £3600

+ Intrest ⇒ £3600 × 1.025 = £3690

Investment B:

compound intrest for 2 years.

$3500 \times (1.03)^2 = £3713.15$

∴ £3713.15 − £3690 = £23.15

(£3713.15 ↑ B, £3690 ↑ A)

Answer £ 23.15

Turn over for the next question

7

Do not write outside the box

8 (a) Show that the lines $y = 3x + 7$ and $2y - 6x = 8$ are parallel.

Do **not** use a graphical method.

[3 marks]

$y = mx + c$ ↑ gradient

gradient of $y = 3x + 7 \Rightarrow 3$

gradient of $2y - 6x = 8$

$2y = 8 + 6x$

$y = 3x + 4 \Rightarrow$ gradient is 3.

They have the same gradient. ∴ They are parallel.

8 (b) Is the point (–5, –6) above, below or on the line $y = 3x + 7$?

Tick **one** box.

✓	Above		Below		On the line

You **must** show your working.

Do **not** use a graphical method.

[2 marks]

$x = -5 \Rightarrow y = 3(-5) + 7$

$= -15 + 7$

$= -8$

when $x = -5$; $y = -8$. Hence $(-5, -6)$ is above the line.

Do not write outside the box

9 The cost of a ticket increases by 10% to £19.25

Work out the original cost.

[3 marks]

110% = £19.25 ÷11
10% = £1.75
100% = £17.50 ×10

Answer £ 17.50

10 The nth term of a sequence is $12n - 5$

Work out the numbers in the sequence that

have two digits

and

are **not** prime.

[3 marks]

n	1	2	3	4	5	6	7	8
12n−5	7	19	31	43	55	67	79	91
	↑	↑	↑	↑	↑	↑	↑	↑
	1 digit	P	P	P	55 = 5×11	P	P	91 = 7×13

Answer 55 , 91

11

Do not write outside the box

11 $\mathbf{a} = \begin{pmatrix} 6 \\ -10 \end{pmatrix}$ $\mathbf{b} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ $\mathbf{c} = \begin{pmatrix} -4 \\ 7 \end{pmatrix}$

11 (a) Work out $\mathbf{a} + \mathbf{b} + \mathbf{c}$

[2 marks]

$\begin{pmatrix} 6 \\ -10 \end{pmatrix} + \begin{pmatrix} -1 \\ 2 \end{pmatrix} + \begin{pmatrix} -4 \\ 7 \end{pmatrix} = \begin{pmatrix} 6-1-4 \\ -10+2+7 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

Answer $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

11 (b) Show that $\mathbf{a} + 2\mathbf{c}$ is parallel to $\mathbf{b}$

[2 marks]

$\begin{pmatrix} 6 \\ -10 \end{pmatrix} + 2\begin{pmatrix} -4 \\ 7 \end{pmatrix} = \begin{pmatrix} 6-8 \\ -10+14 \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$

$\begin{pmatrix} -2 \\ 4 \end{pmatrix} = 2\begin{pmatrix} -1 \\ 2 \end{pmatrix} = 2\mathbf{b}$

Multiple of b, so is parallel to b.

Do not write outside the box

12

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

A force of 40 Newtons is applied to an area of 3.2 square metres.

Work out the pressure.

Give the units of your answer.

[2 marks]

Pressure = $\frac{40\text{N}}{3.2\text{m}^2} = 12.5\text{N/m}^2$

Answer 12.5N/m^2

13 Tick **all** the statements that are true for any rhombus.

[1 mark]

- [✓] The diagonals are lines of symmetry
- [✓] The diagonals bisect each other
- [✓] The diagonals are perpendicular
- [ ] The diagonals are equal in length

Turn over for the next question

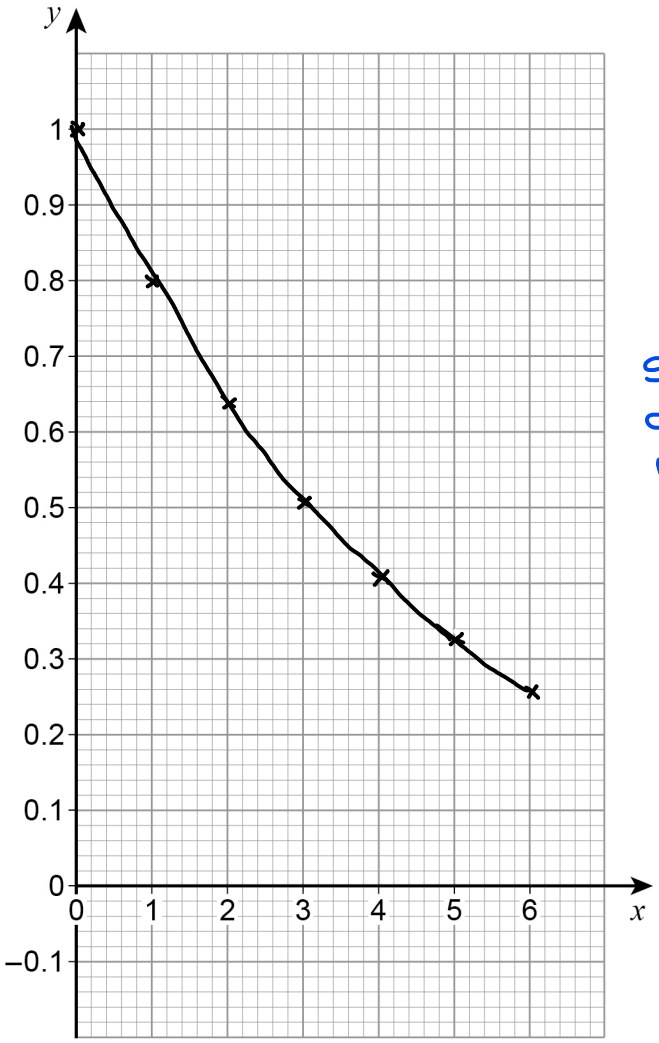
7

Do not write outside the box

14 Draw the graph of $y = 0.8^x$ for values of x from 0 to 6

[3 marks]

x	0	1	2	3	4	5	6
y	1	0.80	0.64	0.51	0.41	0.33	0.26



Smooth curve connecting all points.

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15 Amy has x beads.

Billy has three more beads than Amy.

Carly has four times as many beads as Billy.

B: $x+3$

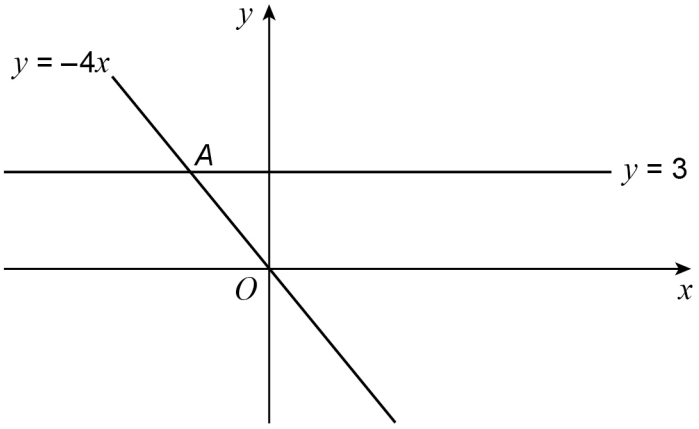
C: 4 B: $4(x+3)$

Circle the expression for the number of beads that Carly has.

[1 mark]

$4x + 3$ $3x + 4$ $4(x + 3)$ $x + 12$

16 Two straight lines intersect at point *A*.



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Circle the coordinates of *A*.

[1 mark]

$(-\frac{3}{4}, 3)$ $(-4, 3)$ $(-12, 3)$ $(-\frac{4}{3}, 3)$

$-4x = 3$ ← equate the two line equations.

$x = -\frac{3}{4}$

5

Do not write outside the box

17 Here are two methods to make a 4-digit code.

Codes can have repeated digits.

Method A

For the first two digits use an odd number between 30 and 100

For the last two digits use a multiple of 11

Method B

Use four digits in the order even odd even odd

Do **not** use the digit zero

Which method gives the **greater** number of possible codes?

You **must** show your working.

[3 marks]

Method A: $\frac{100-30}{2} = 35$ odd numbers between 30 and 100

9 2-digit multiples of 11.

$9 \times 35 = 315$ possible codes.

Method B: possible numbers: 1, 2, 3, 4, 5, 6, 7, 8, 9

even = 4 odd = 5

$4 \times 5 \times 4 \times 5 = 400$ codes

Answer Method B

Do not write outside the box

18 Show that, for $x \neq 0$

$$\frac{x+4}{3x} - \frac{5}{2x}$$

can be written in the form $\frac{ax+b}{cx}$ where a, b and c are integers.

[3 marks]

$\frac{2(x+4)}{6x} - \frac{3(5)}{6x} = \frac{2(x+4)-3(5)}{6x}$ ← make denominators the same

$\frac{2x+8-15}{6x} = \frac{2x-7}{6x}$

Answer $\frac{2x-7}{6x}$

19 The equation of a straight line is $3x + 2y = 24$

Circle the point where the line crosses the x-axis.

[1 mark]

(0, 8) (12, 0) (0, 12) **(8, 0)**

When line crosses x-axis, $y = 0$

$3x + 2(0) = 24$

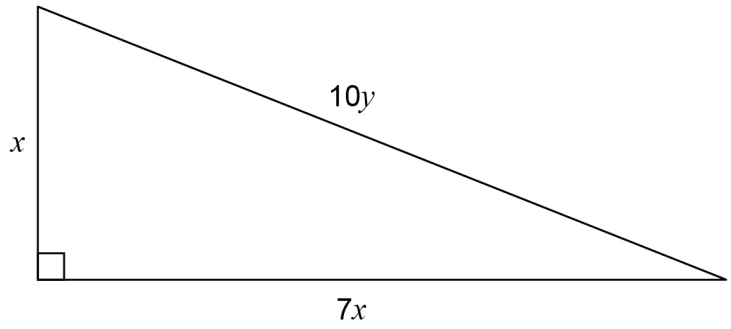
$3x = 24$

$x = 8$ so, (8, 0)

7

Do not write outside the box

20 All dimensions are in centimetres.



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Use Pythagoras' theorem to work out the exact value of $\frac{x}{y}$

[3 marks]

$x^2 + (7x)^2 = (10y)^2$ ← $a^2+b^2=c^2$

$x^2 + 49x^2 = 100y^2$

$50x^2 = 100y^2$

$x^2 = \frac{100y^2}{50} = 2y^2$ ✓ ✓

$x = \sqrt{2}\,y$

So, $\frac{x}{y} = \sqrt{2}$

Answer $\sqrt{2}$

Do not write outside the box

21 The mass of an ornament is m grams.

The height of the ornament is h centimetres.

m is directly proportional to the cube of h.

m = 1600 when h = 8

21 (a) Work out an equation connecting m and h.

[3 marks]

$m \propto h^3$ so $m = kh^3$

$m = 1600,\ h = 8 \Rightarrow 1600 = k(8)^3$

$k = \frac{1600}{512} = 3.125$

So $m = 3.125(h)^3$

Answer $m = 3.125h^3$

21 (b) Work out the mass of an ornament of height 12 centimetres.

[2 marks]

$h = 12 \Rightarrow m = 3.125(12)^3$

$= 3.125 \times 1728$

$= 5400\text{g}$

Answer 5400 grams

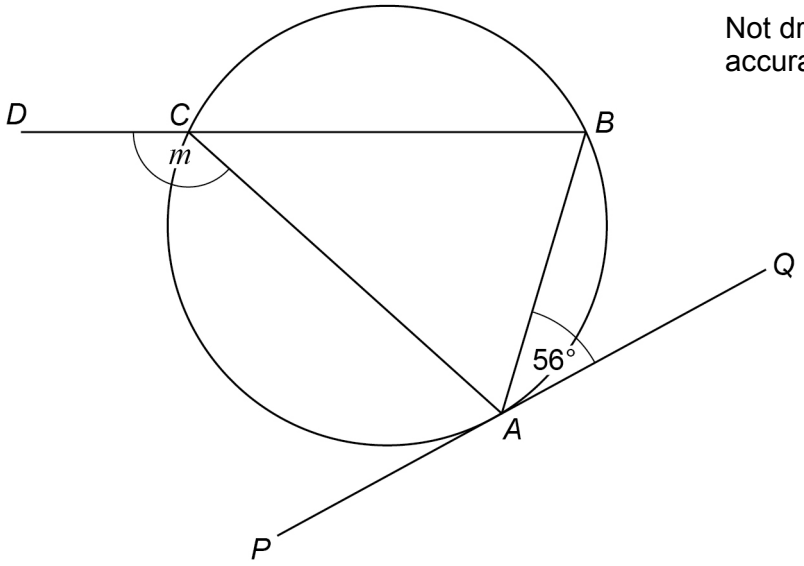
Turn over for the next question

8

22 *A*, *B* and *C* are points on a circle.

DCB is a straight line.

PAQ is a tangent to the circle.



Sam is trying to work out the size of angle m.

Here is his working.

angle *ACB* = 56°	angles in the same segment are equal
m = 180° – 56°	angles at a point on a straight line add up to 180°
m = 124°	

Make a criticism of his working.

[1 mark]

He used the incorrect theorem in the first line of his working. Should have used alternate segment theorem.

Do not write outside the box

23 A sequence of numbers is formed by the iterative process

$$u_{n+1} = \frac{3}{u_n + 1}, \qquad u_1 = 4$$

Work out the values of u_2 and u_3

[2 marks]

$u_2 = \frac{3}{4+1} = \frac{3}{5} = 0.6$

$u_3 = \frac{3}{0.6+1} = \frac{3}{1.6} = 1.875$

$u_2 =$ 0.6

$u_3 =$ 1.875

Turn over for the next question

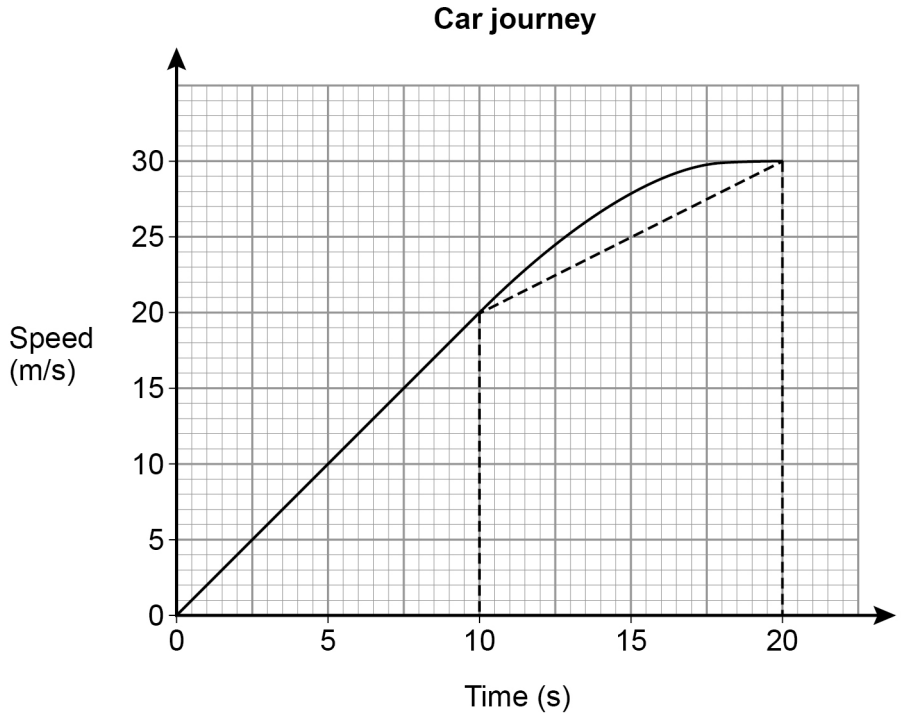
3

Do not write outside the box

24 The speed-time graph shows 20 seconds of a car journey.

Harry wants to estimate the distance the car travels in this time.

He uses a triangle and a trapezium, as shown, to estimate the area under the graph.



24 (a) Complete Harry's method to estimate the distance the car travels.

[3 marks]

distance travelled = area under the graph.

× area of triangle = $\frac{10 \times 20}{2} = 100\text{m}$

$\frac{\text{base} \times \text{height}}{2}$

× area of trapezium = $\frac{20+30}{2} \times 10 = 250\text{m}$

$\frac{a+b}{2} \times h$

× distance = 100 + 250 = 350m

Answer 350 m

Do not write outside the box

24 (b) For this journey, which of these is true for Harry's method?

Tick **one** box.

[1 mark]

☐ It works out an overestimate of the distance

☑ It works out an underestimate of the distance

There is an area between the graph and the trapezium that has not been calculated.

☐ It could work out an overestimate or an underestimate of the distance

Turn over for the next question

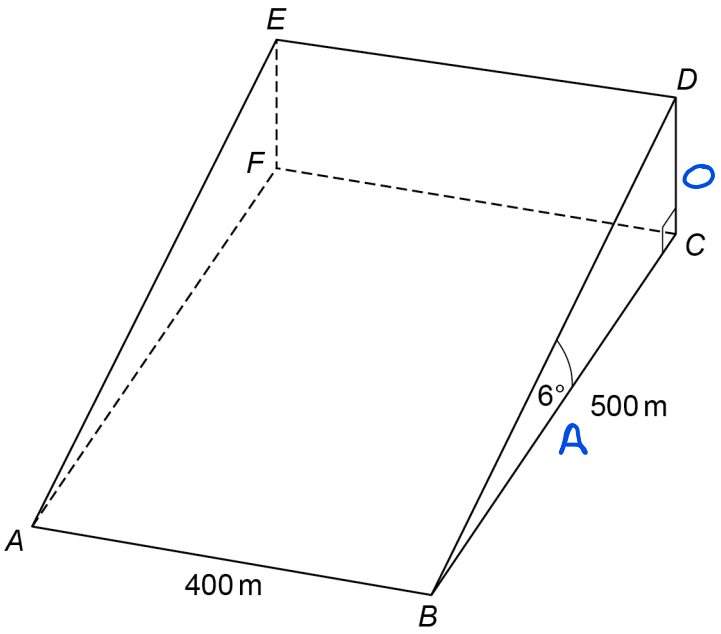
4

Do not write outside the box

25 *ABCDEF* is a triangular prism which represents part of a hill.

ABCF is the horizontal rectangular base.

D is vertically above *C*.



25 (a) Work out the height *CD*. **[2 marks]**

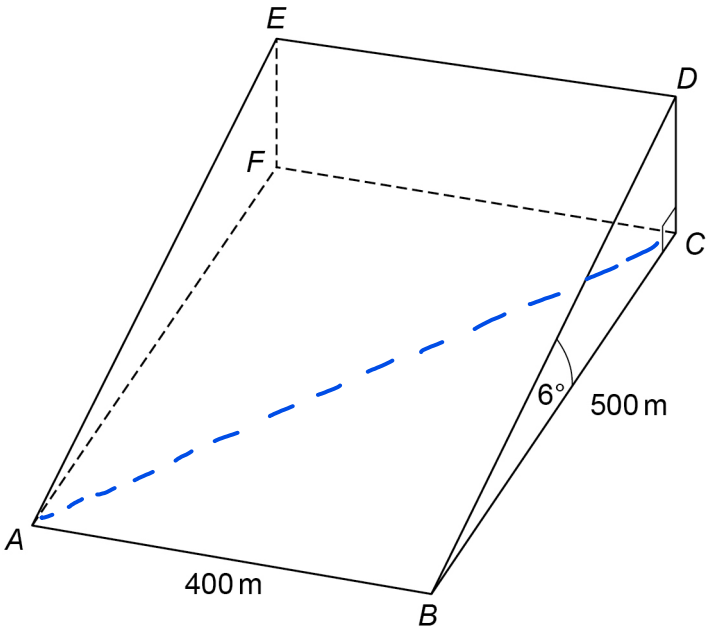
SOH CAH TOA $\tan x = \frac{O}{A}$

$\tan 6 = \frac{CD}{500}$ $CD = 500 \tan 6 = 52.6\text{m}$

Answer 52.6 m

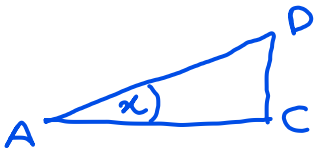
Do not write outside the box

25 (b) Jamil walks in a straight line from *A* to *D*.



Work out the size of angle *DAC*.

You **must** show your working.



[4 marks]

$(AC)^2 = 400^2 + 500^2$

$= \sqrt{400^2 + 500^2} = 640.3\text{m}$

$\tan(x) = \frac{CD}{AC} = \frac{52.6}{640.3} = 0.082$

$x = \tan^{-1}(0.082) = 4.7^\circ$

Answer 4.7 degrees

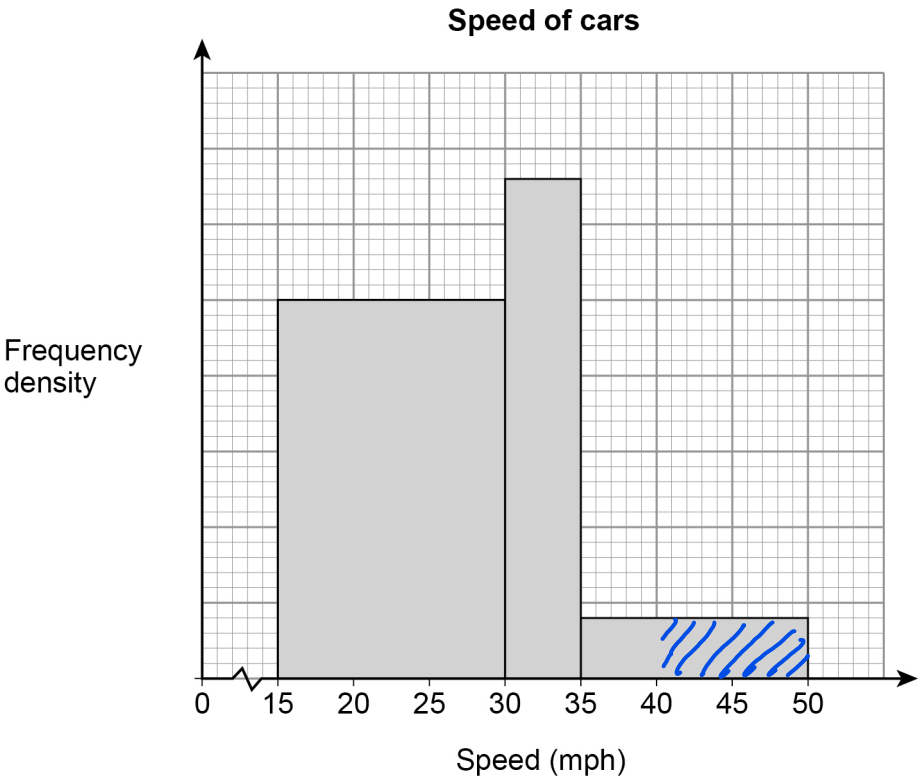
6

Turn over ►

Do not write outside the box

26 The histogram shows information about the speed of cars as they pass a checkpoint.

The scale on the frequency density axis is missing.



The histogram shows information about 480 cars.

26 (a) How many cars does the first bar represent?

[4 marks]

width of bar 1 : 15 ; bar 2 : 5 ; bar 3 : 15

Let height of 1 large square on graph be x,

$15(5x) + 5(6.6x) + 15(0.8x) = 480$ ← areas of each bar

$75x + 33x + 12x = 480$

$120x = 480$

$x = \frac{480}{120} = 4$

Area of 1st bar : $15(5 \times 4) = 15(20) = 300$

Answer 300

Do not write outside the box

26 (b) Cars with a speed greater than 40 mph are over the speed limit.

Use the histogram to estimate the number of cars that are over the speed limit.

[2 marks]

Total no. of cars in bar 3: $0.8 \times 4 \times 15 = 48$

40 mph is $\frac{1}{3}$ into $35 < \text{speed} < 50$. So, $\frac{2}{3}$ of cars in bar 3 are over the speed limit.

$\frac{2}{3} \times 48 = 32$

Answer 32

Turn over for the next question

6

Do not write outside the box

27 A bag contains 30 discs.

10 are red and 20 are blue.

One disc is taken out at random and replaced by **two** of the other colour.

Another disc is then taken out at random and replaced by **two** of the other colour.

Another disc is then taken out at random.

Work out the probability that all three discs taken out are **red**.

[3 marks]

red, red, red

$\frac{10}{30} \times \frac{9}{31} \times \frac{8}{32} = \frac{10 \times 9 \times 8}{30 \times 31 \times 32} = \frac{3}{124}$

when the first red is taken out, there are 9 red left and 29 in total. Two blue are added, so total is 31.

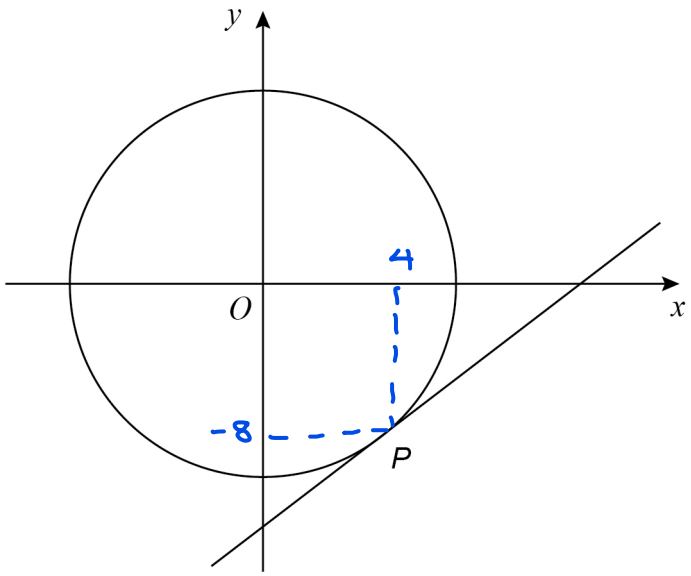
Answer $\frac{3}{124}$

Do not write outside the box

28 P is a point on the circle with equation $x^2 + y^2 = 80$

P has x-coordinate 4 and is below the x-axis.

Not drawn accurately



Work out the equation of the tangent to the circle at P.

[5 marks]

$x = 4 \Rightarrow (4)^2 + y^2 = 80$

$y^2 = 80 - 16 = 64$

$y = \pm 8$

at P: $(4, -8)$

gradient of OP $= \frac{0-(-8)}{0-4} = \frac{8}{-4} = -2$

gradient of tangent is negative reciprocal because it is ⊥ to OP.

gradient of tangent $= \frac{1}{2}$

$P(4, -8) \Rightarrow -8 = \frac{1}{2}(4) + c$

$-8 = 2 + c \qquad c = -10$

Answer $y = \frac{1}{2}x - 10$

END OF QUESTIONS

8

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