

Please check the examination details below before entering your candidate information


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Centre Number					Candidate Number				
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Pearson Edexcel International GCSE

Time 2 hours

Paper reference **4PM1/01R**

Further Pure Mathematics
PAPER 1R



Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ►

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International GCSE in Further Pure Mathematics Formulae sheet

MensurationSurface area of sphere = $4\pi r^2$ Curved surface area of cone = $\pi r \times$ slant heightVolume of sphere = $\frac{4}{3}\pi r^3$ **Series****Arithmetic series**Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$ **Geometric series**Sum to n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$ Sum to infinity, $S_\infty = \frac{a}{1-r}$ $|r| < 1$ **Binomial series** $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots$ for $|x| < 1, n \in \mathbb{Q}$ **Calculus****Quotient rule (differentiation)**

$$\frac{d}{dx} \left(\frac{f(x)}{g(x)} \right) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

Trigonometry**Cosine rule**In triangle ABC : $a^2 = b^2 + c^2 - 2bc \cos A$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Logarithms

$$\log_a x = \frac{\log_b x}{\log_b a}$$

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Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

- 1 (a) On the grid below, draw the graph of the line with equation

(i) $4x + 5y = 20$ (ii) $3y - 4x = -12$

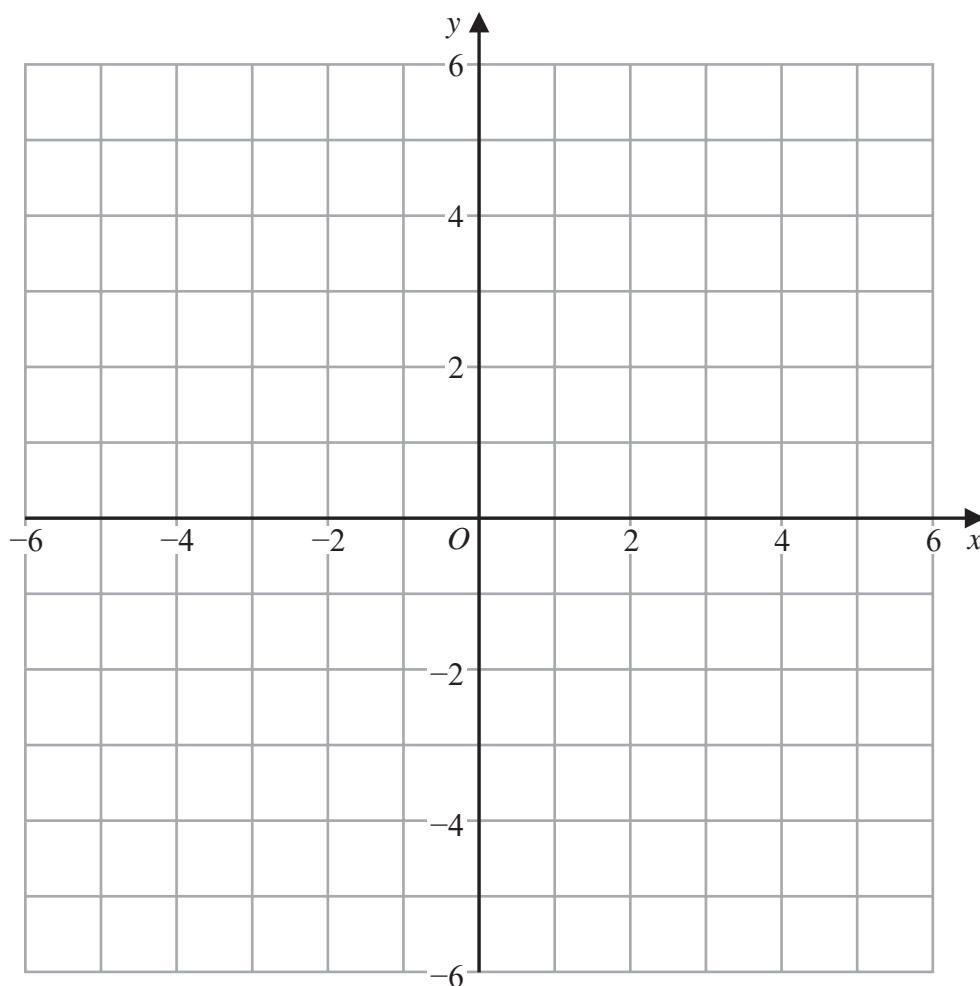
(2)

- (b) Show, by shading on the grid, the region R defined by the inequalities

$$4x + 5y \leq 20 \quad 3y - 4x \geq -12 \quad y \leq 3 \quad x \geq 1$$

Label the region R .

(2)



Turn over for a spare grid if you need to redraw your graph.



Question 1 continued

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2 The n th term of an arithmetic series is u_n

Given that $u_5 = 46$ and that $u_{20} = 181$

(a) find

(i) the common difference of this series,

(ii) the first term of this series.

(4)

(b) Evaluate $\sum_{n=21}^{50} u_n$

(4)

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Question 2 continued

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(Total for Question 2 is 8 marks)



3 The point A has coordinates $(1, 7)$ and the point B has coordinates $(9, 3)$

The line l is the perpendicular bisector of AB

(a) Find an equation of l

(5)

The line l crosses the x -axis at the point C

(b) Find the area of the triangle ABC

(5)

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Question 3 continued

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(Total for Question 3 is 10 marks)



Question 4 continued

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(Total for Question 4 is 8 marks)



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Question 5 continued

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(Total for Question 5 is 8 marks)



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6 The points P , Q , R and S are the vertices of a quadrilateral $PQRS$ such that

$$\vec{PQ} = 2\mathbf{i} + 3\mathbf{j} \quad \vec{PR} = -\mathbf{i} + 18\mathbf{j} \quad \vec{PS} = -3\mathbf{i} + 15\mathbf{j}$$

(a) Show that $PQRS$ is a parallelogram.

(4)

(b) Find a unit vector parallel to \vec{QS} as a simplified expression in terms of \mathbf{i} and \mathbf{j}

(4)

The point T lies on QS such that $QT:TS = 5:8$

(c) Find \vec{PT} as a simplified expression in terms of \mathbf{i} and \mathbf{j}

(2)

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Question 6 continued

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Question 6 continued

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Question 6 continued

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(Total for Question 6 is 10 marks)



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7

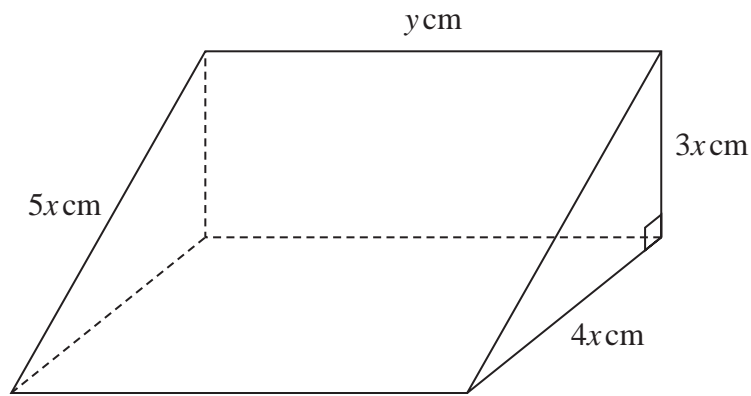


Diagram **NOT** accurately drawn

Figure 2

Figure 2 shows a block of wood in the shape of a right triangular prism.

The cross section of the prism is a right-angled triangle with sides of length $3x\text{ cm}$, $4x\text{ cm}$ and $5x\text{ cm}$.

The length of the prism is $y\text{ cm}$.

The total surface area of the five faces of the prism is 144 cm^2

The volume of the prism is $V\text{ cm}^3$

(a) Show that

$$V = 72x - 6x^3 \tag{5}$$

Given that x can vary,

(b) use calculus to find the value of x for which V is a maximum, justifying that this value gives a maximum value of V (4)

(c) Find the maximum value of V (2)

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Question 7 continued

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Question 7 continued

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Question 7 continued

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(Total for Question 7 is 11 marks)



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8 The curve C has equation $y = 2x^2 - \sin x$

The point A on C has x coordinate π

Show that an equation of the normal to C at the point A is

$$x + (4\pi + 1)y - \pi(8\pi^2 + 2\pi + 1) = 0 \quad (8)$$

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Question 8 continued

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(Total for Question 8 is 8 marks)



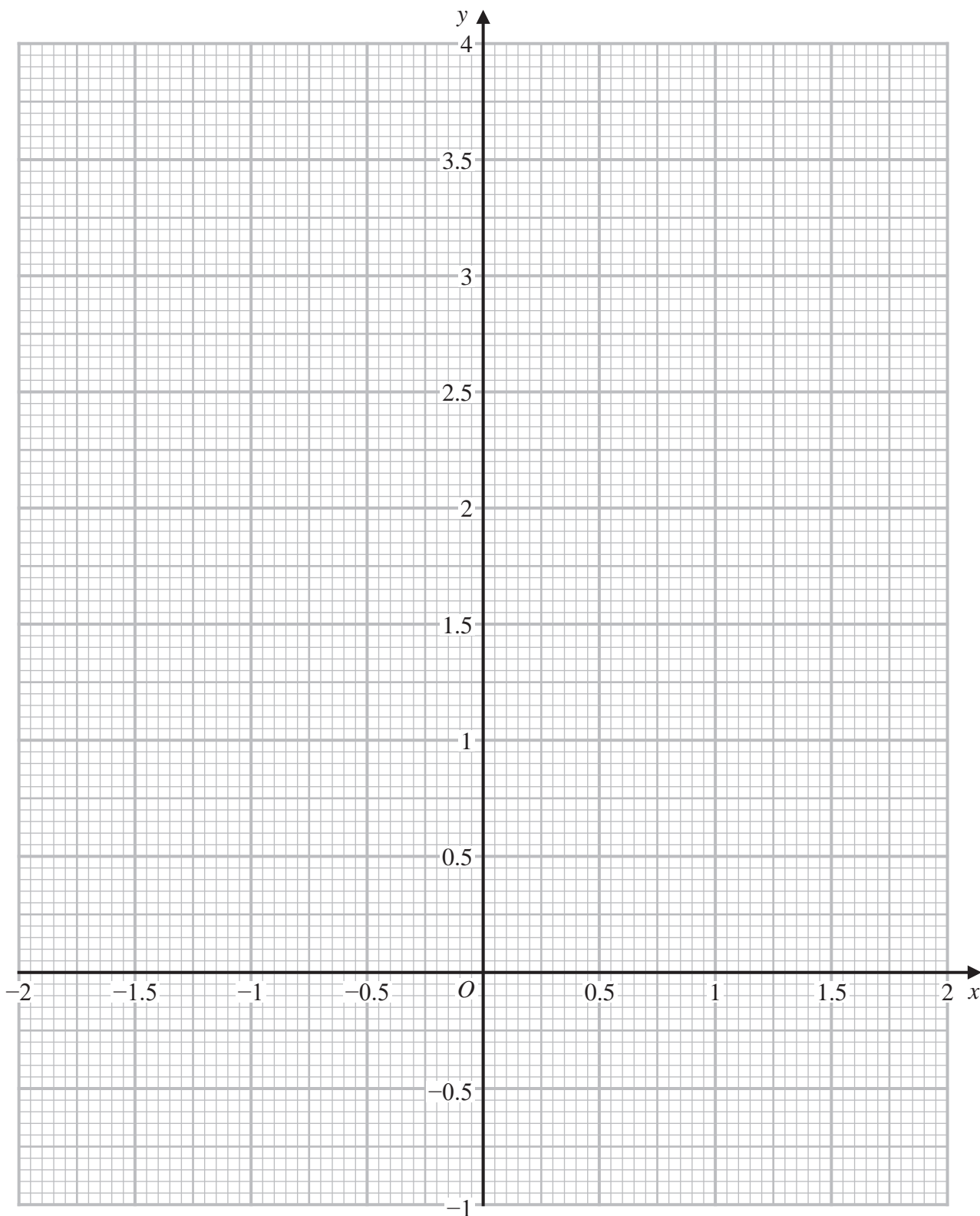
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Question 9 continued

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Question 9 continued

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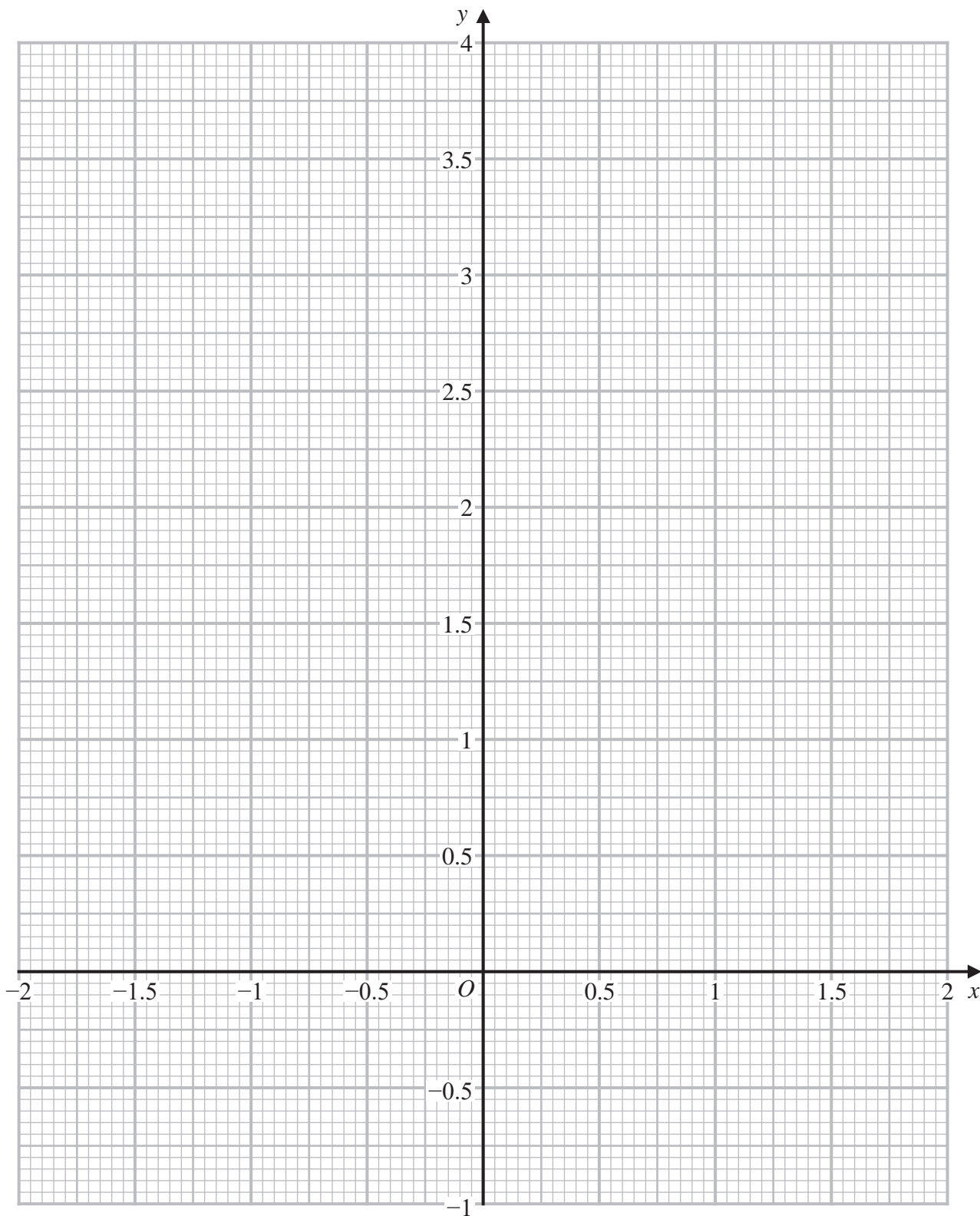
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Question 9 continued

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(Total for Question 9 is 9 marks)



10 (a) Use the factor theorem to show that $(4x - 3)$ is a factor of

$$16x^3 + 11x - 15 \quad (2)$$

(b) Using formulae given on page 2, show that

(i) $\sin 2\theta = 2 \sin \theta \cos \theta$

(ii) $\cos 2\theta = 2 \cos^2 \theta - 1$ (5)

(c) Show that the equation

$$27 \cos \theta \cos 2\theta + 19 \sin \theta \sin 2\theta - 15 = 0$$

becomes the equation

$$16x^3 + 11x - 15 = 0$$

by using the substitution $x = \cos \theta$

(4)

(d) Hence show that any solution of the equation

$$27 \cos \theta \cos 2\theta + 19 \sin \theta \sin 2\theta - 15 = 0$$

is given by $\cos \theta = \frac{3}{4}$

(4)

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Question 10 continued

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Question 10 continued

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Question 10 continued

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(Total for Question 10 is 15 marks)



11

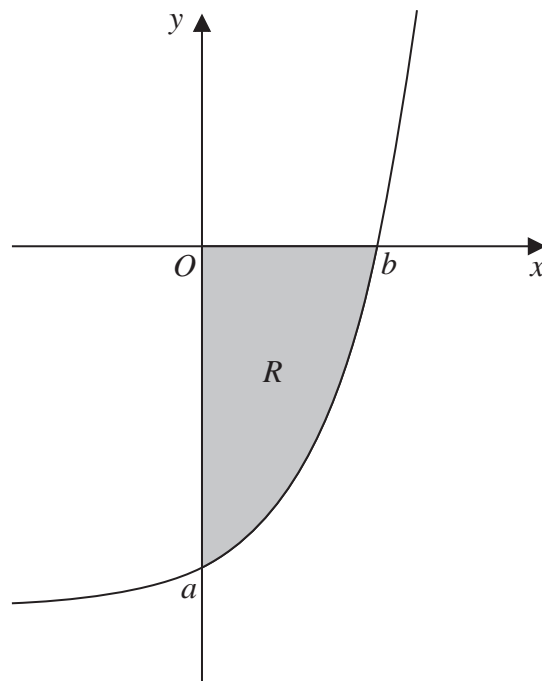
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accurately drawn

Figure 3

The finite region R , shown shaded in Figure 3, is bounded by the curve with equation $y = e^{2x} - 9$ and the coordinate axes.

The curve crosses the coordinate axes at the points with coordinates $(0, a)$ and $(b, 0)$

- (a) (i) Find the value of a
 (ii) Show that $b = \ln 3$

(3)

The region R is rotated through 360° about the x -axis.

- (b) Use calculus to find the volume of the solid generated.

Give your answer in the form $\pi(p \ln 3 + q)$, where p and q are integers.

(6)

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Question 11 continued

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Question 11 continued

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Question 11 continued

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