Model Answers

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel International GCSE

Centre Number | Candidate Number

Monday 7 January 2019

Morning (Time: 2 hours) | Paper Reference **4MA1/1HR**

Mathematics A

Level 1/2
Paper 1HR
Higher Tier

You must have:
Ruler graduated in centimetres and millimetres, protractor, compasses, pen, HB pencil, eraser, calculator. Tracing paper may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- **Calculators may be used.**
- You must **NOT** write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶

P59022A

1/1/1/1/

P 5 9 0 2 2 A 0 1 2 8

Pearson

International GCSE Mathematics

Formulae sheet – Higher Tier

Arithmetic series

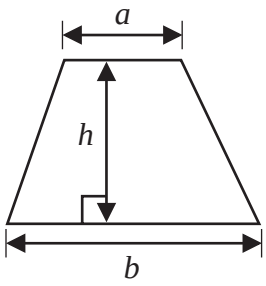
Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

The quadratic equation

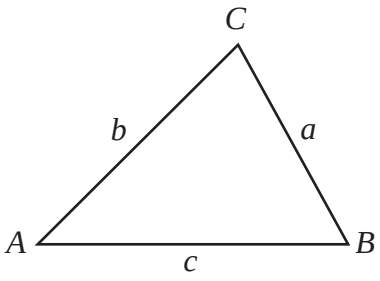
The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$ are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Area of trapezium $= \frac{1}{2}(a + b)h$



Trigonometry



In any triangle *ABC*

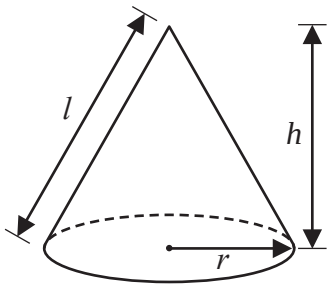
Sine Rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine Rule $a^2 = b^2 + c^2 - 2bc\cos A$

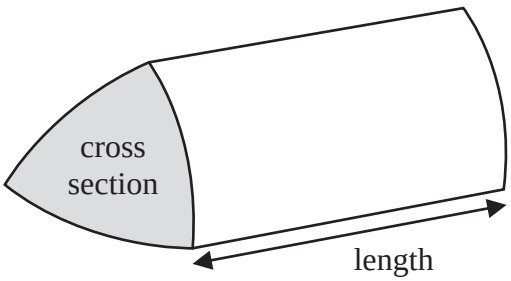
Area of triangle $= \frac{1}{2}ab\sin C$

Volume of cone $= \frac{1}{3}\pi r^2 h$

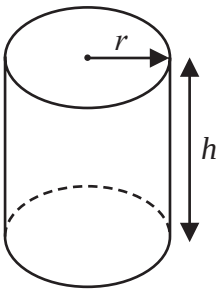
Curved surface area of cone $= \pi r l$



Volume of prism
= area of cross section × length

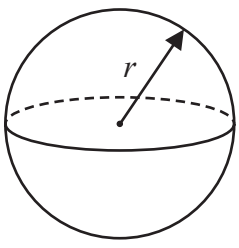


Volume of cylinder $= \pi r^2 h$
Curved surface area of cylinder $= 2\pi r h$



Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$



Answer ALL TWENTY THREE questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 Show that $1\frac{2}{3} + 2\frac{3}{4} = 4\frac{5}{12}$

$1\frac{2}{3} = \frac{1\times3+2}{3} = \frac{3+2}{3} = \frac{5}{3}$

$2\frac{3}{4} = \frac{2\times4+3}{4} = \frac{8+3}{4} = \frac{11}{4}$

$\frac{5}{3}\ {}^{\times4}\ +\ \frac{11}{4}\ {}^{\times3} \Rightarrow \frac{20}{12} + \frac{33}{12} = \frac{20+33}{12} = \frac{53}{12}$

$= \frac{48+5}{12} = \frac{48}{12} + \frac{5}{12}$

$= 4\frac{5}{12}$

(Total for Question 1 is 3 marks)

2 There are 60 children in a club.

In the club, the ratio of the number of girls to the number of boys is 3 : 1 ⇒ 4 parts

$\frac{3}{5}$ of the girls play a musical instrument.

$\frac{4}{5}$ of the boys play a musical instrument.

What fraction of the 60 children play a musical instrument?

No. of girls = $\frac{3}{4} \times 60 = 45$

who play = $\frac{3}{5} \times 45 = 3 \times 9 = 27$

No. of boys = $\frac{1}{4} \times 60 = 15$

who play = $\frac{4}{5} \times 15 = 4 \times 3 = 12$

Total who play = $12 + 27 = 39$

As a fraction = $\frac{39}{60} = \frac{13}{20}$ (÷3)

(Total for Question 2 is 4 marks)

3

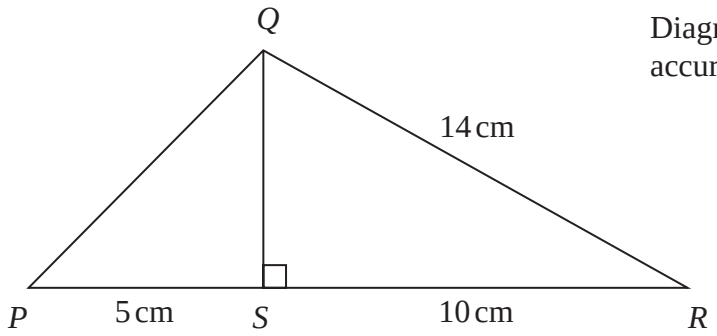


Diagram **NOT** accurately drawn

In triangle *PQR*,

S is the point on *PR* such that angle *RSQ* = 90°
RQ = 14 cm
RS = 10 cm
SP = 5 cm

Work out the length of *PQ*.

$RQ^2 = RS^2 + SQ^2$ (pythagoras theorem : $a^2 = b^2 + c^2$)

$14^2 = 10^2 + SQ^2$

$SQ^2 = 14^2 - 10^2$

$SQ^2 = 96$

In Δ PQS:

$PQ^2 = SQ^2 + SP^2$

$PQ^2 = 96 + 5^2$

$PQ^2 = 121$

$PQ = \sqrt{121} = 11$ cm

11 cm

(Total for Question 3 is 4 marks)

4 *a*, *a*, *b* and 40 are four numbers.

a is the least number.
40 is the greatest number.

The range of the four numbers is 14
The median of the four numbers is 30

Work out the value of *a* and the value of *b*.

range = maximum − minimum

$14 = 40 - a$

$a = 40 - 14 = 26$

a, a, b, 40

median = $\frac{a+b}{2}$

$\frac{a+b}{2} = 30$

$26 + b = 30 \times 2$

$b = 60 - 26 \Rightarrow b = 34$

a = 26

b = 34

(Total for Question 4 is 3 marks)

5 The Shanghai Maglev Train takes 8 minutes to travel a distance of 30.5 kilometres.

Work out the average speed of the train.
Give your answer in kilometres per hour.

Average speed = $\frac{\text{Total distance}}{\text{Total time}}$

Time taken in hours = $\frac{8}{60}$ ← 60 minutes in a hour.

Average speed = $\frac{30.5}{\left(\frac{8}{60}\right)}$ = 228.75

228.75 kilometres per hour

(Total for Question 5 is 3 marks)

6 The diagram shows the triangle PQR.

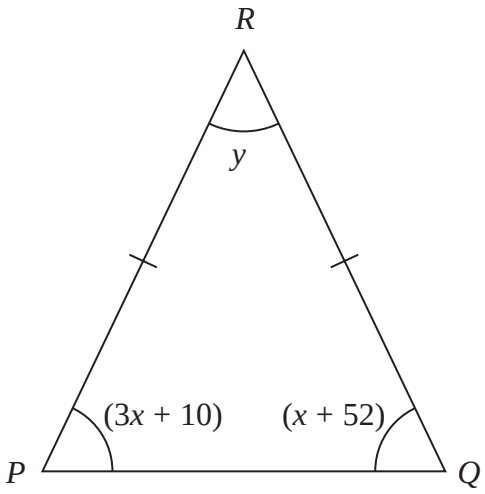


Diagram **NOT** accurately drawn

In the diagram, all the angles are in degrees.

$RP = RQ$

Find the value of y.
Show clear algebraic working.

$3x+10 = x+52$ (Isosceles base angles are equal)

$3x - x = 52 - 10$

$2x = 42$

$x = 21$

$(3x+10) + y + (x+52) = 180$ (Angles in a triangle add up to 180)

$3(21) + 10 + y + (21) + 52 = 180$

$63 + 10 + y + 21 + 52 = 180$

$y + 146 = 180$

$y = 180 - 146$

$= 34$

$y =$ 34

(Total for Question 6 is 4 marks)

7 The diagram shows two water towers in Kuwait.

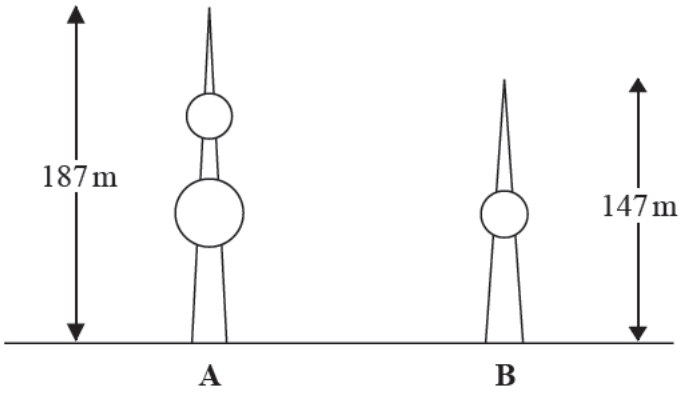


Diagram **NOT** accurately drawn

The real height of tower **A** is 187 m.
The real height of tower **B** is 147 m.

Ahmed makes a scale model of both towers.

The height of tower **A** on the scale model is 90 cm.

Work out the height of tower **B** on the scale model.
Give your answer correct to the nearest centimetre.

187 m → 90 cm
÷187 ↓ ↓
1 m → $\frac{90}{187}$ cm
×147 ↓ ↓
147 m → $\frac{90}{187} \times 147 = 70.75$
7 > 5
∴ round up

71 cm

(Total for Question 7 is 3 marks)

8 Solve the simultaneous equations

$$4x + 2y = 9 \quad \text{—①}$$
$$x - 4y = 9 \quad \text{—②}$$

Show clear algebraic working.

② × 4 $\quad 4x - 16y = 36$ —③

Cancel x to find y

① − ③ $\quad 4x + 2y = 9$
$\quad - 4x - 16y = 36$
$\quad 18y = -27$
$\quad y = -\frac{3}{2}$

Subs $y = -\frac{3}{2}$ in ②

$x - 4\left(-\frac{3}{2}\right) = 9$

$x + 6 = 9 \Rightarrow x = 9 - 6 = 3$

$x = 3$

$y = -\frac{3}{2}$

(Total for Question 8 is 3 marks)

9 $N = 480 \times 10^9$

(a) Write N as a number in standard form.

$4.8 \times 10^2 \times 10^9 \Rightarrow 4.8 \times 10^{11}$ $\quad a^b \times a^c = a^{b+c}$

number between 0 and 10

(1)

(b) Write N as a product of powers of its prime factors.
Show your working clearly.

2	4.8×10^{11}
2	2.4×10^{11}
2	1.2×10^{11}
2	6×10^{10}
2	3×10^{10}
2	1.5×10^{10}
2	7.5×10^9
2	3.75×10^9
2	1.875×10^9
2	9.375×10^8
2	4.6875×10^8
2	2.34375×10^8
2	1.171875×10^8
2	5.859375×10^7
3	2.9296875×10^7
5	9.765625×10^6
5	1.953125×10^6
5	3.90625×10^5
5	78125
	15625

5	15625
5	3125
5	625
5	125
5	25
5	5
	1

$2^{14} \times 3 \times 5^{10}$

(3)

(c) Find the largest factor of N that is an odd number.

$3 \times 5^{10} = 29296875$

(1)

(Total for Question 9 is 5 marks)

10 The shape, shown shaded in the diagram, is the region between two semicircles.

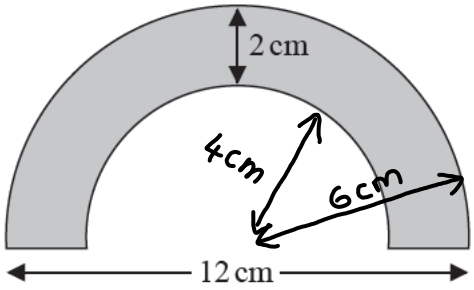


Diagram **NOT** accurately drawn

The diameter of the outer semicircle is 12 cm.
The shape has constant thickness 2 cm.

Calculate the area of the shape.
Give your answer as a multiple of π.

equation to find the area of a semi-circle

Area of the outer Semi-circle $= \frac{1}{2} \times \pi \times r^2 = \frac{1}{2} \times \pi \times 6^2$

$= \frac{1}{2} \times 36 \times \pi = 18\pi$

Area of inner Semi-circle $= \frac{1}{2} \times \pi \times 4^2 = \frac{1}{2} \times 16 \times \pi = 8\pi$

4^2 ← 6 − 2

Difference of areas : $18\pi - 8\pi = 10\pi$

10π cm^2

(Total for Question 10 is 3 marks)

11 There are 12 boys and 8 girls in a class.
The boys and the girls have some coins.

The mean number of coins that the boys have is 5.5
The girls have a total of 18 coins.

Work out the mean number of coins the 20 children have.

$$\frac{\text{Total coins with boys}}{\text{No. of boys}} = \text{mean number of coins with boys}$$

No. of coins with boys $\rightarrow \frac{x}{12} = 5.5 \Rightarrow x = 12 \times 5.5 = 66$

Total coins $= 18 + 66 = 84$ coins (g, b)

$$\text{Mean} = \frac{\text{Total coins}}{\text{Total children}} = \frac{84}{20} = \frac{42}{10} = 4.2 \quad (\div 2)$$

(Total for Question 11 is 3 marks)

12 Here are the first four terms of a sequence of fractions.

$$\frac{1}{1} \quad \frac{2}{3} \quad \frac{3}{5} \quad \frac{4}{7}$$

The numerators of the fractions form the sequence of whole numbers 1 2 3 4 …
The denominators of the fractions form the sequence of odd numbers 1 3 5 7 …

(a) Write down an expression, in terms of n, for the nth term of this sequence of fractions.

numerator : n

Denominator : $2n-1$

$\therefore$ n^{th} term $= \dfrac{n}{2n-1}$

(2)

(b) Using algebra, prove that when the square of any odd number is divided by 4 the remainder is 1

→ odd numbers have an n^{th} term

$2n-1$

→ Squaring; $(2n-1)^2 = (2n-1)(2n-1)$

$= 4n^2 - 2n - 2n + 1$

$= 4n^2 - 4n + 1$

→ Dividing by 4

$$4 \overline{)\, 4n^2 - 4n + 1}$$

$n^2 - n$

$4n^2 - 4n$

1 ← remainder for any odd number

(3)

(Total for Question 12 is 5 marks)

13 A curve **C** has equation $y = x^3 - x^2 - 8x + 12$

↑ constants derive to 0.

(a) Find $\frac{dy}{dx}$

$\frac{dy}{dx} = 3x^{3-1} - 2x^{2-1} - 8x^{1-1}$

$= 3x^2 - 2x - 8$

$\frac{dy}{dx} = 3x^2 - 2x - 8$

(2)

The curve **C** has two turning points.

(b) Work out the x coordinates of the two turning points.
Show your working clearly.

turning points $\longrightarrow \frac{dy}{dx} = 0$

$3x^2 - 2x - 8 = 0$

$3x^2 - 6x + 4x - 8 = 0$

$3x(x-2) + 4(x-2) = 0$

$(x-2)(3x+4) = 0$

$x = 2 \qquad 3x = -4$

$x = \frac{-4}{3}$

(3)

(c) Show that the x-axis is a tangent to the curve **C**.

Subs $x = 2$ in C:

$(2)^3 - (2)^2 - 8(2) + 12$

$= 8 - 4 - 16 + 12$

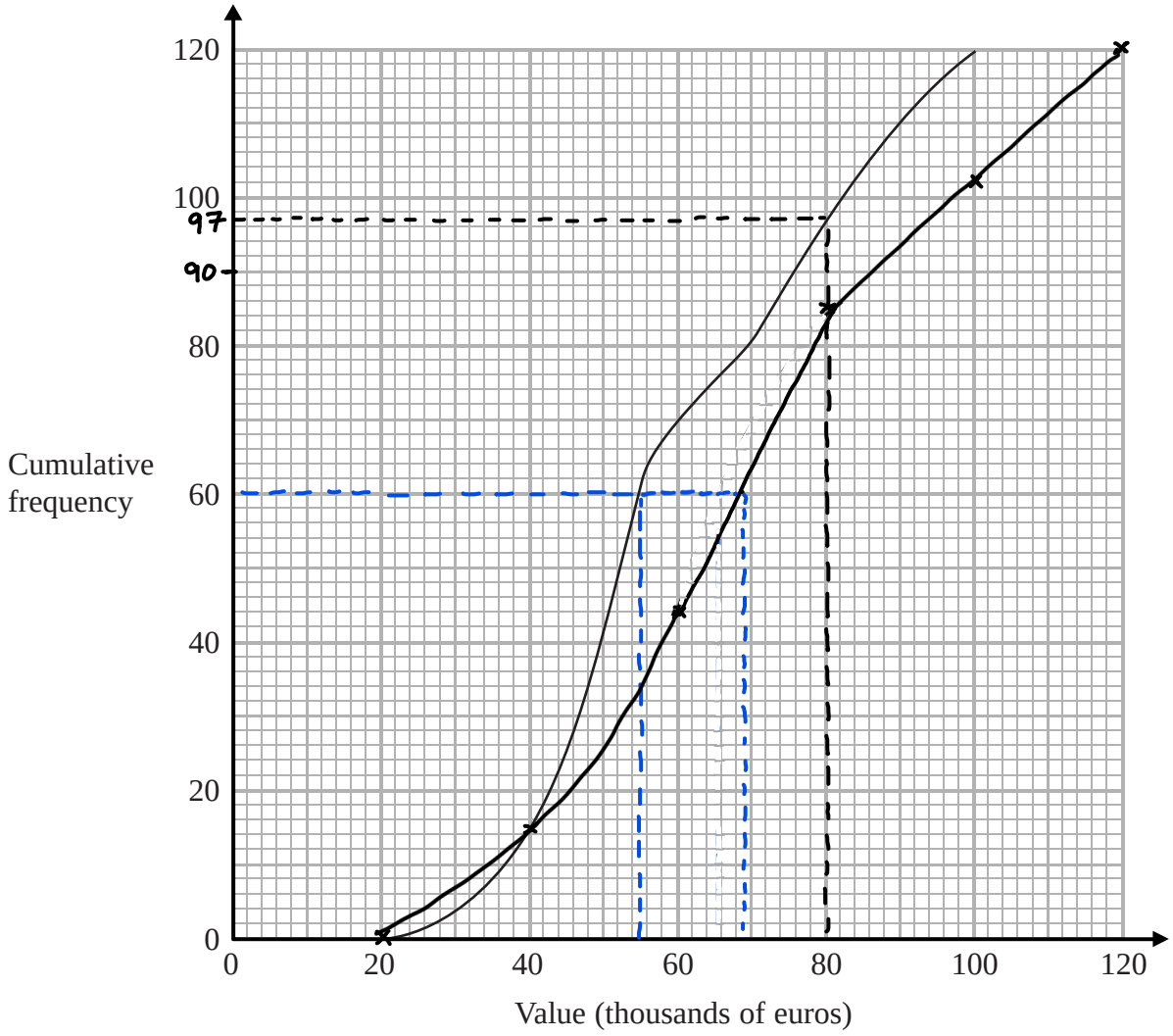
$= 0$

(2,0) is a turning point where the tangent x-axis touches the curve.

(2)

(Total for Question 13 is 7 marks)

14 The cumulative frequency diagram gives information about the values, in thousands of euros, of 120 apartments in 2015



(a) Find an estimate for the number of these apartments with a value of 80 thousand euros or less in 2015

No. of apartments = 97

< 80,000

(1)

The table gives information about the values, in thousands of euros, of the same 120 apartments in 2018

Value in thousands of euros (v)	Cumulative frequency
$0 < v \leqslant 20$	0
$0 < v \leqslant 40$	15
$0 < v \leqslant 60$	44
$0 < v \leqslant 80$	85
$0 < v \leqslant 100$	102
$0 < v \leqslant 120$	120

(b) On the grid opposite, draw a cumulative frequency diagram for this information.

(2)

(c) Find an estimate for the increase in the median value for these apartments from 2015 to 2018

2015

$\frac{1}{2} \times 120$th

60th

55

2018

$\frac{1}{2} \times 120$th

60th

69

$69 - 55 = 14$

14 thousand euros

(2)

(Total for Question 14 is 5 marks)

15 (a) Simplify $(3x^2y^5)^4$

$(a^b)^c = a^{bxc}$

$(3)^4(x^2)^4(y^5)^4 = 81x^8y^{20}$

(2)

(b) Expand and simplify $4n(n-3)(n+5)$

$4n(n^2+5n-3n-15)$

$4n(n^2+2n-15)$ Simplify

$4n^3+8n^2-60n$ Factorise

$(4n^2+20n)(n-3)$

(2)

(c) Factorise $4c^2-9d^2$

$a^2-b^2 = (a+b)(a-b)$

$(2c)^2-(3d)^2$

$(2c+3d)(2c-3d)$

(1)

(d) Simplify fully $\dfrac{x^2-7x+12}{4x-x^2}$

$\dfrac{x^2-3x-4x+12}{x(4-x)} = \dfrac{x(x-3)-4(x-3)}{x(4-x)}$

$= \dfrac{(x-4)(x-3)}{-(x)(x-4)} = \dfrac{-(x-3)}{x} = \dfrac{3-x}{x}$

(3)

(Total for Question 15 is 8 marks)

16 There are 12 beads in a bag.

7 of the beads are red.
3 of the beads are green.
2 of the beads are yellow.

Lucy takes at random a bead from the bag and keeps it.
Then Julian takes at random a bead from the bag.

(a) Work out the probability that they each take a yellow bead.

$P(yy) = \frac{2}{12} \times \frac{1}{11} = \frac{2}{132} = \frac{1}{66}$

AND

No. of available beads have decreased as not replaced.

(2)

(b) Work out the probability that the beads they take are **not** the same colour.

Total maximum probability 1

$1 - [P(gg) + P(rr) + P(yy)]$

$1 - \left[\frac{3}{12} \times \frac{2}{11} + \frac{7}{12} \times \frac{6}{11} + \frac{2}{132}\right]$

$1 - \left[\frac{6}{132} + \frac{42}{132} + \frac{2}{132}\right]$

$1 - \left[\frac{50}{132}\right]$

$\frac{132}{132} - \frac{50}{132} = \frac{132-50}{132} = \frac{82}{132} = \frac{41}{66}$

$\frac{132}{132} = \frac{1}{1} = 1$

(3)

(Total for Question 16 is 5 marks)

17 Here are a solid sphere and a solid cylinder.

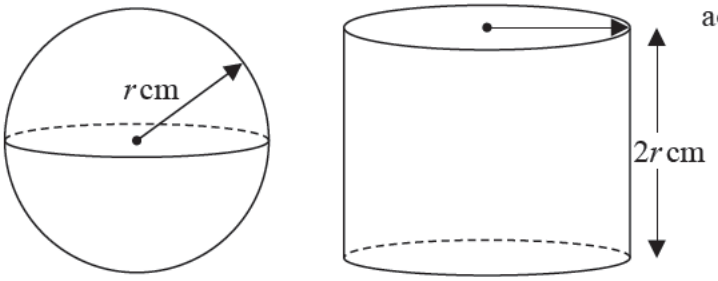


Diagram **NOT** accurately drawn

The radius of the sphere is r cm.
The radius of the cylinder is r cm.
The height of the cylinder is $2r$ cm.

The total surface area of the cylinder is $k\pi$ cm^2

(a) Find an expression for k in terms of r.

Surface area of a cylinder $= 2\pi r^2 + 2\pi rh$

Two circular disks on the top and bottom.

$$k\pi = 2\pi r^2 + 2\pi r(2r)$$
$$k\pi = 2\pi r^2 + 4\pi r^2$$
$$\frac{k\pi}{\pi} = \frac{6\pi r^2}{\pi}$$
$$k = 6r^2$$

(2)

(b) Show that the ratio

total surface area of the cylinder : total surface area of the sphere

is the same as the ratio

volume of the cylinder : volume of the sphere

Total Surface area of the cylinder : Total Surface area of the sphere

$6\pi r^2 : 4\pi r^2 \Rightarrow 6:4 \Rightarrow 3:2$ $\div \pi r^3$

Volume of cylinder : Volume of the sphere

$\pi r^2 h : \frac{4}{3}\pi r^3$

$\pi r^2 (2r) : \frac{4}{3}\pi r^3$

$2\pi r^3 : \frac{4}{3}\pi r^3$ $\div \pi r^3$

$2 : \frac{4}{3} \Rightarrow 6:4 \Rightarrow 3:2$

(3)

(Total for Question 17 is 5 marks)

18 Show that $\frac{\sqrt{8}}{\sqrt{8}-2}$ can be written in the form $n + \sqrt{n}$, where n is an integer.

Show your working clearly.

Rationalise the denominator

$$\frac{\sqrt{8}}{\sqrt{8}-2} \times \frac{\sqrt{8}+2}{\sqrt{8}+2}$$

$$= \frac{\sqrt{8}(\sqrt{8}+2)}{(\sqrt{8}-2)(\sqrt{8}+2)} = \frac{\sqrt{8}\sqrt{8} + 2\sqrt{8}}{\sqrt{8}\sqrt{8} + 2\sqrt{8} - 2\sqrt{8} - 4}$$

$$= \frac{8+2\sqrt{8}}{8-4} = \frac{8+2\sqrt{8}}{4} = \frac{8}{4} + \frac{(2)2\sqrt{2}}{4}$$

$\sqrt{8} = \sqrt{4\times 2} = 2\sqrt{2}$

$$= 2 + \sqrt{2}$$

(Total for Question 18 is 3 marks)

19

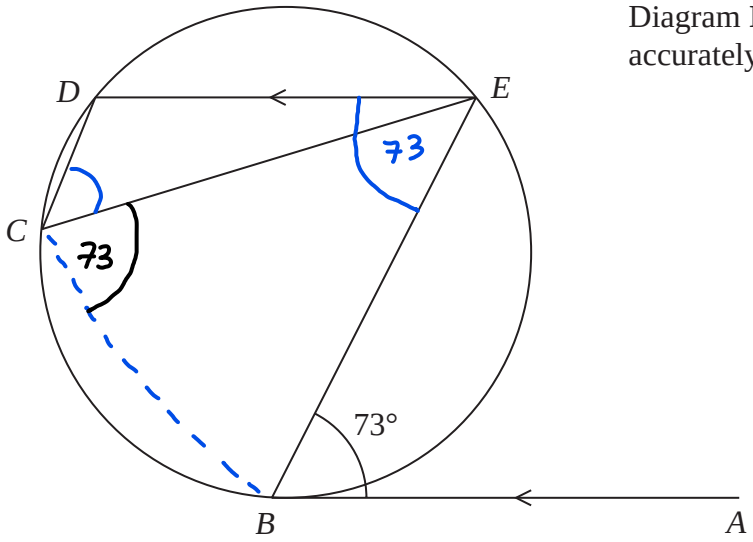


Diagram **NOT** accurately drawn

B, *C*, *D* and *E* are points on a circle.

AB is the tangent at *B* to the circle.
AB is parallel to *ED*.
Angle *ABE* = 73°

Work out the size of angle *DCE*.
Give a reason for each stage of your working.

$B\hat{C}E = 73^\circ$ (Alternate Segment theorem)

$B\hat{C}D + D\hat{E}B = 180^\circ$ (opposite angles of a cyclic quadrilateral sum to 180).

$B\hat{C}D = 180^\circ - 73^\circ = 107^\circ$

$B\hat{C}D = D\hat{C}E + B\hat{C}E$

$107^\circ = D\hat{C}E + 73^\circ$

$D\hat{C}E = 107^\circ - 73^\circ = 34^\circ$

34 °

(Total for Question 19 is 5 marks)

20 Here is a cube *ABCDEFGH*.

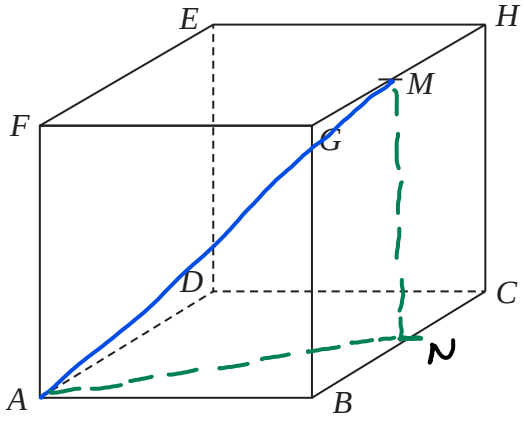


Diagram **NOT** accurately drawn

M is the midpoint of the edge *GH*.

Find the size of the angle between the line *MA* and the plane *ABCD*.
Give your answer correct to 1 decimal place.

Assuming side length of cube = 2cm

Assuming N is the midpoint of BC.

you can assume any length and use them appropriately in the question.

In ΔAMN:

$AN^2 = AB^2 + BN^2$ Pythagoras theorem $a^2 = b^2 + c^2$

$AN^2 = 4 + 1 = 5$

$AN = \sqrt{5}$ cm

$MN = 2$ cm

$M\hat{A}N = \theta$

$\tan(\theta) = \frac{MN}{AN}$

$\tan\theta = \frac{2}{\sqrt{5}}$

$\theta = \tan^{-1}\left(\frac{2}{\sqrt{5}}\right) = 41.81$

$\rightarrow 1 < 5$

$\therefore$ round down

41.8 °

(Total for Question 20 is 4 marks)

21 Here is a triangle *XYZ*.

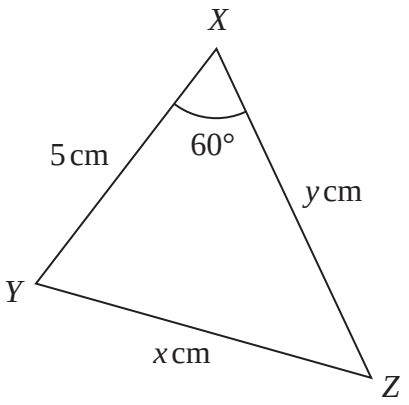


Diagram **NOT** accurately drawn

The perimeter of the triangle is *k* cm.

Given that $x = y - 1$ $\quad x + 1 = y$
find the value of *k*.
Show your working clearly.

Perimeter $= 5 + x + y = 5 + x + (x+1) = k$

Finding x:

cosine rule

$x^2 = 5^2 + y^2 - 2(5)(y)\cos(60)$

$x^2 = 5^2 + (x+1)^2 - 2(5)(x+1)\left(\frac{1}{2}\right)$ $\quad \cos(60)$

$x^2 = 25 + (x^2 + 2x + 1) - 5(x+1)$

$x^2 = 25 + x^2 + 2x + 1 - 5x - 5$

$x^2 = x^2 - 3x + 21$ $\quad -x^2$

$0 = -3x + 21$

$3x = 21 \Rightarrow x = \frac{21}{3} = 7$

Perimeter $= 5 + (7) + (7+1)$

$= 5 + 7 + 8$

$= 20 = k$

$k =$ 20

(Total for Question 21 is 5 marks)

22 *ABCDEF* is a regular hexagon.

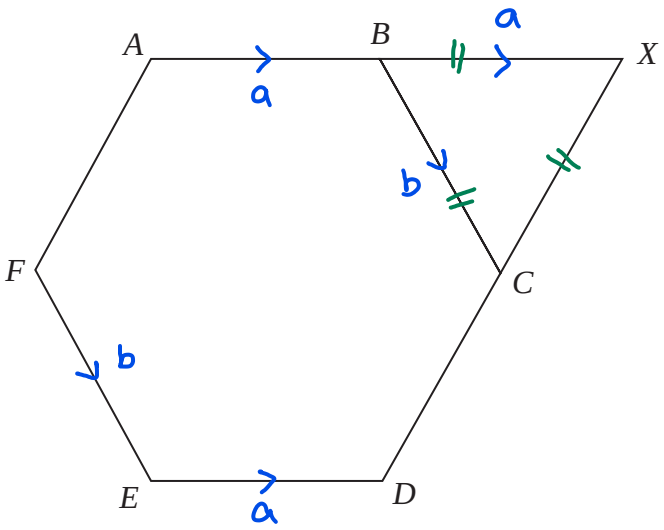


Diagram **NOT** accurately drawn

ABX and *DCX* are straight lines.

$\overrightarrow{AB}$ = **a** $\overrightarrow{BC}$ = **b**

Find $\overrightarrow{EX}$ in terms of **a** and **b**.
Give your answer in its simplest form.

$\overrightarrow{CX} = \overrightarrow{CB} + \overrightarrow{BX}$ $\overrightarrow{CX} = \overrightarrow{DC}$

$= -\underset{\sim}{b} + \underset{\sim}{a}$

$\overrightarrow{EX} = \overrightarrow{ED} + \overrightarrow{DC} + \overrightarrow{CX}$

$= \underset{\sim}{a} + (-\underset{\sim}{b} + \underset{\sim}{a}) + (-\underset{\sim}{b} + \underset{\sim}{a})$

$= \underset{\sim}{a} + \underset{\sim}{a} + \underset{\sim}{a} - \underset{\sim}{b} - \underset{\sim}{b} = 3\underset{\sim}{a} - 2\underset{\sim}{b}$

(Total for Question 22 is 4 marks)

23 The function f is defined as $f(x) = \dfrac{\sqrt{x^2 + k^2}}{x}$ for $x > 0$ and where k is a positive number.

(a) Find the value of p for which $f^{-1}(p) = k$

$$y = \frac{\sqrt{x^2+k^2}}{x}$$

Subject x

$$xy = \sqrt{x^2+k^2}$$
$$x^2y^2 = x^2+k^2$$
$$x^2y^2 - x^2 = k^2$$
$$x^2(y^2-1) = k^2$$
$$x^2 = \frac{k^2}{y^2-1}$$
$$x = \frac{k}{\sqrt{y^2-1}}$$

Replace x with y and y with x.

$$f^{-1}(x) = \frac{k}{\sqrt{x^2-1}}$$

$$\frac{k}{\sqrt{p^2-1}} = k$$
$$\frac{k}{k} = \sqrt{p^2-1}$$
$$1 = p^2 - 1$$
$$1+1 = p^2$$
$$\sqrt{2} = p$$

$p = \sqrt{2}$ **(3)**

The function g is defined as $g(x) = x^2$ for $x > 0$

(b) Given that $gf(a) = k$ for $k > 1$
find an expression for a in terms of k.

$$f(a) = \frac{\sqrt{(a)^2+k^2}}{a} = \frac{\sqrt{a^2+k^2}}{a}$$

$$gf(a) = \left(\frac{\sqrt{a^2+k^2}}{a}\right)^2$$

$$k = \frac{a^2+k^2}{a^2} \Rightarrow ka^2 = a^2 + k^2$$

$$ka^2 - a^2 = k^2$$
$$a^2(k-1) = k^2 \Rightarrow a^2 = \frac{k^2}{k-1} \Rightarrow a = \frac{\sqrt{k^2}}{\sqrt{k-1}}$$

$a = \dfrac{k}{\sqrt{k-1}}$ **(3)**

(Total for Question 23 is 6 marks)

TOTAL FOR PAPER IS 100 MARKS

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