

GCSE (9–1) Mathematics

J560/04 Paper 4 (Higher Tier)

Thursday 25 May 2017 – Morning

Time allowed: 1 hour 30 minutes



You may use:

- A scientific or graphical calculator
- Geometrical instruments
- Tracing paper



First name										
Last name										
Centre number						Candidate number				

INSTRUCTIONS

- Use black ink. You may use an HB pencil for graphs and diagrams.
- Complete the boxes above with your name, centre number and candidate number.
- Answer **all** the questions.
- Read each question carefully before you start to write your answer.
- Where appropriate, your answers should be supported with working. Marks may be given for a correct method even if the answer is incorrect.
- Write your answer to each question in the space provided.
- Additional paper may be used if required but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the barcodes.

INFORMATION

- The total mark for this paper is **100**.
- The marks for each question are shown in brackets [].
- Use the π button on your calculator or take π to be 3.142 unless the question says otherwise.
- This document consists of **20** pages.

Answer **all** the questions.

1 Calculate.

(a) $\sqrt{\frac{4.8^2 + 3.6^2}{4}}$ use calculator

$$= \sqrt{\frac{23.04 + 12.96}{4}} = \sqrt{\frac{36}{4}} = \sqrt{9}$$

(a) 3 [2]

(b) $\frac{1}{(2 \times 10^4) + (5 \times 10^3)}$

$$= \frac{1}{20,000 + 5,000} = \frac{1}{25,000} = 0.00004$$

(b) 0.00004 [2]

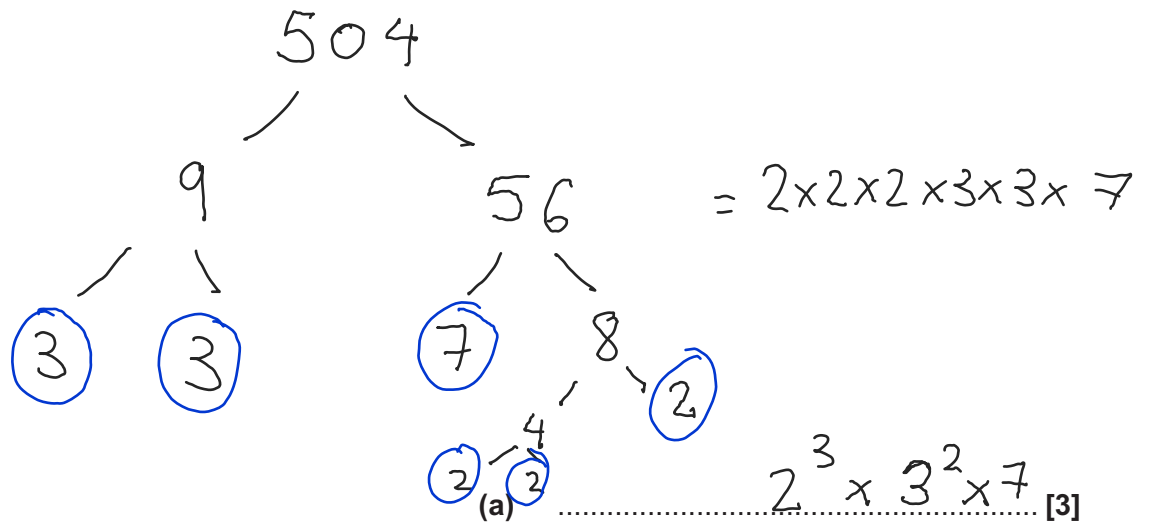
2 The length, L , of a steel rod is 8.3 m, correct to 1 decimal place.

Complete the error interval for length L .

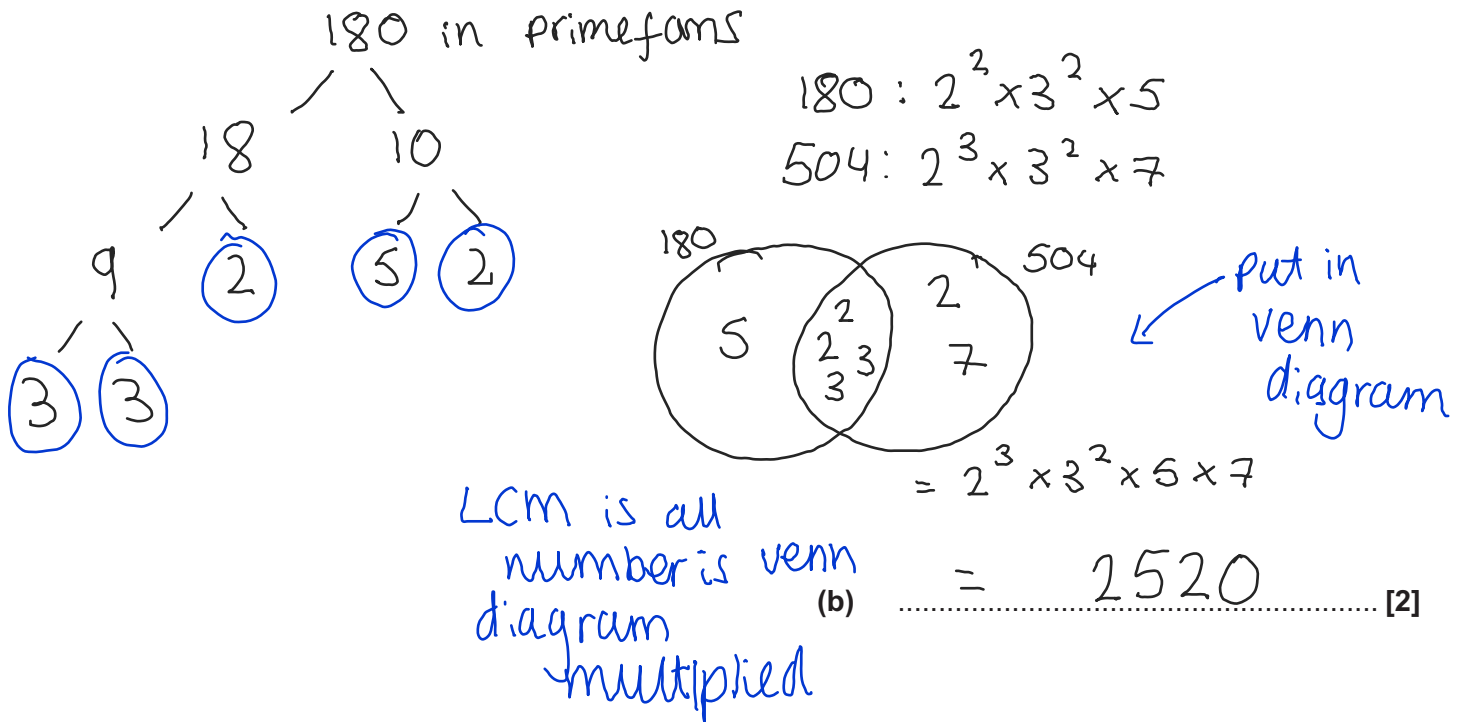
all values in
this interval
round to 8.3

..... 8.25 $\leq L <$ 8.35 [2]

3 (a) Write 504 as the product of its prime factors.

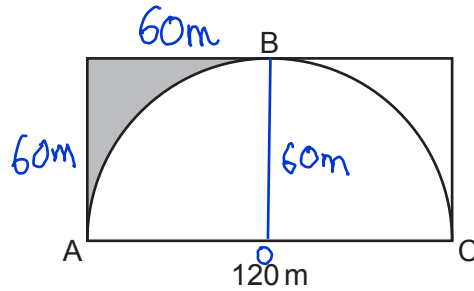


(b) Find the lowest common multiple (LCM) of 180 and 504.



- 6 The diagram shows a semi-circle inside a rectangle of length 120 m. The semi-circle touches the rectangle at A, B and C.

Not to scale



Calculate the **perimeter** of the shaded region.
Give your answer correct to 3 significant figures.

$$OB \text{ is radius} = 120\text{m} \div 2 = 60\text{m}$$

$$\text{Circumference of AB: } \pi \times 120 \times \frac{1}{4} \quad \leftarrow \text{AB is a quarter of a circle}$$

$$= 30\pi$$

$$\text{Perimeter} = 60\text{m} + 60\text{m} + 30\pi$$

$$= 214.\underline{2}477\dots$$

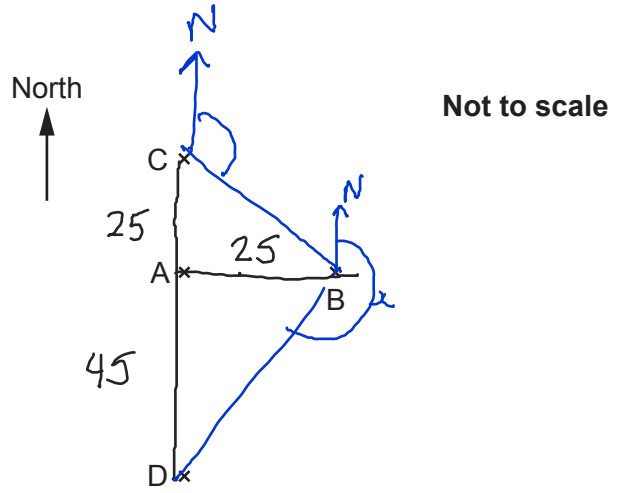
round down

..... 214 m [5]

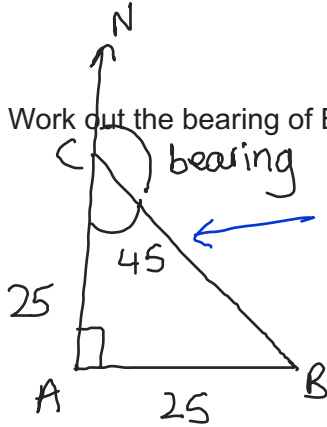
6

7 A, B, C and D are four towns.

B is 25 kilometres due East of A.
 C is 25 kilometres due North of A.
 D is 45 kilometres due South of A.



(a) Work out the bearing of B from C.

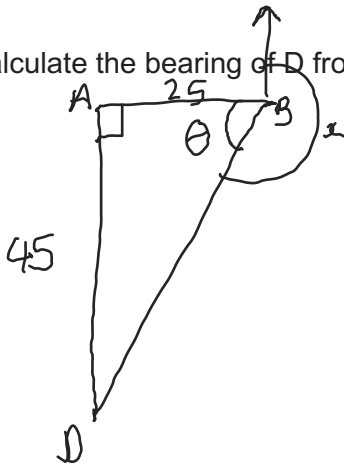


isosceles triangle so $\angle ACB = 45^\circ$

$$\text{Bearing} = 180 - 45$$

(a) 135 $^\circ$ [2]

(b) Calculate the bearing of D from B.



$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan \theta = \frac{45}{25}$$

$$\theta = \tan^{-1} \left(\frac{45}{25} \right) = 60.9$$

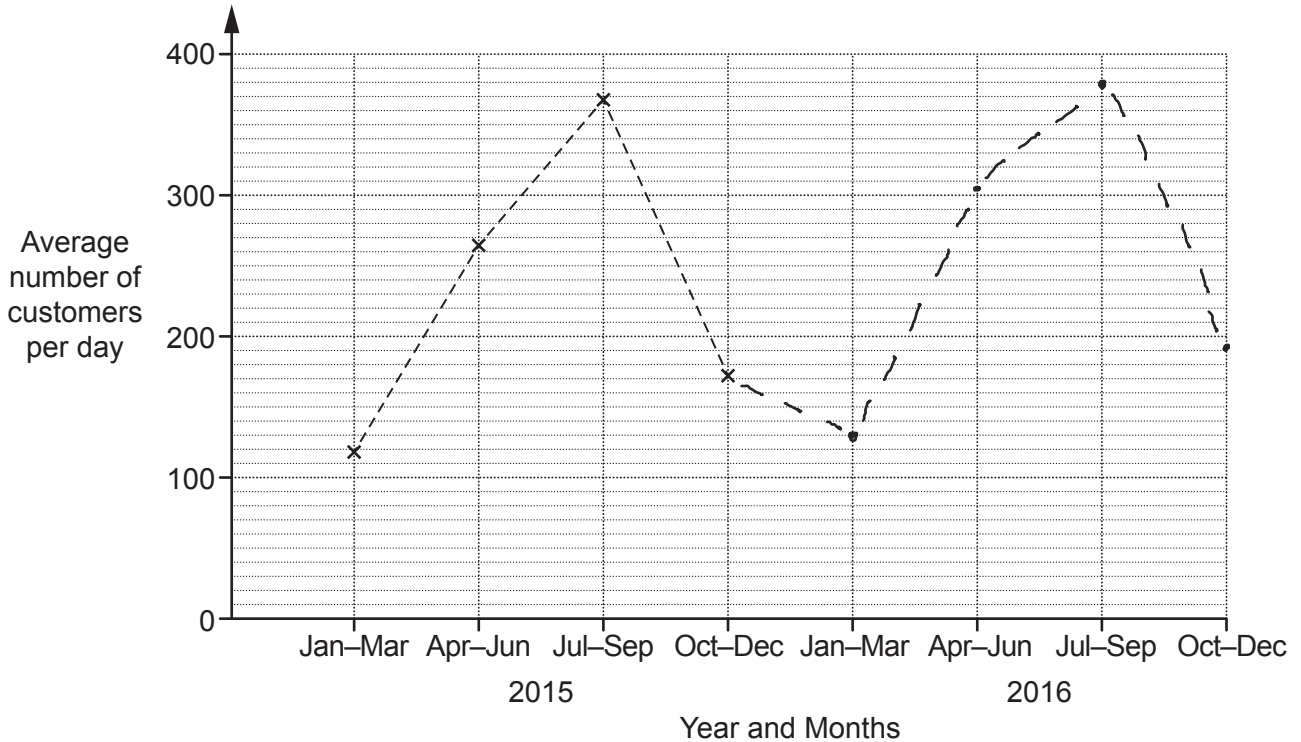
$$\begin{aligned} \text{Bearing (x)} &= 270 - 60.9 \\ &= 209.1 \end{aligned}$$

(b) 209 $^\circ$ [4]

- 8 The table shows the average number of customers per day entering a shop.

	2015				2016			
Months	Jan-Mar	Apr-Jun	July-Sep	Oct-Dec	Jan-Mar	Apr-Jun	July-Sep	Oct-Dec
Average number of customers per day	119	264	368	172	130	304	381	192

- (a) Complete the time series graph below.



[2]

- (b) Make two different comments comparing the number of customers entering the shop in 2015 and 2016.

Comment 1 Overall, more people entered the shop
in 2016 than 2015.

Comment 2 In both years, June to Sept had the
most amount of customers

[2]

- 9 Each week Dan drives two routes, route X and route Y.

One week he drives route X three times and route Y twice. (1)
He drives a total of 134 miles that week.

Another week he drives route X twice and route Y five times. (2)
He drives a total of 203 miles that week.

- (a) Find the length of each route.

$$\begin{aligned} \textcircled{1} \quad 3x + 2y &= 134 \quad \times 2 \\ \textcircled{2} \quad 2x + 5y &= 203 \quad \times 3 \end{aligned}$$

$$\begin{array}{r} \textcircled{2} \times 3 : \quad 6x + 15y = 609 \quad - \\ \textcircled{1} \times 2 : \quad 6x + 4y = 268 \quad - \\ \hline \end{array}$$

$$\begin{aligned} 11y &= 341 \\ &\div 11 \\ y &= 31 \end{aligned}$$

sub into (1)

$$3x + 2(31) = 134$$

$$3x + 62 = 134$$

$$3x = 72$$

$$x = 24$$

(a) route X = 24 miles
route Y = 31 miles [5]

- (b) State an assumption that has been made in answering part (a).

There is no additional driving only routes X and Y. [1]

- 10 On 1st November 2015 there were 4200 trees planted in a wood. 1
 On 1st November 2016, only 3948 of these trees were still alive. 2

It is assumed that the number of trees still alive is given by

$$N = ar^t$$

where N is the number of trees still alive t years after 1st November 2015.

- (a) Write down the value of a .

① $t=0$

$$N = ar^t$$

$$4200 = ar^0 = 1$$

$$a = 4200$$

(a) 4200 [1]

- (b) Show that r is 0.94. [2]

$t=1$ - ② $N = ar^t$

$$3948 = 4200 \times r^1$$

$$0.94 = \frac{3948}{4200} = r$$

- (c) Show that on 1st November ~~2030~~ the number of trees still alive is predicted to have decreased by over 60% compared with 1st November 2015. [3]

$N_{2030} : t=15$

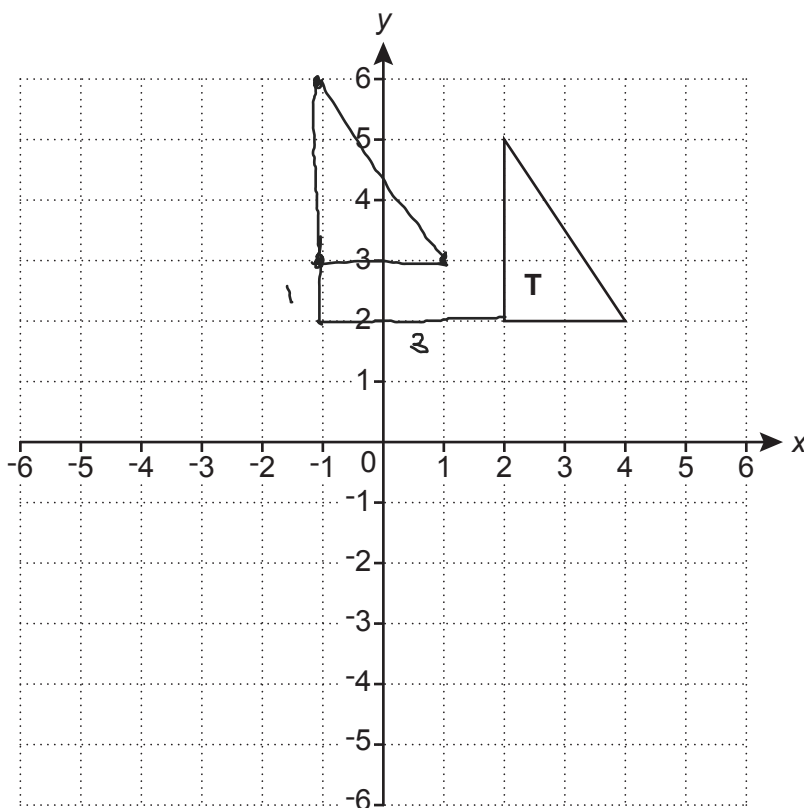
$$N = 4200 \times 0.94^{15} = 1660$$

Percentage decrease: $\frac{\text{Initial} - \text{Final}}{\text{Initial}} \times 100$

$$\frac{4200 - 1660}{4200} \times 100 = 60.5\%$$

$$60.5\% > 60\%$$

11 Triangle T is drawn on a coordinate grid.



(a) Translate triangle T using the vector $\begin{pmatrix} -3 \\ 1 \end{pmatrix}$. *3 left 1 up* [2]

(b) Describe fully the **single** transformation that represents the following.

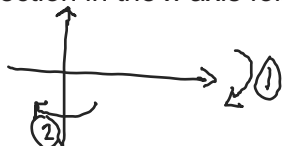
(i) A rotation with centre (0, 0) of 180° followed by a rotation with centre (0, 0) of 90° clockwise.

$180 + 90 = 270$ clockwise

Rotation centre (0,0) 270 clockwise

[2]

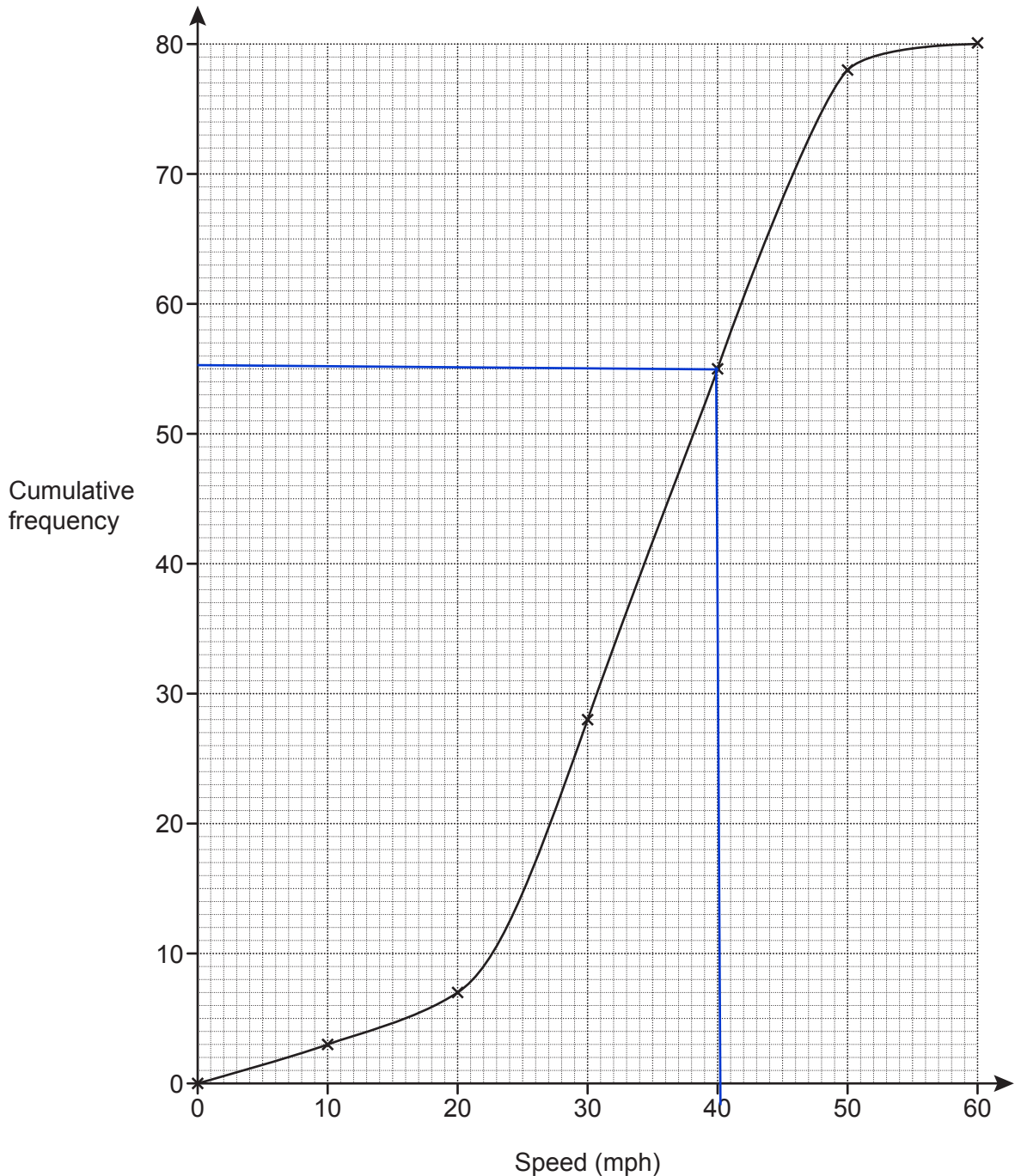
(ii) A reflection in the x-axis followed by a reflection in the y-axis.



Rotation centre (0,0) 180°

[3]

- 12 The cumulative frequency graph shows the speeds, in miles per hour (mph), of vehicles passing a 40 mph speed limit sign on a road.



A speed camera will be installed if more than 30% of vehicles go over the speed limit of 40 mph.

Use information from the graph to decide if a speed camera should be installed.

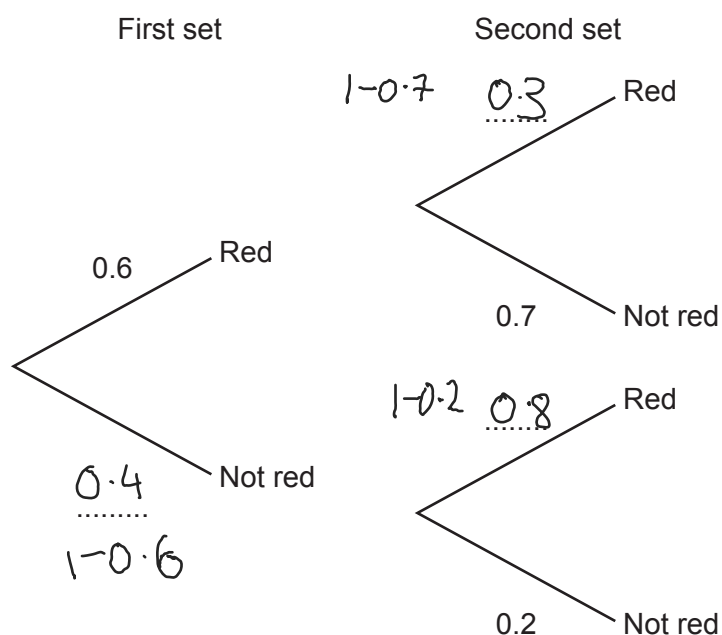
[4]

$$\begin{aligned}
 &55 \text{ ppl go } 40 \text{ mph or less} \\
 &80 - 55 = 25 \text{ ppl go more than } 40 \text{ mph} \\
 &\frac{25}{80} \times 100 = 31.25\% \\
 &31.25 > 30
 \end{aligned}$$

Turn over

So a speed camera should be installed

- 13 Rashid drives his car along a road passing through two sets of traffic lights. The tree diagram shows the probabilities of the lights being **red** when he reaches them.



- (a) Complete the tree diagram. [1]

- (b) Write down the probability that the first set is not red.

(b) 0.4 [1]

- (c) Given that the first set is red, write down the probability that the second set is not red.

(c) 0.7 [1]

- (d) Work out the probability that both sets are **not red**.

$$P(\text{not red and not red})$$

$$= 0.4 \times 0.2$$

(d) 0.08 [2]

- (e) Work out the probability that at least one set is **not red**.

$$P(\text{Both red}) = 0.6 \times 0.3 = 0.18$$

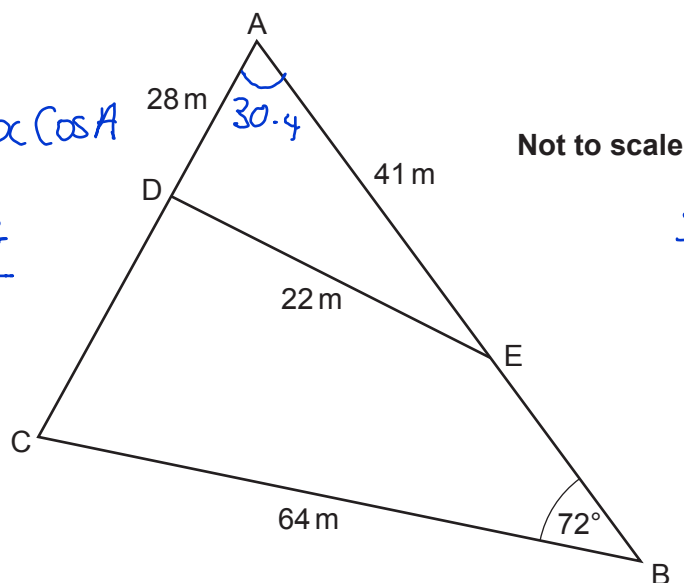
$$P(\text{At least one not red}) = 1 - P(\text{Both red})$$

$$= 1 - 0.18$$

(e) 0.82 [3]

- 14 The diagram shows triangle ABC with D on AC and E on AB. DE is a straight line.

cosine rule:
 $a^2 = b^2 + c^2 - 2bc \cos A$
 OR
 $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$



Sine Rule:
 $\frac{a}{\sin A} = \frac{b}{\sin B}$

AD = 28m, AE = 41m, DE = 22m and BC = 64m.

Calculate the length CD.

① Find $\angle DAE$: $\cos A = \frac{28^2 + 41^2 - 22^2}{2 \times 28 \times 41} = \frac{283}{328}$

$$A = \cos^{-1}\left(\frac{283}{328}\right) = 30.4^\circ$$

② sine rule to find AC

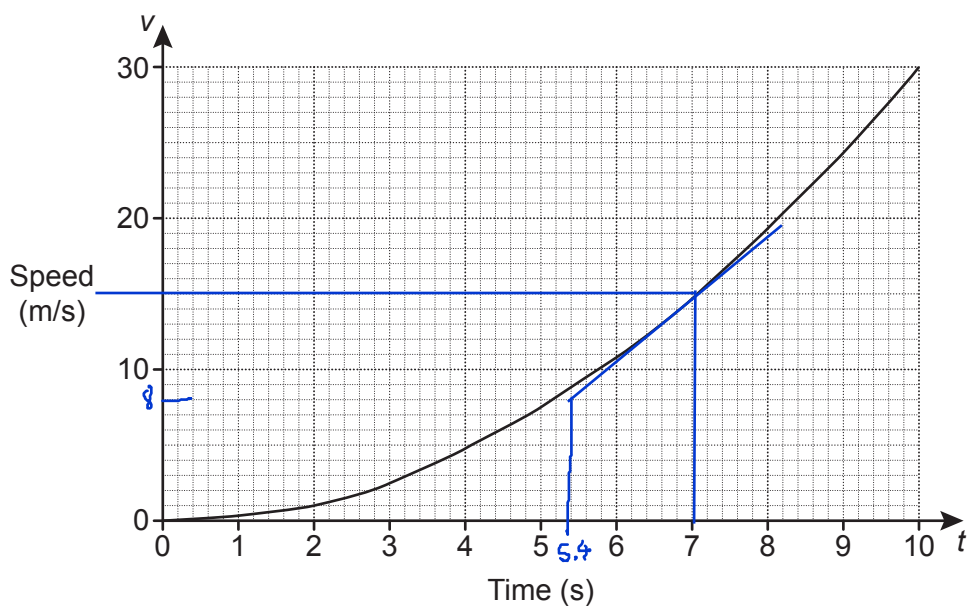
$$\frac{AC}{\sin 72} = \frac{64}{\sin 30.4}$$

$$AC = \frac{64 \sin 72}{\sin 30.4} \approx 120.3$$

$$\begin{aligned} CD &= AC - AD \\ &= 120.3 - 28 = \end{aligned}$$

..... 92.3 m [6]

- 15 The graph shows the speed, v metres per second (m/s), of a car at time t seconds.



- (a) Find the speed of the car at $t = 7$.

(a) 15 m/s [1]

- (b) It is claimed that the car has accelerated from 0 to 60 miles per hour in the first 10 seconds.

Does the graph support this claim? Show your reasoning.
Use 1 mile = 1.6 kilometres.

[5]

The car accelerated to 30 m/s

$$\begin{aligned}
 &30 \text{ m per sec} : \\
 &30 \times 60 = 1800 \text{ meters per min} \quad \left. \begin{array}{l} \left. \begin{array}{l} 1 \text{ min} = 60 \text{ sec} \\ 60 \text{ min} = 1 \text{ hour} \end{array} \right\} \\ \leftarrow \end{array} \right. \\
 &1800 \times 60 = 108000 \text{ m per hour} \\
 &108000 = 108 \text{ km} \quad \leftarrow \div 1000
 \end{aligned}$$

$$\begin{aligned}
 &1 \text{ miles} = 1.6 \text{ km} \\
 &67.5 = 108 \text{ km} \quad \leftarrow \times 67.5 \quad = 67.5 \text{ mph}
 \end{aligned}$$

$67.5 > 60 \text{ mph}$ so Yes the car
did accelerate this much

(c) Use the graph to estimate the acceleration at $t = 7$.

gradient: $\frac{y_1 - y_2}{x_1 - x_2}$

$$\frac{15 - 8}{7 - 5.4} = 4.375$$

— Draw tangent at $t = 7$
 $(7, 15)$ $(5.4, 8)$

4.375

(c) m/s² [3]

(d) The speed of this car is directly proportional to the square of the time.

Find a formula linking v and t .

$$v \propto t^2$$

$$v = kt^2$$

$$t = 7, v = 15$$

$$15 = k \times 7^2$$

$$49k = 15$$

$$k = \frac{15}{49}$$

(d) $v = \frac{15}{49} t^2$ [3]

(e) Georgina says that the graph shows that the speed of the car will continue to increase after 10 seconds.

Make one comment to show that this statement is incorrect.

Graph only shows what will happen in up to 10sec.
 30m/s may be the maximum speed of the
 car so the speed may not increase [1]

- 16 Write $x^2 - 10x + 16$ in the form $(x + a)^2 + b$.

complete the square

$$\begin{aligned} & (x - \frac{10}{2})^2 - (\frac{10}{2})^2 + 16 \\ = & (x - 5)^2 - 25 + 16 \end{aligned}$$

$$= (x - 5) - 9 \dots\dots\dots [3]$$

- 17 Describe fully the graph which has the equation $x^2 + y^2 = 9$.

circle r^2

$$\sqrt{9} = 3$$

Circle centre $(0, 0)$ with radius 3 [2]

18 (a) Solve by factorisation.

$$2x^2 + 5x - 12 = 0$$

← coefficient is prime so
 $(2x+a)(x+b)$ to
 get $2x^2$

$$(2x - 3)(x + 4) = 0$$

$$ab = -12$$

$$-3, 4$$

$$ax + 2bx = 5$$

$$a + 2b = 5$$

$$a = -3 \quad b = 4$$

$$2x - 3 = 0$$

$$2x = 3$$

$$x = \frac{3}{2}$$

$$x + 4 = 0$$

$$x = -4$$

to = 0, 1 of
 the brackets
 MUST = 0

(a) $x = \frac{3}{2}$ or $x = -4$ [3]

(b) Solve this equation.

Give each value correct to 2 decimal places.

$$3x^2 + 2x - 3 = 0$$

Quadratic
 Formula

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4 \times 3 \times -3}}{2 \times 3}$$

$$x = \frac{-2 \pm \sqrt{40}}{6}$$

$$\oplus x = 0.7207$$

$$\ominus x = -1.387$$

(b) $x = 0.72$ or $x = -1.39$ [3]

19 (a) Here are the first four terms of a sequence.

$$\frac{1}{2} \quad \frac{4}{3} \quad \frac{9}{4} \quad \frac{16}{5}$$

Find the n th term of this sequence.

$$1, 4, 9, 16 = n^2$$

$$1, 2, 3, 4, 5 = n+1$$

(a) $\frac{n^2}{n+1}$ [2]

(b) Here are the first four terms of a quadratic sequence, the n th term of this quadratic sequence is $an^2 + bn + c$.

$$2 \quad 12 \quad 28 \quad 50$$

Find the values of a , b and c .

$$\begin{array}{cccc}
 2 & 12 & 28 & 50 \\
 \text{+10} & \text{+16} & \text{+22} & \\
 \text{+6} & \text{+6} & &
 \end{array}$$

seq		2, 12, 28, 50	-	$3n^2$
$3n^2$		3, 12, 27, 48		$3n^2 + n - 2$

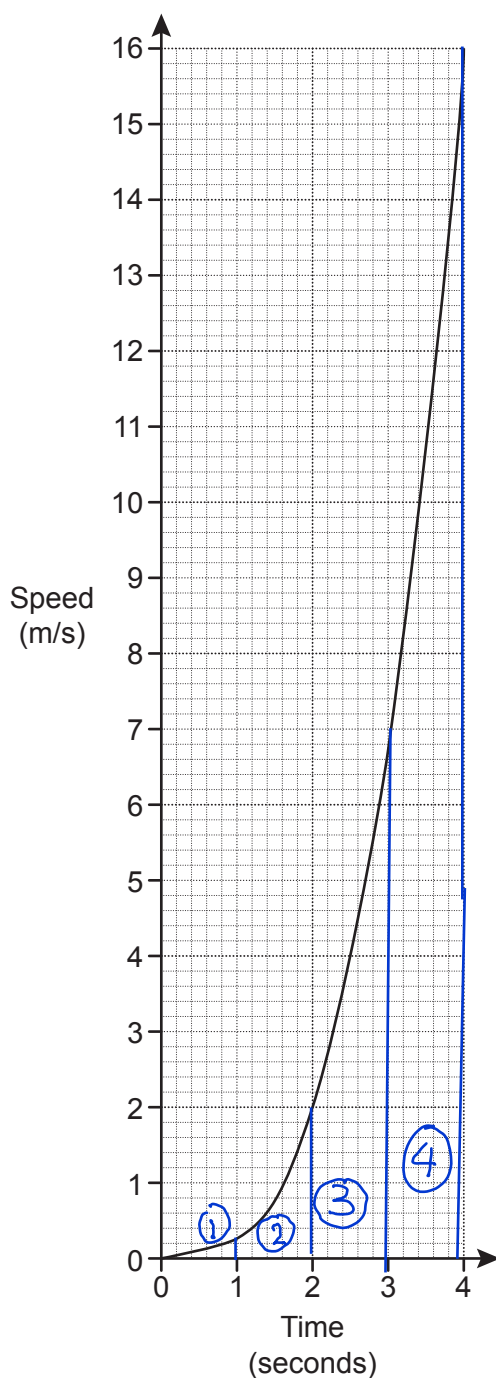
$$\begin{array}{ccccccc}
 -2 & & -1 & 0 & 1 & 2 & \\
 \uparrow & & \uparrow & \uparrow & \uparrow & \uparrow & \\
 -1 & & +1 & +1 & & & \\
 & & & & & & \text{nth term of this sequence}
 \end{array}$$

$n-2$

(b) $a = 3$
 $b = 1$
 $c = -2$

[4]

- 20 The graph shows the speed, in metres per second, of a particle over the first four seconds of motion.



Use the graph to estimate the distance travelled by the particle in the four seconds.

Distance = Area under curve

using trapeziums

① $\frac{1}{2} \times 1 \times 0.2 = 0.1 \text{ m}$

② $\frac{1}{2} \times (0.2 + 2) \times 1 = 1.1 \text{ m}$

③ $\frac{1}{2} (2 + 7) \times 1 = 4.5 \text{ m}$

④ $\frac{1}{2} (7 + 16) \times 1 = 11.5$

$0.1 + 1.1 + 4.5 + 11.5$

17.2

..... metres [2]

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