Please check the examination details below before entering your candidate information

| Candidate surname | Other names |
|---|---|

**Pearson Edexcel International GCSE**

Centre Number | Candidate Number

# Monday 20 January 2020

Morning (Time: 2 hours) | Paper Reference **4PM1/02**

# Further Pure Mathematics

**Level 2**
**Paper 2**

**Calculators may be used.**

Total Marks

## Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

## Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

*Turn over* ▶

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Pearson

# International GCSE in Further Pure Mathematics Formulae sheet

## Mensuration

**Surface area of sphere** = $4\pi r^2$

**Curved surface area of cone** = $\pi r \times$ slant height

**Volume of sphere** = $\frac{4}{3}\pi r^3$

## Series

### Arithmetic series

Sum to $n$ terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

### Geometric series

Sum to $n$ terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

### Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

## Calculus

### Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

## Trigonometry

### Cosine rule

In triangle $ABC$: $a^2 = b^2 + c^2 - 2bc\cos A$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$ $\qquad$ $\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$ $\qquad$ $\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$ $\qquad$ $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$

## Logarithms

$\log_a x = \frac{\log_b x}{\log_b a}$

**Answer all ELEVEN questions.**

**Write your answers in the spaces provided.**

**You must write down all the stages in your working.**

**1** A particle $P$ is moving along a straight line that passes through the fixed point $O$. At time $t$ seconds, $t \geqslant 0$, the displacement, $s$ metres, of $P$ from $O$ is given by

$$s = t^3 + 4t^2 - 27t + 4$$

Find the value of $t$ at the instant when the velocity of $P$ is 8 m/s.

**(4)**

**(Total for Question 1 is 4 marks)**

**2** Find the set of values of $x$ for which

(a) $3 + 2x \leqslant x + 2$

**(1)**

(b) $8x^2 + 10x < 3$

**(4)**

(c) **both** $3 + 2x \leqslant x + 2$ **and** $8x^2 + 10x < 3$

**(1)**

**Question 2 continued**

**(Total for Question 2 is 6 marks)**

**3**

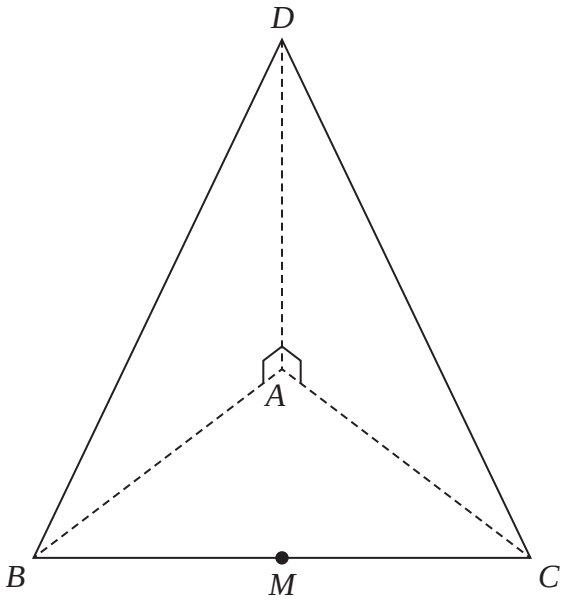



Diagram **NOT** accurately drawn

**Figure 1**

Figure 1 shows a triangular pyramid *ABCD*.

The base, *ABC*, of the pyramid is a horizontal isosceles triangle with $AB = AC = 10$ cm and $BC = 16$ cm. The midpoint of *BC* is *M*.

The face *BCD* of the pyramid is an isosceles triangle with $BD = CD = 26$ cm and *D* is vertically above *A*.

$\angle BAD = \angle CAD = 90°$

(a) Calculate the length, in cm, of *AM*.

**(2)**

Calculate, in degrees to the nearest degree,

(b) the size of $\angle BCD$,

**(3)**

(c) the size of the angle between the planes *BCA* and *BCD*.

**(4)**

**Question 3 continued**

**Question 3 continued**

**Question 3 continued**

**(Total for Question 3 is 9 marks)**

**4** The points $A$, $B$, $C$ and $D$ are the vertices of a quadrilateral $ABCD$ such that

$\overrightarrow{AB} = 7\mathbf{i} + p\mathbf{j}$ $\overrightarrow{AC} = 11\mathbf{i} - p\mathbf{j}$ $\overrightarrow{AD} = 4\mathbf{i} - 2p\mathbf{j}$

(a) Show that, for all values of $p$, $ABCD$ is a parallelogram.

**(3)**

Given that $|\overrightarrow{BD}| = 3\sqrt{10}$

(b) find the possible values of $p$.

**(3)**

Given that $p > 0$

(c) find a unit vector which is parallel to $\overrightarrow{BD}$.

**(1)**

**Question 4 continued**

**Question 4 continued**

**Question 4 continued**

**(Total for Question 4 is 7 marks)**

**5** Given that $\alpha$ and $\beta$ are such that $\alpha + \beta = \frac{7}{2}$ and $\alpha\beta = 2$

(a) form a quadratic equation with integer coefficients that has roots $\alpha$ and $\beta$,

**(2)**

(b) form a quadratic equation with integer coefficients that has roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$.

**(6)**

**Question 5 continued**

**(Total for Question 5 is 8 marks)**

**6**

Diagram **NOT** accurately drawn

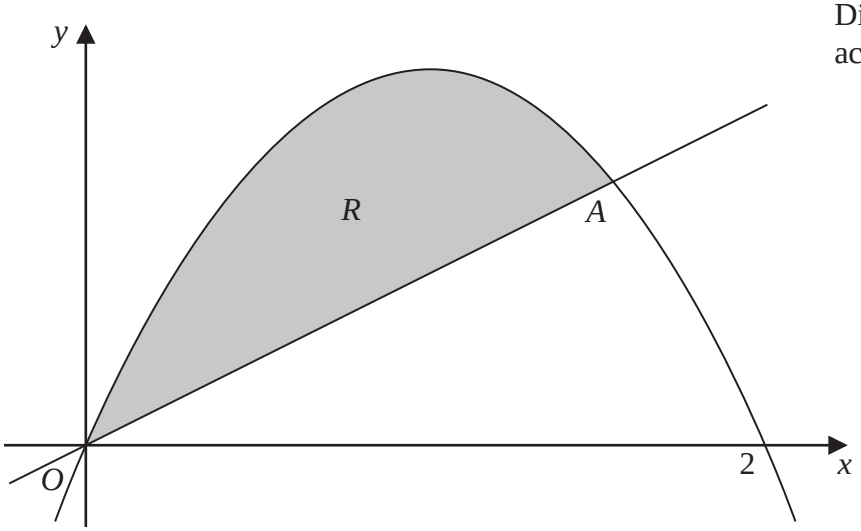



**Figure 2**

The region $R$, shown shaded in Figure 2, is bounded by the curve with equation $y = 2x - x^2$ and the line with equation $2y - x = 0$

The curve and the line intersect at the origin $O$ and the point $A$.

(a) Show that the point $A$ has coordinates $\left(\frac{3}{2}, \frac{3}{4}\right)$.

**(2)**

The region $R$ is rotated through 360° about the $x$-axis.

(b) Use algebraic integration to find, in terms of $\pi$, the volume of the solid formed.

**(6)**

**Question 6 continued**

**Question 6 continued**

**Question 6 continued**

**(Total for Question 6 is 8 marks)**

**7** The 7th term of a geometric series is 192 and the 8th term of this geometric series is 1152

(a) Find, as a fraction in its simplest form, the 4th term of this geometric series.

**(3)**

A different geometric series $G$ has a common ratio $r$ and $n$th term $t_n$

Given that $t_3 = 24$ and $t_2 + t_3 + t_4 = -36$

(b) show that $r$ satisfies the equation

$$2r^2 + 5r + 2 = 0$$

**(5)**

Given further that $G$ is convergent with sum to infinity $S$,

(c) find the value of $S$.

**(4)**

**Question 7 continued**

**Question 7 continued**

**Question 7 continued**

**(Total for Question 7 is 12 marks)**

**8** Given that $y = e^{3x} \sin 2x$

show that $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 13y = 0$

**(8)**

**Question 8 continued**

**Question 8 continued**

**Question 8 continued**

**(Total for Question 8 is 8 marks)**

**9** A curve $C$ has equation

$$y = \frac{qx - 2}{x - p} \qquad x \neq p$$

The curve crosses the $y$-axis at the point $A$.

The line $l$ with equation $y = x + 2$ is the normal to $C$ at $A$.

(a) (i) Show that $p = 1$

(ii) Find the value of $q$.

**(7)**

(b) Using the axes on the opposite page, sketch $C$, showing clearly the asymptotes and the coordinates of the points where $C$ crosses the coordinate axes.

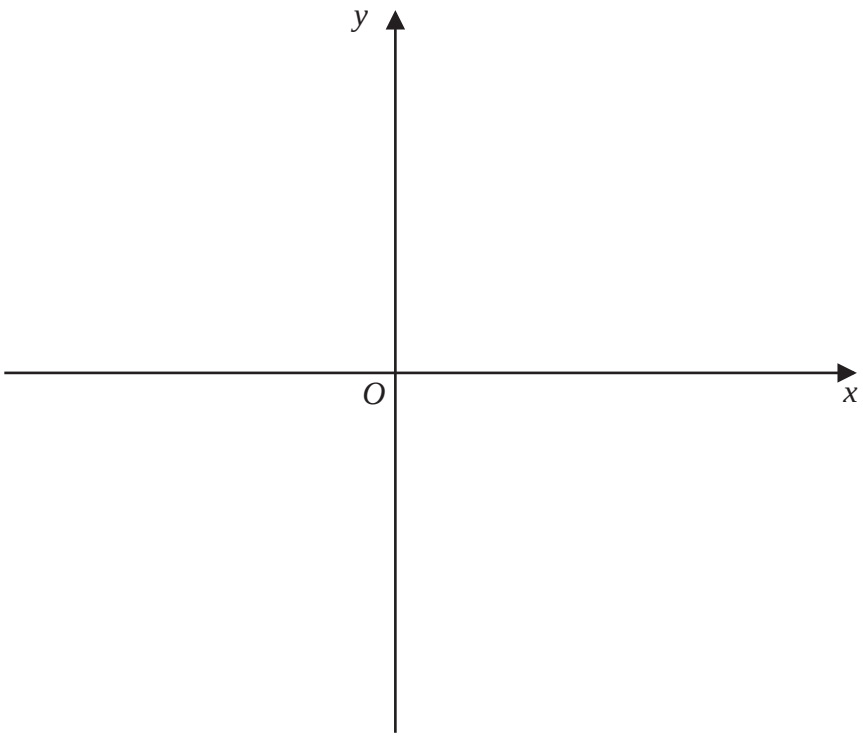
**(5)**

The line $l$ meets $C$ again at the point $D$.

(c) Find the $x$ coordinate of $D$.

**(4)**

**Question 9 continued**



**Question 9 continued**

**Question 9 continued**

**(Total for Question 9 is 16 marks)**

**10** The volume of a sphere is increasing at a constant rate of $40\,cm^3/s$.

Find the rate of increase, in $cm^2/s$, of the surface area of the sphere at the instant when the radius is 4 cm.

**(9)**

**Question 10 continued**

**(Total for Question 10 is 9 marks)**

**11** (a) Express the equation

$$3\sin(A - B) = \sin(A + B)$$

in the form $\tan A = k\tan B$, giving the value of the integer $k$.

**(4)**

(b) Given that $\theta \neq \dfrac{(2n+1)\pi}{2}$ where $n \in \mathbb{Z}$,

show that $\dfrac{\cos^4\theta - \sin^4\theta}{\cos^2\theta} = 1 - \tan^2\theta$

**(3)**

(c) Using the exact values of $\sin x°$, $\cos x°$ and $\tan x°$ for $x = 30, 45, 60$

show that

(i) $\cos 15° = \dfrac{\sqrt{6} + \sqrt{2}}{4}$

**(2)**

(ii) $\tan 255° = \dfrac{3 + \sqrt{3}}{3 - \sqrt{3}}$

**(4)**

**Question 11 continued**

**Question 11 continued**

**(Total for Question 11 is 13 marks)**

**TOTAL FOR PAPER IS 100 MARKS**