Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International GCSE

Thursday 8 June 2023

Morning (Time: 2 hours)

Paper reference **4PM1/02R**

Further Pure Mathematics
PAPER 2R

Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page. Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over

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P 7 4 2 8 4 A 0 1 3 2

Pearson

International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere $= 4\pi r^2$

Curved surface area of cone $= \pi r \times \text{slant height}$

Volume of sphere $= \frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric series

Sum to n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

Calculus

Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

Trigonometry

Cosine rule

In triangle ABC: $a^2 = b^2 + c^2 - 2bc\cos A$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$

$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$

Logarithms

$\log_a x = \frac{\log_b x}{\log_b a}$

Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 $$\mathrm{f}(x) = 2x^2 + (k+8)x + k$$

Show that for all values of k, the equation $\mathrm{f}(x) = 0$ has distinct real roots.

(4)

(Total for Question 1 is 4 marks)

2 Find the set of values of x for which

(a) $2(x+1) < 5x - 2$ **(2)**

(b) $3x^2 - x \leqslant 10$ **(3)**

(c) **both** $2(x+1) < 5x - 2$ **and** $3x^2 - x \leqslant 10$ **(1)**

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Question 2 continued

(Total for Question 2 is 6 marks)

3

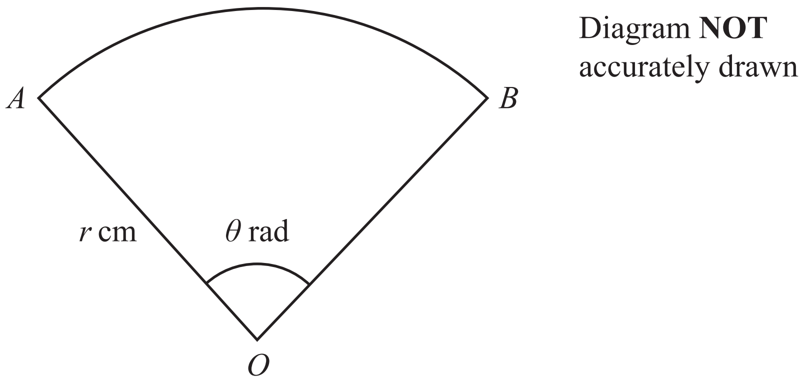

Figure 1

Figure 1 shows the sector OAB of a circle with centre O.

The radius of the circle is r cm and the angle AOB is θ radians.

The area of the sector is 675 cm^2

(a) Show that the perimeter of the sector, P cm, is given by

$$P = 2r + \frac{1350}{r}$$

(3)

Given that r can vary,

(b) find, using calculus, the minimum value of P
Give your answer in the form $a\sqrt{b}$ where a is an integer and b is a prime number.

(5)

(c) Justify that the value of P you found in (b) is a minimum.

(2)

P 7 4 2 8 4 A 0 6 3 2

Question 3 continued

(Total for Question 3 is 10 marks)

4 O, A and B are fixed points such that

$$\overrightarrow{OA} = 5\mathbf{i} + 7\mathbf{j} \qquad \overrightarrow{AB} = a\mathbf{i} + 16\mathbf{j} \qquad \text{and} \qquad \left|\overrightarrow{OB}\right| = 5\sqrt{29}$$

(a) Find the possible values of a

(4)

Given that $a > 0$

(b) find a unit vector that is parallel to $\overrightarrow{AB}$

(2)

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Question 4 continued

(Total for Question 4 is 6 marks)

5 A particle P is moving along the x-axis.

At time t seconds, $t \geqslant 0$, the velocity, v m/s, of P is given by

$$v = 2t^2 - 19t + 35$$

(a) Find the acceleration of P when $t = 5$

(2)

The particle comes to instantaneous rest at the points A and B at times t_1 seconds and t_2 seconds respectively, where $t_1 < t_2$

(b) Find the value of t_1 and the value of t_2

(2)

(c) Use calculus to find the distance AB

(3)

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Question 5 continued

(Total for Question 5 is 7 marks)

6 $$f(x) = 2x^2 + 5x - p$$

The equation $f(x) = 0$ has roots α and β

Given that $\alpha^3 + \beta^3 = -\frac{215}{8}$

(a) find the value of p

(5)

Without solving the equation $f(x) = 0$

(b) form a quadratic equation, with integer coefficients, that has roots

$$\frac{\alpha + \beta}{\alpha^2} \text{ and } \frac{\alpha + \beta}{\beta^2}$$

(5)

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Question 6 continued

Question 6 continued

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Question 6 continued

(Total for Question 6 is 10 marks)

7

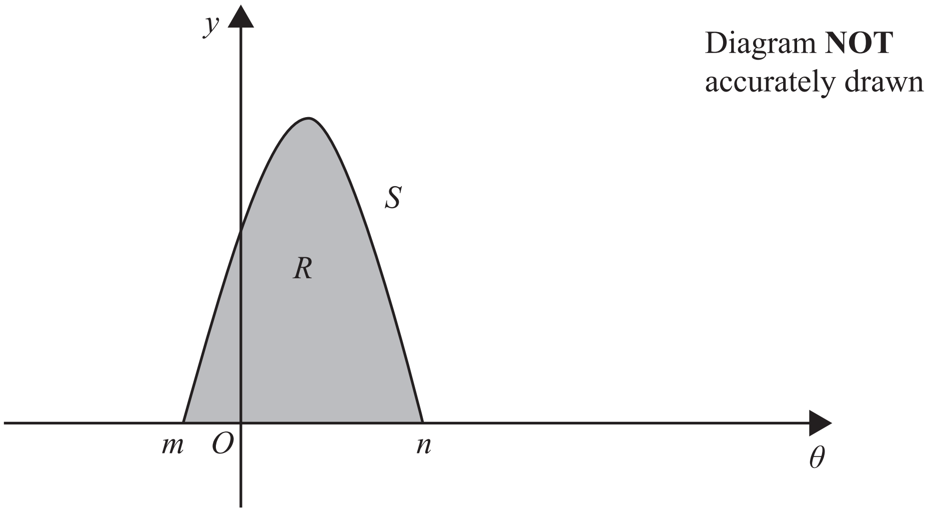

Figure 2

Figure 2 shows part of the curve S with equation $y = \left(\cos 3\theta + \sqrt{3}\sin 3\theta\right)^{\frac{1}{2}}$

where $m \leqslant \theta \leqslant n$

The curve S meets the x-axis at the point with coordinates $(m, 0)$ and at the point with coordinates $(n, 0)$

(a) Find the exact value of m and the exact value of n

(3)

The finite region R, shown shaded in Figure 2, is bounded by the curve S, and the x-axis in the region $m \leqslant \theta \leqslant n$

The region R is rotated through 2π radians about the x-axis.

(b) Use calculus to find the exact volume of the solid generated.

(4)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 7 marks)

8 The points A and B have coordinates $(1, 5)$ and $(9, 9)$ respectively.

(a) Find an equation of line AB, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(3)

The line l is perpendicular to AB and passes through the point X which lies on AB such that $AX : XB = 3:1$

(b) Show that an equation of l is $y = -2x + 22$

(5)

The point C has coordinates $(6, p)$

Given that C lies on l

(c) find the value of p

(1)

$ABCD$ is a parallelogram where the x coordinate of D is negative.

(d) Find the coordinates of the point D

(3)

(e) Find the area of the parallelogram $ABCD$

(4)

Question 8 continued

Question 8 continued

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Question 8 continued

(Total for Question 8 is 16 marks)

9 A curve C has equation $y = \dfrac{3 - 2x}{x + 6}$ where $x \neq -6$

(a) Write down an equation of the asymptote to C that is parallel to the

(i) x-axis (ii) y-axis

(2)

(b) Find the coordinates of the point where C crosses the

(i) x-axis (ii) y-axis

(2)

(c) Using the axes opposite, sketch the graph of C, showing clearly its asymptotes and the coordinates of the points where C crosses the coordinate axes.

(3)

(d) Show that the gradient of the tangent to C is always negative.

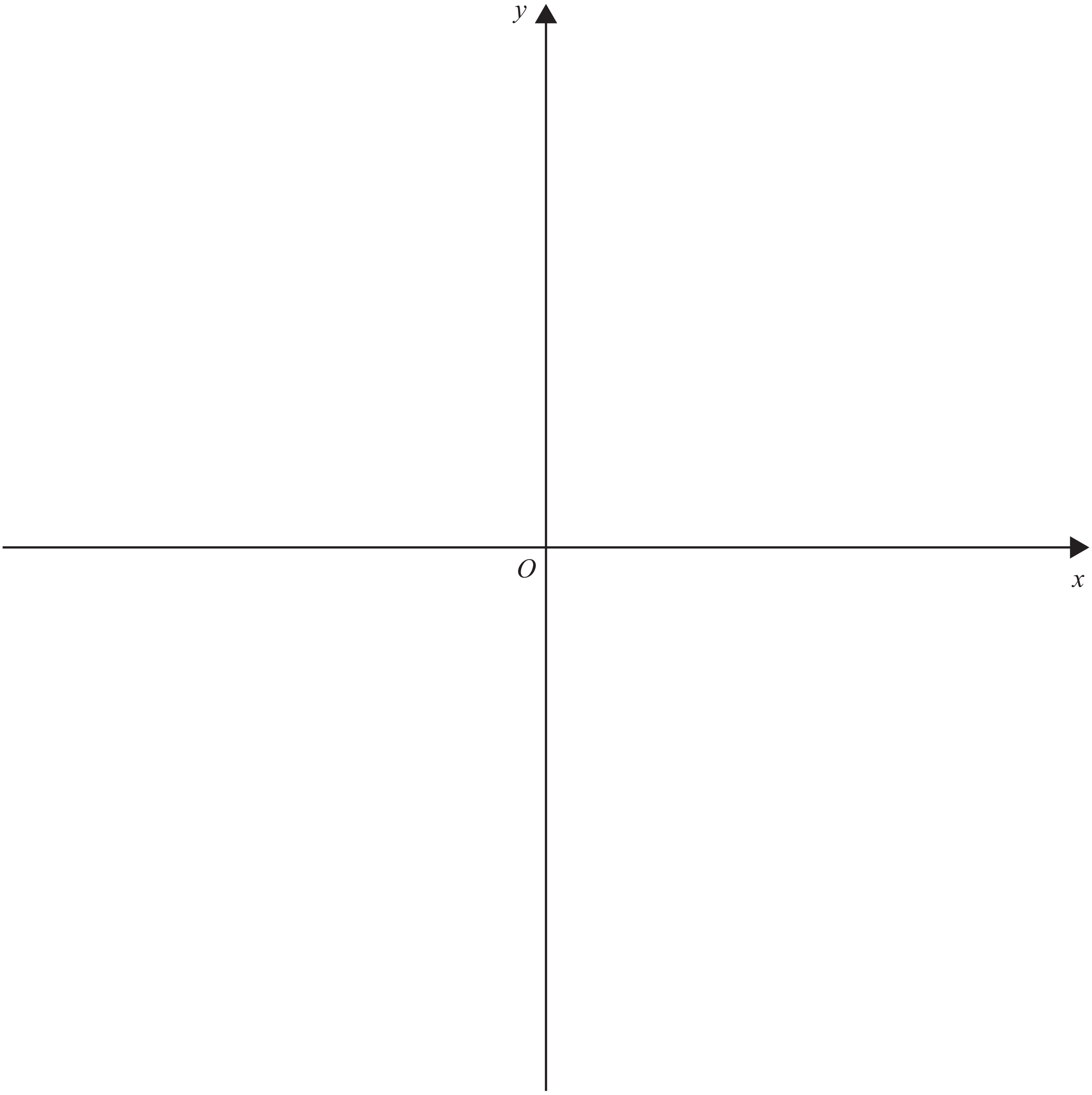
(3)

A tangent to C has equation $y = -\dfrac{3}{5}x + k$ where $k > 0$

(e) Find the value of k

(5)

Question 9 continued



Question 9 continued

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Question 9 continued

(Total for Question 9 is 15 marks)

10 Solve the equation

$$\log_4 x^3 + 8\log_x 64 = 22$$

(7)

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Question 10 continued

(Total for Question 10 is 7 marks)

11 (a) Use a formula on page 2 to show that $\sin^2 A = \frac{1}{2}(1 - \cos 2A)$

(3)

(b) Show that $\sin^4 x + \cos^4 x = \frac{3 + \cos 4x}{4}$

(5)

(c) Hence solve, in degrees to one decimal place, the equation

$$8\sin^4\left(\frac{\theta}{2}\right) + 8\cos^4\left(\frac{\theta}{2}\right) = 5\sin(2\theta) + 6 \quad \text{for} \quad 0^\circ \leqslant \theta < 180^\circ$$

(4)

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Question 11 continued

Question 11 continued

(Total for Question 11 is 12 marks)

TOTAL FOR PAPER IS 100 MARKS