


Please check the examination details below before entering your candidate information

Candidate surname					Other names				
Pearson Edexcel		Centre Number			Candidate Number				
International GCSE		<input type="text"/> <input type="text"/> <input type="text"/> <input type="text"/> <input type="text"/>			<input type="text"/> <input type="text"/> <input type="text"/> <input type="text"/> <input type="text"/>				
Monday 20 January 2020									
Morning (Time: 2 hours)					Paper Reference 4PM1/02R				
Further Pure Mathematics									
Paper 2R									
Calculators may be used.								Total Marks	

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ►

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Pearson

International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere = $4\pi r^2$

Curved surface area of cone = $\pi r \times$ slant height

Volume of sphere = $\frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric series

Sum to n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r}$ $|r| < 1$

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad \text{for } |x| < 1, n \in \mathbb{Q}$$

Calculus

Quotient rule (differentiation)

$$\frac{d}{dx} \left(\frac{f(x)}{g(x)} \right) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

Trigonometry

Cosine rule

In triangle ABC : $a^2 = b^2 + c^2 - 2bc \cos A$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Logarithms

$$\log_a x = \frac{\log_b x}{\log_b a}$$

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Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1

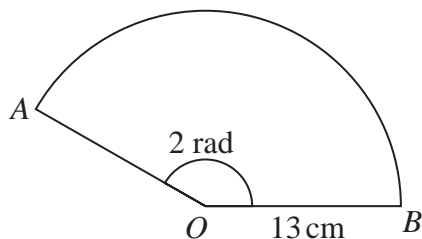


Diagram **NOT**
accurately drawn

Figure 1

Figure 1 shows the sector AOB of a circle with centre O .
The radius of the circle is 13 cm and angle $AOB = 2$ radians.

- (a) Find the length of the arc AB . (1)
- (b) Find the area of the sector AOB . (2)

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(Total for Question 1 is 3 marks)



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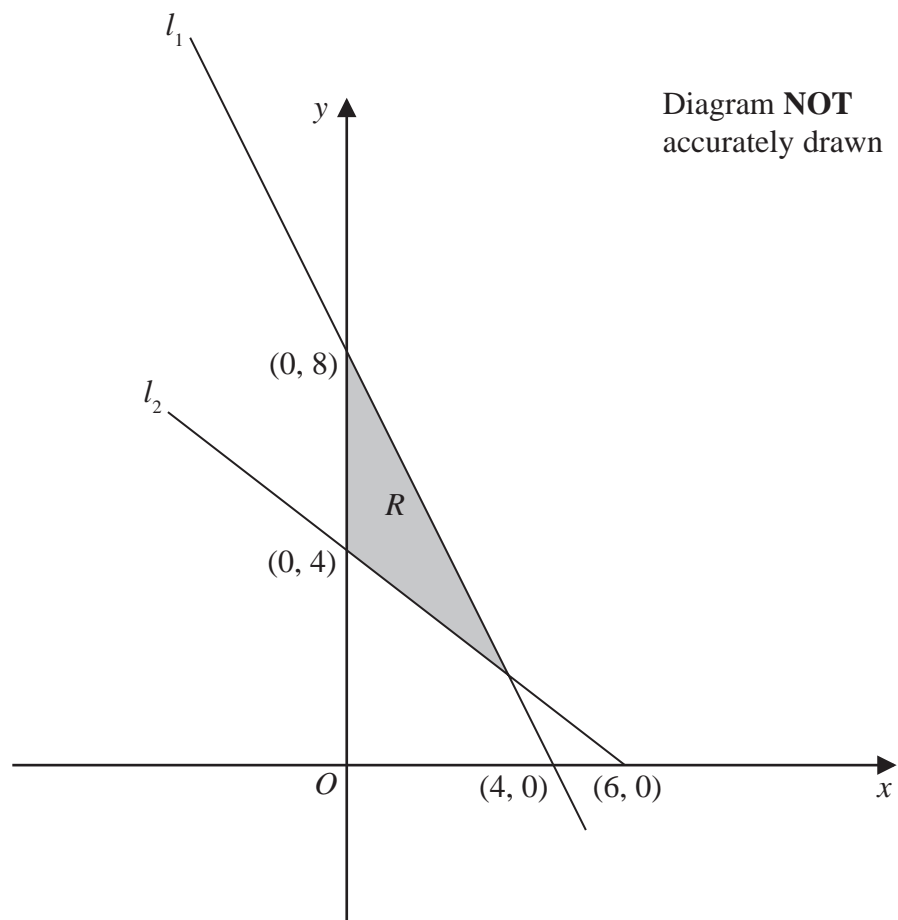


Figure 2

Figure 2 shows the shaded region R bounded by the line l_1 , the line l_2 and the y -axis.

The points with coordinates $(0, 8)$ and $(4, 0)$ lie on l_1

The points with coordinates $(0, 4)$ and $(6, 0)$ lie on l_2

(a) Find, in the form $ax + by = c$, where a , b and c integers, an equation of

(i) l_1

(ii) l_2

(3)

(b) Hence write down three inequalities that define the region R .

(3)

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Question 2 continued

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(Total for Question 2 is 6 marks)



3 In triangle ABC , $AB = 11$ cm and $BC = 12$ cm.

The area of triangle $ABC = 33$ cm²

Find, in cm to 3 significant figures, the two possible lengths of AC .

(5)

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Question 3 continued

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(Total for Question 3 is 5 marks)



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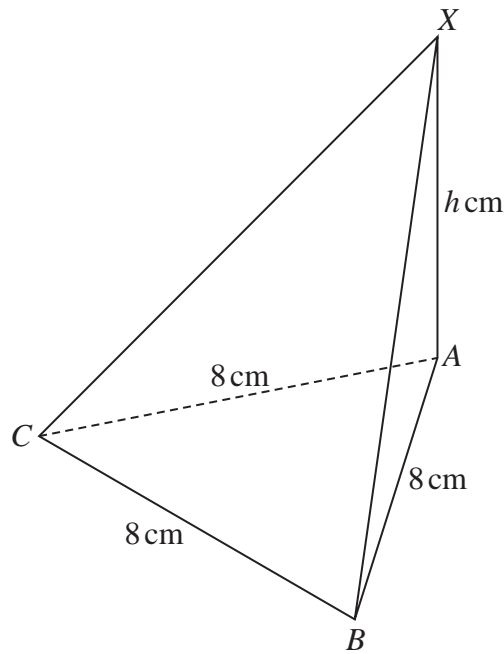


Diagram **NOT** accurately drawn

Figure 3

Figure 3 shows a triangular pyramid $ABCX$.
 The base ABC of the pyramid is an equilateral triangle where $AB = BC = CA = 8$ cm.
 The vertex X of the pyramid is such that AX is perpendicular to the base of the pyramid and $AX = h$ cm.

The volume of the pyramid is $48\sqrt{3}$ cm³

- (a) Show that $h = 9$ (3)
- (b) Find, in degrees to one decimal place, the size of angle BXC . (3)
- (c) Find, in degrees to one decimal place, the size of the angle between the plane BCX and the base ABC of the pyramid. (3)

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Question 4 continued

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Question 4 continued

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Question 4 continued

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(Total for Question 4 is 9 marks)



5 (a) Show that $(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) = \alpha^3 + \beta^3$ (2)

The roots of the equation $2x^2 + 3x + 6 = 0$ are α and β

Without solving the equation,

(b) find the value of $\alpha^3 + \beta^3$ (2)

(c) Show that $(\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2 = \alpha^4 + \beta^4$ (2)

(d) Form a quadratic equation with integer coefficients that has roots $(\alpha^3 - \beta)$ and $(\beta^3 - \alpha)$ (6)

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Question 5 continued

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Question 5 continued

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Question 5 continued

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(Total for Question 5 is 12 marks)



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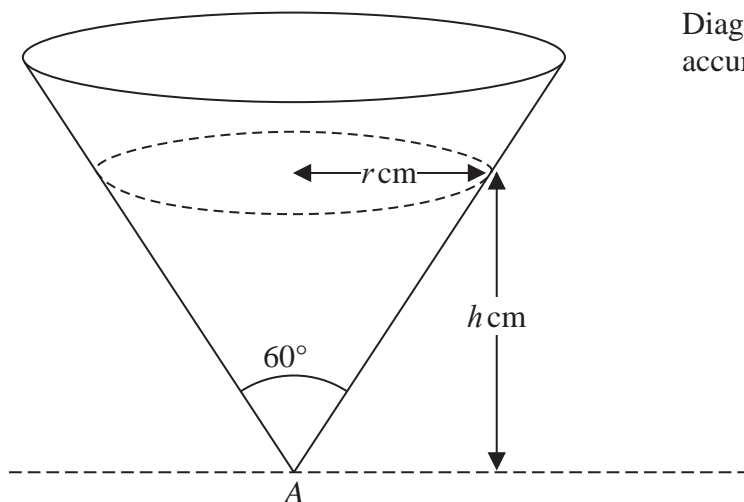


Diagram **NOT**
accurately drawn

Figure 4

Figure 4 shows a hollow right circular cone fixed with its axis of symmetry vertical.

The cone is inverted and contains liquid, which is dripping out of a small hole at the vertex A of the cone at a constant rate of $0.9 \text{ cm}^3/\text{s}$.

At time t seconds after the liquid starts to drip from the cone, the height of the liquid is $h \text{ cm}$ above A . The volume of liquid in the cone at time t seconds is $V \text{ cm}^3$

The vertical angle of the cone is 60°

(a) Show that $V = \frac{1}{9}\pi h^3$ (2)

(b) Find, in cm/s to 3 significant figures, the rate at which the height of the liquid is decreasing when the height of the liquid in the cone above the vertex is 1.2 cm . (4)

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Question 6 continued

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Question 6 continued

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Question 6 continued

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(Total for Question 6 is 6 marks)



7 The geometric series G has first term a , common ratio r and n th term u_n

Given that $u_4 = e^{x+2}$ and that $u_7 = e^{\frac{2x+1}{2}}$

(a) show that $r = e^{-\frac{1}{2}}$

(3)

(b) Hence find a in terms of e and x .

(3)

Given that the sum to infinity of G can be written as $\frac{e^p}{e^{\frac{1}{2}} - 1}$

(c) find an expression for p in terms of x .

(3)

Given that $u_{18} > 1.6$ and that x is an integer,

(d) find the least value of x .

(4)

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Question 7 continued

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Question 7 continued

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Question 7 continued

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(Total for Question 7 is 13 marks)



- 8 (a) Write down the value of k such that $\sin 2A = k \sin A \cos A$ (1)

$$g(A) = 2 + 3\cos A - \sin A - 3\sin 2A - 2\cos^2 A$$

Given that $g(A)$ can be written in the form $(p \cos A - \sin A)(q - r \sin A)$ where p , q and r are integers,

- (b) find the value of p , the value of q and the value of r . (3)

- (c) Hence solve, in radians to 3 significant figures where appropriate, the equation

$$g(2\theta) = 0 \quad \text{for} \quad 0 \leq \theta < \pi \quad (6)$$

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Question 8 continued

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Question 8 continued

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Question 8 continued

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(Total for Question 8 is 10 marks)



9 Given that $\frac{1}{(2-x)^3}$ can be written as $p(1-qx)^{-3}$

(a) find the value of p and the value of q . (2)

(b) Expand $\frac{1}{(2-x)^3}$ in ascending powers of x up to and including the term in x^3 and express each coefficient as an exact fraction in its lowest terms. (3)

$$f(x) = \frac{a+bx}{(2-x)^3} \text{ where } a \text{ and } b \text{ are integers}$$

The first three terms of the expansion of $f(x)$ are $\frac{3}{8} - \frac{43}{16}x + cx^2$

(c) Find the value of a and the value of b . (3)

(d) Find the exact value of c . (2)

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Question 9 continued

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Question 9 continued

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Question 9 continued

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(Total for Question 9 is 10 marks)



10 The equation of a curve C is $y = f(x)$ where $f'(x) = 3x^2 - 4x - p$ and $p \neq 0$

The points with coordinates $(2, 0)$ and $(-1, 9)$ lie on C .

(a) Show that C has equation $y = x^3 - 2x^2 - 4x + 8$ (6)

The straight line l has equation $y = 8 - 4x$

(b) Use algebraic integration to find the exact area of the finite region bounded by C and l . (6)

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Question 10 continued

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Question 10 continued

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(Total for Question 10 is 12 marks)



11 The curve C has equation $y = \frac{3x - 2}{x + 1}$

(a) Write down an equation of the asymptote to C which is parallel to the

- (i) x -axis (ii) y -axis

(2)

(b) Find the coordinates of the point where C crosses the

- (i) x -axis (ii) y -axis

(2)

(c) Sketch C , showing clearly the asymptotes and the coordinates of the points where C crosses the coordinate axes.

(3)

The straight line l has equation $y = mx + 4$

Given that there are **no** points of intersection between l and C ,

(d) show algebraically that the range of possible values of m can be written as

$$a - 2\sqrt{b} < m < a + 2\sqrt{b}$$

where a and b are integers whose values need to be found.

(7)

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Question 11 continued

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Question 11 continued

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Question 11 continued

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