Pearson Edexcel

Mark Scheme (Results)

Summer 2023

Pearson Edexcel International GCSE
In Further Pure Mathematics (4PM1)
Paper 2

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the last candidate in exactly the same way as they mark the first.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme - not according to their perception of where the grade boundaries may lie.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification/indicative content will not be exhaustive.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, a senior examiner must be consulted before a mark is given.
- Crossed out work should be marked **unless** the candidate has replaced it with an alternative response.

- **Types of mark**
    - M marks: method marks
    - A marks: accuracy marks
    - B marks: unconditional accuracy marks (independent of M marks)

- **Abbreviations**
    - cao – correct answer only
    - ft – follow through
    - isw – ignore subsequent working
    - SC - special case
    - oe – or equivalent (and appropriate)
    - dep – dependent
    - indep – independent
    - awrt – answer which rounds to
    - eeoo – each error or omission

- **No working**
  If no working is shown then correct answers normally score full marks
  If no working is shown then incorrect (even though nearly correct) answers score no marks.
- **With working**
  If the final answer is wrong, always check the working in the body of the script (and on any diagrams), and award any marks appropriate from the mark scheme.
  If it is clear from the working that the “correct” answer has been obtained from incorrect working, award 0 marks.
  If a candidate misreads a number from the question. Eg. Uses 252 instead of 255; method marks may be awarded provided the question has not been simplified. Examiners should send any instance of a suspected misread to review.
  If there is a choice of methods shown, then award the lowest mark, unless the answer on the answer line makes clear the method that has been used.
  If there is no answer achieved then check the working for any marks appropriate from the mark scheme.

- **Ignoring subsequent work**
  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. Incorrect cancelling of a fraction that would otherwise be correct.
  It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect eg algebra.
  Transcription errors occur when candidates present a correct answer in working, and write it incorrectly on the answer line; mark the correct answer.

- **Parts of questions**
  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded to another.

General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:

$$\left(x^2+bx+c\right)=\left(x+p\right)\left(x+q\right),\text{ where }|pq|=|c| \quad \text{leading to } x=....$$

$$\left(ax^2+bx+c\right)=\left(mx+p\right)\left(nx+q\right)\text{ where }|pq|=|c|\text{ and }|mn|=|a| \quad \text{leading to } x=....$$

2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for *a*, *b* and *c*, leading to $x=....$

3. Completing the square:

$$x^2+bx+c=0:\ \left(x\pm\tfrac{b}{2}\right)^2\pm q\pm c=0,\quad q\neq 0 \qquad \text{leading to } x=....$$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $\left(x^n\rightarrow x^{n-1}\right)$

2. Integration:

Power of at least one term increased by 1. $\left(x^n\rightarrow x^{n+1}\right)$

Use of a formula:

Generally, the method mark is gained by **either**

quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

or, where the formula is not quoted, the method mark can be gained by implication from the substitution of correct values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers may be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show...."**Exact answers:**

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)
Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.

2306
4PM1 Paper 2
Mark Scheme

Question	Scheme	Marks
1	$\frac{\left(a+2\sqrt{5}\right)}{\left(3-\sqrt{5}\right)}\times\frac{\left(3+\sqrt{5}\right)}{\left(3+\sqrt{5}\right)}=\frac{3a+a\sqrt{5}+6\sqrt{5}+10}{9-5}\left(=\frac{3a+10+\left(6+a\right)\sqrt{5}}{4}\right)$	M1
	$\left(\frac{3a+10+\left(6+a\right)\sqrt{5}}{4}=\frac{11+b\sqrt{5}}{2}\right)$ $\Rightarrow\frac{3a+10}{4}=\frac{11}{2}$ oe $\Rightarrow a=4\Rightarrow\frac{6+a}{4}=\frac{b}{2}$ oe $\Rightarrow b=5$	M1M1A1A1 [5]
	ALT $\left(\frac{\left(a+2\sqrt{5}\right)}{\left(3-\sqrt{5}\right)}=\frac{11+b\sqrt{5}}{2}\Rightarrow\right)2\left(a+2\sqrt{5}\right)=\left(3-\sqrt{5}\right)\left(11+b\sqrt{5}\right)$ $2a+4\sqrt{5}=33+3b\sqrt{5}-11\sqrt{5}-5b\left(=\left(33-5b\right)+\left(3b-11\right)\sqrt{5}\right)$	M1
	$\Rightarrow 4=3b-11\Rightarrow b=5$ $\Rightarrow 2a=33-5b\Rightarrow a=4$	M1M1A1A1 [5]
		Total 5 marks

Mark	Notes
M1	For multiplying both numerator & denominator of $\dfrac{\left(a+2\sqrt{5}\right)}{\left(3-\sqrt{5}\right)}$ through by $\dfrac{\left(3+\sqrt{5}\right)}{\left(3+\sqrt{5}\right)}$ to give $\dfrac{3a+a\sqrt{5}+6\sqrt{5}+10}{9-5}$. Allow one error on the numerator. The denominator must be correct.
M1	For correctly equating **their** coefficients with $\dfrac{11+b\sqrt{5}}{2}$. **Although this is not a dependent mark, there must be at least one equation in *a* and *b***
M1	For a complete and correct attempt to solve one of their equations to find a value for *a* **or** a value for *b*. **Although this is not a dependent mark, there must be at least one equation in *a* and *b*.**
A1	For either $a = 4$ or $b = 5$
A1	For both $a = 4$ and $b = 5$
Students may also multiply by $-3-\sqrt{5}$ This produces the correct answer and is the same as the main MS, but all the terms are negative. Mark to exactly the same principles.	
ALT	
M1	Correctly removes the denominators from the given equation and multiplies out as shown to give the equation $2a+4\sqrt{5}=33+3b\sqrt{5}-11\sqrt{5}-5b$ Allow one error.
M1	For correctly equating **their** coefficients on either side of the equation. **Although this is not a dependent mark, there must be at least one equation in *a* and *b***
M1	For a correct and complete attempt to solve one of their equations to find a value for *a* **or** a value for *b*. **Although this is not a dependent mark, there must be at least one equation in *a* and *b***
A1	For either $a = 4$ or $b = 5$
A1	For both $a = 4$ and $b = 5$

Question	Scheme	Marks
2	$n=1\Rightarrow a=8^{(1-2)}=8^{-1}$ oe eg $\frac{1}{8}$	B1
	$n=2\Rightarrow ar=8^{(1-4)}=8^{-3}\Rightarrow r=\frac{8^{-3}}{8^{-1}}=8^{-2}$ oe eg $\frac{1}{64}$	M1A1
	$(S_\infty=)\frac{8^{-1}}{1-8^{-2}}=\frac{2^{-3}}{\frac{63}{2^6}}=\frac{8}{63}$ oe eg $\frac{\frac{1}{8}}{1-\frac{1}{64}}\left(=\frac{\frac{1}{8}}{\frac{63}{64}}=\frac{1}{8}\times\frac{64}{63}\right)$	M1dM1
	$\frac{8}{63}$ oe or $p=8,\ q=63$ oe	A1
		Total 6 marks

Mark	Notes
B1	For $a=8^{-1}$ oe
M1	For substituting n = 2 into the expression for nth term to find a value for ar **and** dividing by a to find r. This mark can be implied by a correct value for r.
A1	For $r=8^{-2}$ oe
M1	For applying the correct formula for the sum to infinity of a convergent geometric series for their values of a and r, providing $\lvert r\rvert<1$ They must be using values they've attained or stated for a and r.
dM1	For a correct attempt to use an index law with their expression to obtain the required form or a correct attempt to divide their fractions. Dependent on previous method mark.
A1	For the correct answer in the required form any equivalent with p and q integers is acceptable.

In this question, the final dM1 may implied from a correct substitution of their values of r and a, evaluated correctly, if working isn't shown.
You may have to check their final answer.

Eg

$r=\frac{1}{4}, a=\frac{1}{8}$

$(S_\infty=)\frac{\frac{1}{8}}{1-\frac{1}{4}}=\frac{1}{6}$ is M1 dM1 A0 because the $\frac{1}{6}$ is correct for their a and their r

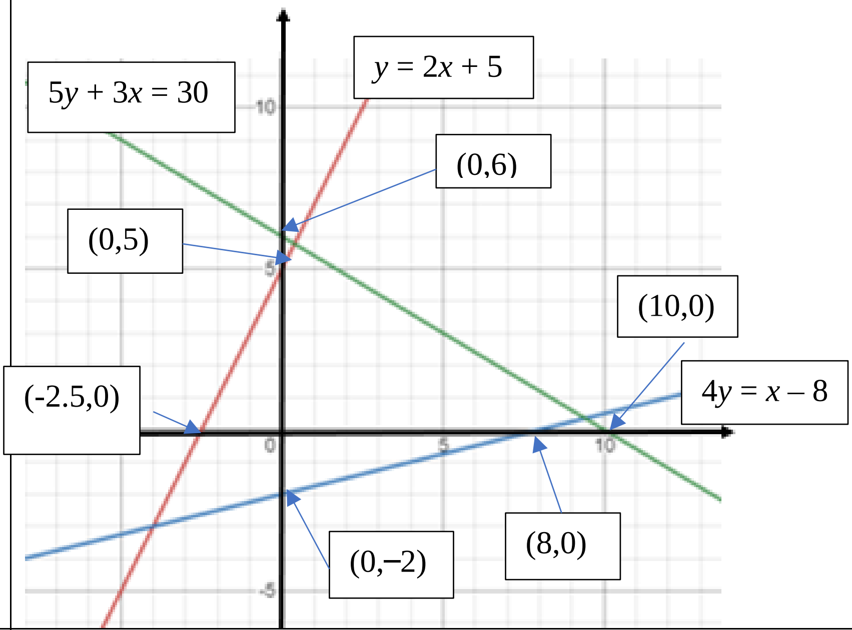
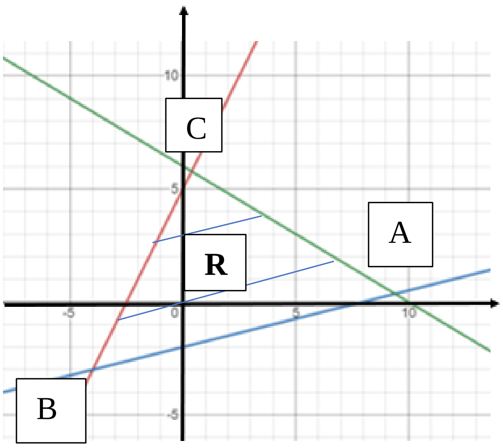
Question	Scheme	Marks
3	**Mark parts (i) and (ii) together.** (Let the obtuse angle $AOB = \alpha$ - students do not need to state this, allow use of any symbol or letter, including θ) $53.2 = r\alpha$ $372.4 = \frac{r^2\alpha}{2}$ $\alpha = \frac{53.2}{r},\quad 372.4 = \frac{r^2}{2}\times\frac{53.2}{r}$ oe $\Rightarrow r = 14$ $\alpha = \frac{53.2}{14} = 3.8$ $\theta = \left(2\pi - 3.8 =\right)2.48$	 B1 B1 M1A1 M1 A1 [6]
ALT1 final 4 marks	First 2 marks as ALT2 $r = \frac{53.2}{\alpha}, 372.4 = \frac{\left(\frac{53.2}{\alpha}\right)^2}{2}\times\alpha$ oe $\Rightarrow \alpha = \frac{372.4}{1415.12} = 3.8$ $r\left(=\frac{53.2}{3.8}\right) = 14$ $\theta = \left(2\pi - 3.8 =\right)2.48$	B1B1 M1 M1(A1 on ePen) A1(M1 on ePen) A1 [6]
ALT2	$2\pi r - r\theta = 53.2$ oe $\pi r^2 - \frac{1}{2}r^2\theta = 372.4$ oe $\theta = 2\pi - \frac{53.2}{r} \Rightarrow \pi r^2 - \frac{1}{2}\left(2\pi - \frac{53.2}{r}\right)r^2 = 372.4$ oe $26.6r = 372.4$ oe $r = 14$ $\theta = 2.48$	B1 B1 M1 M1(A1 on ePen) A1(M1 on ePen) A1 [6]

ALT3	First 2 marks as ALT2 $r=\frac{53.2}{2\pi-\theta}$ $r^2\left(\pi-\frac{\theta}{2}\right)=372.4\Rightarrow\left(\frac{53.2}{2\pi-\theta}\right)^2=\frac{372.4}{\pi-\frac{\theta}{2}}$ oe eg $\left(\frac{53.2}{2\pi-\theta}\right)^2=\frac{744.8}{2\pi-\theta}$ $\frac{53.2^2}{744.8}=2\pi-\theta\left(\Rightarrow\theta=2\pi-\frac{53.2^2}{744.8}\right)$ $\theta=2.48$ $r=$ awrt 14.0	B1B1 M1 M1(A1 on ePen) A1(M1 on ePen) A1 [6]
Total 6 marks		

Mark	Notes
B1	Correctly uses the formula for length of an arc $53.2 = r \times$ angle in radians or correctly uses the formula for length of an arc $53.2 = 2\pi r \times \dfrac{\text{angle in degrees}}{360}$
B1	Correctly uses the formula for area of a sector $372.4 = \dfrac{r^2}{2} \times$ angle in radians or correctly uses the formula for area of a sector $372.4 = \pi r^2 \times \dfrac{\text{angle in degrees}}{360}$
M1	Eliminates α from both equations, allow up to one error. If they have worked in degrees, they **must** reach an equation where π must not be present (ie it has cancelled).
A1	For *r* =14
M1	For using their *r* in a correct equation to find a value for α If they are working in degrees, there **must** follow a correct attempt to convert their angle into radians.
A1	For the correct value of $\theta = 2.48$
ALT1 B1B1	As main scheme
M1	Eliminates *r* from both equations and reaches an equation in α , allow one error
M1 (A1 on ePen)	Solves their equation (allow one error) to find a value for α
A1(M1 on ePen)	For the correct value for r
A1	For the correct value of $\theta = 2.48$
ALT2 B1	Correctly uses the formula for length of an arc
B1	Correctly uses the formula for area of a sector
M1	Rearranges for θ (allow one error) and substitutes correctly into the other equation to reach an equation for r
M1 (A1 on ePen)	For an equation of the form *ar = b ab > 0*
A1(M1 on ePen)	r = 14
A1	$\theta = 2.48$
ALT3 B1B1	As ALT2
M1	Rearranges for r (allow one error) and substitutes correctly into the other equation to reach an equation for θ
M1 (A1 on ePen)	For reaching an equation of the form $d = 2\pi - \theta$ $\quad d > 0$
A1(M1 on ePen)	$\theta = 2.48$
A1	$r =$ awrt 14.0

Question	Scheme	Marks
4(a)	**Mark parts (i) and (ii) together.**	
	$2y-x-4=0 \Rightarrow y=\dfrac{x+4}{2}$ oe	
	$\dfrac{x^2}{4}+2=\dfrac{x+4}{2}$ oe $\Rightarrow \dfrac{x^2+8}{4}=\dfrac{x+4}{2}$ oe $\Rightarrow x^2+8=2x+8$ oe	M1
	$\Rightarrow x^2-2x=0 \Rightarrow x(x-2)=0$ oe	M1
	$\Rightarrow x=(0,)\ 2$	
	$\Rightarrow y=2,\ 3$	
	So coordinates at point A are (0, 2) *	A1* cso
	and at point B (2, 3)	A1
		[4]
	NB SC1 – for correct and complete substitution of x = 0 into the equation of curve S or line l to show that y = 2, where there is no other work, other work is incorrect or no marks would otherwise be gained. Award this as 1st M mark. THERE ARE NO OTHER MARKS AVAILABLE FOR PART (a) IF THIS IS ALL STUDENTS DO	
(b)	$V=\pi\int_2^{"3"}(4y-8)\ \mathrm{d}y-\pi\int_2^{"3"}(2y-4)^2\ \mathrm{d}y$	M1
	$\left[\Rightarrow \pi\int_2^{"3"}(-4y^2+20y-24)\ \mathrm{d}y \quad \text{or} \quad \pi\int_2^{"3"}(4y-8)\ \mathrm{d}y-\pi\int_2^{"3"}(4y^2-16y+16)\ \mathrm{d}y\right]$	M1
	$=(\pi)\left[-\dfrac{4y^3}{3}+\dfrac{20y^2}{2}-24y\right]_{("2")}^{("3")}$	
	or $(\pi)\left[\dfrac{4y^2}{2}-8y\right]_{("2")}^{("3")}-(\pi)\left[\dfrac{4y^3}{3}-\dfrac{16y^2}{2}+16y\right]_{("2")}^{("3")}$	
	$=(\pi)\left[\left(\dfrac{4\times"3"^3}{3}+\dfrac{20\times"3"^2}{2}-24\times"3"\right)-\left(\dfrac{4\times"2"^3}{3}+\dfrac{20\times"2"^2}{2}-24\times"2"\right)\right]$	M1
	$=\dfrac{2}{3}\pi$	A1
		[4]
	ALT	
	$V=\pi\int_2^{"3"}(4y-8)\ \mathrm{d}y-\pi\times\dfrac{1}{3}\times2^2\times1$	M1
	$=\pi\left[\dfrac{4y^2}{2}-8y\right]_{"2"}^{"3"}\left(-\dfrac{4}{3}\pi\right)$	M1
	$=\pi\left[\left(2\times"3"^2-8\times"3"\right)-\left(2\times"2"^2-8\times"2"\right)\right]\left(-\dfrac{4}{3}\pi\right)$	M1
	$=\dfrac{2}{3}\pi$	A1
		Total 8 marks

Part	Mark	Notes
(a)	**M1**	Correctly equates the equation of S with the equation of l
	M1	Forms a quadratic and a minimally acceptable attempt to solve their quadratic. For a 2TQ the **solution** of their quadratic for must be correct, **but any zero solution doesn't need to be shown as it's obvious from the given diagram.** If they achieve a 3TQ and attempt to solve – see general guidance.
	A1* cso	For correct substitution of $x = 0$ to show coordinates of A (0, 2) or $x = 0$, $y = 2$. Allow coordinates without brackets. SC1 (see MS) if this is the only work done.
	A1	For the coordinates of B (2, 3) $x = 2$, $y = 3$ Allow coordinates without brackets.
Note on labelling: **If the labels (i) and (ii) are not present, the marks can be awarded if** • **The coordinates appear in the correct order or** • **They are labelled with *A* and *B*** **If there is ambiguity award A1 A0 if both correct and A0 A0 if only one is correct**		
(b)	**M1**	For a correct statement for the volume of rotation with 2 and their upper limit used correctly and correctly including π. The lower limit **must** be 2.
	M1	For a minimally acceptable attempt to integrate (see general guidance) **and** no power of y must decrease. π and limits do not need to be present to gain this mark. There must be a minimum of 2 terms to integrate.
	M1	For substituting their limits into their changed expression the correct way round. There must be at least one clear substitution of each limit. π does not need to be present to gain this mark.
	A1	For the correct final volume $= \frac{2}{3}\pi$ accept $0.\dot{6}\pi, 0.67\pi$ or better
	ALT	
	M1	For a correct statement for the volume of rotation of the curve with 2 and their upper limit used correctly including π minus the correct formula used for the volume of a cone. The lower limit **must** be 2. **This mark may also be awarded if the integral is subtracted the wrong way round.**
	M1	For a minimally acceptable attempt to integrate (see general guidance) **and** no power of y must decrease. π, limits and $-\frac{4}{3}\pi$ do not need to be present to gain this mark. There must be a minimum of 2 terms to integrate.
	M1	For substituting their limits into their changed expression. There must be at least one clear explicit substitution of each limit. This mark can be implied by a correct final answer. π and $-\frac{4}{3}\pi$ do not need to be present to gain this mark.
	A1	For the correct final volume $= \frac{2}{3}\pi$ accept $0.\dot{6}\pi, 0.67\pi$ or better **If a negative value is found and changed at the end to a positive value, this final A mark cannot be awarded.**
	Note: any candidate incorrectly rotating around *x* axis, 2nd & 3rd M marks are available for integrating an expression & substituting in. Max mark M0 M1 M1 A0.	

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>5 (a)</td><td>
</td><td>B3</td></tr>
<tr><td>(b)</td><td>
</td><td>B1</td></tr>
<tr><td>(c)</td><td>
<table>
<tr><th></th><th>A</th><th>B</th><th>C</th></tr>
<tr><td>x</td><td>9.4</td><td>– 4</td><td>0.4</td></tr>
<tr><td>y</td><td>0.4</td><td>– 3</td><td>5.8</td></tr>
<tr><td>P</td><td>$2\times 9.4-5\times 0.4$
$=16.8$</td><td>$2\times -4-5\times -3$
$=7$</td><td>$2\times 0.4-5\times 5.8$
$=-28.2$
[Exact value is]
$\left[-\frac{365}{13}\approx -28.1\right]$</td></tr>
</table>
Allow ± 0.1 on the coordinates in each case.</td><td>M1

M1A1</td></tr>
<tr><td colspan="3" align="right">Total 7 marks</td></tr>
</table>

<table>
<tr><th>Part</th><th>Mark</th><th>Notes</th></tr>
<tr><td colspan="3">The tolerance for all marks in this question is $\pm$ half a small square.</td></tr>
<tr><td rowspan="3">(a)</td><td>B1</td><td>For one of the lines correctly drawn to within tolerance (as a minimum, examiners should check intersections with axes, candidates do not need to mark these).</td></tr>
<tr><td>B1</td><td>For two of the lines drawn to within tolerance (as a minimum, examiners should check intersections with axes, candidates do not need to mark these).</td></tr>
<tr><td>B1</td><td>For all three lines correctly drawn to within tolerance (as a minimum, examiners should check intersections with axes, candidates do not need to mark these).</td></tr>
<tr><td colspan="3">As a minimum, lines must intersect with other for marks in (a) and (b) to be awarded</td></tr>
<tr><td>(b)</td><td>B1ft</td><td>For the correct enclosed region shaded in or out or for R clearly labelled.
The ft mark can only be awarded if 3 distinct lines have been drawn and it’s clear they’ve shaded on the correct ‘side’ for each of their lines.
If there’s no labelling and it’s not clear which line is which, this mark cannot be awarded.</td></tr>
<tr><td colspan="3">Part c of this question states “using your graph”……
Therefore solutions which obviously use exact coordinates of intersection points having used a graphical calculator or from working algebraically can only score M0 M1 A0.</td></tr>
<tr><td rowspan="3">(c)</td><td>M1</td><td>For reading from the graph at least one point of intersection using their lines. The pair used for this and the next method mark must be within the tolerance of ± 0.1 of the values shown in the table. Any solutions which work out the values algebraically will not gain this mark or the final accuracy mark, but may gain the next method mark.
Occasionally, students are working out the non-integer coordinates from algebra or from a calculator but reading (-4, -3) from the graph and using this to find the value of P. In this case, we can apply bod (benefit of the doubt) and this mark can be awarded if subbed in to find the value of P.</td></tr>
<tr><td>M1</td><td>For a correct substitution to find the value of P from at least one set of their coordinates of the point of intersection.
This is not a dependent mark, so they can use any one of their pairs of values even if it doesn’t fall in tolerance.</td></tr>
<tr><td>A1</td><td>For the correct least value of -28.2 Allow a value between – 28.9 and −27.5 so long as this follows through from their values.
Do not allow a value out of range to be rounded to a value within range.</td></tr>
</table>

Question	Scheme	Marks
6 **(a)**	$(V=)5\times\frac{1}{2}\times r^2\times\sin\left(\frac{\pi}{3}\right)=\frac{5\sqrt{3}}{4}r^2$ *	B1*cso [1]
(b)	$\left(\frac{\mathrm{d}V}{\mathrm{d}r}=\right)\frac{5\sqrt{3}}{2}r$ oe	M1
	(When the area of $BCDF$ = 60 cm^2, $BC = DF = r =$)12 cm	B1
	$\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}r}\times\frac{\mathrm{d}r}{\mathrm{d}t}$ oe	M1
	$\left(\frac{\mathrm{d}V}{\mathrm{d}t}=\right)\frac{5\sqrt{3}}{2}\times 12\times 0.2=6\sqrt{3}$ oe $\left(\mathrm{cm}^3/\mathrm{s}\right)$	dM1A1 [5]
		Total 6 marks

Part	Mark	Notes
(a)	**B1* cso**	For a correct expression for the volume of a prism $V=5\times\frac{1}{2}\times r^2\times\sin\left(\frac{\pi}{3}\right)$ followed by the given answer stated. No errors. Use of $60°$ for $\frac{\pi}{3}$ is fine.
(b)	**M1**	For differentiating the expression for V to given an expression of the form pr where p is a positive constant. We don't need to see $\left(\frac{\mathrm{d}V}{\mathrm{d}r}=\right)$ if it's clear the candidate has attempted to differentiate the given volume.
	B1	For finding the length BC or DF when the area of the face $BCDF$ = 60 and deducing that r = 12 cm.
	M1	For a correct chain rule involving $\frac{\mathrm{d}V}{\mathrm{d}t}$, $\frac{\mathrm{d}V}{\mathrm{d}r}$ and $\frac{\mathrm{d}r}{\mathrm{d}t}$ [Can be implied by later correct substitution.]
	dM1	For substituting their $\frac{\mathrm{d}V}{\mathrm{d}r}$, their r and the given value of $\frac{\mathrm{d}r}{\mathrm{d}t}=0.2$ Dependent on the first method mark.
	A1	For $\left(\frac{\mathrm{d}V}{\mathrm{d}t}=\right)6\sqrt{3}$ $\left(\mathrm{cm}^3/\mathrm{s}\right)$

Question	Scheme	Marks
7(a)	$\left(1+\frac{x}{3}\right)^{-3} = \left[1+(-3)\left(\frac{x}{3}\right)+\frac{(-3)(-3-1)}{2!}\left(\frac{x}{3}\right)^2+\frac{(-3)(-3-1)(-3-2)}{3!}\left(\frac{x}{3}\right)^3 \ldots\right]$ $=1-x+\frac{2}{3}x^2-\frac{10}{27}x^3$	M1A1 A1 [3]
(b)	$-3<x<3$ or $\lvert x\rvert<3$	B1 [1]
(c)	$\left((3+x)^{-3}=3^{-3}\times\left(1+\frac{x}{3}\right)^{-3}=\frac{1}{27}\times\left(1+\frac{x}{3}\right)^{-3}\right)$ $P=\frac{1}{27}$, $Q=\frac{1}{3}$	B1, B1 [2]
(d)	$\frac{(1+4x)}{(3+x)^3}=(1+4x)\times\frac{1}{27}\times\left(1-x+\frac{2}{3}x^2\right)$ $=\frac{1}{27}\times\left(1-x+\frac{2}{3}x^2+4x-4x^2+....\right)$ $=\frac{1}{27}\left(1+3x-\frac{10x^2}{3}\right)$ or $\frac{1}{27}+\frac{x}{9}-\frac{10x^2}{81}$	M1 A1 [2]
(e)	$\int_0^{0.2}\left(\frac{1}{27}+\frac{x}{9}-\frac{10x^2}{81}\right)dx=\left[\frac{x}{27}+\frac{x^2}{18}-\frac{10x^3}{243}\right]_0^{0.2}$ $=\left(\frac{0.2}{27}+\frac{0.2^2}{18}-\frac{10\times0.2^3}{243}\right)-[0]$ $=0.0093004$	M1 M1 A1 [3]
		Total 11 marks

Part	Mark	Notes
(a)	**M1**	For applying a correct binomial expansion in unsimplified form. Minimum required: • The expansion begins with 1 • The next term is correct • The powers of $\frac{x}{3}$ must be correct eg $\left(\frac{x}{3}\right)^2$ • The denominators are correct. Do not allow missing brackets unless recovered later – this is a general point of marking. Ignore any terms with powers higher than 3.
	A1	Following M1 (this is a general point of marking, A marks can only follow M marks), all conditions above met, must see $1 - x$ and at least the term in x^2 or x^3 correct and simplified. Ignore any terms with powers higher than 3.
	A1	A fully correct and simplified expansion. Ignore any terms with powers higher than 3..
(b)	**B1**	For the correct validity.
(c)	**B1**	For the correct value of P or Q explicitly written or embedded in $\frac{1}{27}\times\left(1+\frac{x}{3}\right)^{-3}$
	B1	For the correct values of P and Q explicitly written or embedded in $\frac{1}{27}\times\left(1+\frac{x}{3}\right)^{-3}$
(d)	**M1**	For attempting to multiply their expansion, which must be of the form $"P"\times$ (their expansion from part (a), with a minimum of 3 terms, by $(1 + 4x)$. An attempt must include 3 correctly multiplied out terms of their expansion before simplification. The $"P"$ may remain factorised. Ignore any terms which to powers of x higher than 2.
	A1	For the correct expansion in either form shown. Allow equivalent coefficients. Ignore powers of x higher than 2.
(e)	**M1**	For an attempt to integrate their expression from part (d), provided it has at least one constant term and at least one algebraic term. See general guidance, no power of x must decrease.
	M1	For substituting in the value of 0.2 into a changed expression and subtracting the correct way. Must see the explicit substitution of 0.2 at least once, if the final answer is not correct. Can be implied if final answer correct. Substitution of 0 does not need to be seen.
	A1	For the value of 0.0093004 [The calculator value is 0.0093316] Accept the value of $\frac{113}{12150}$ if seen.

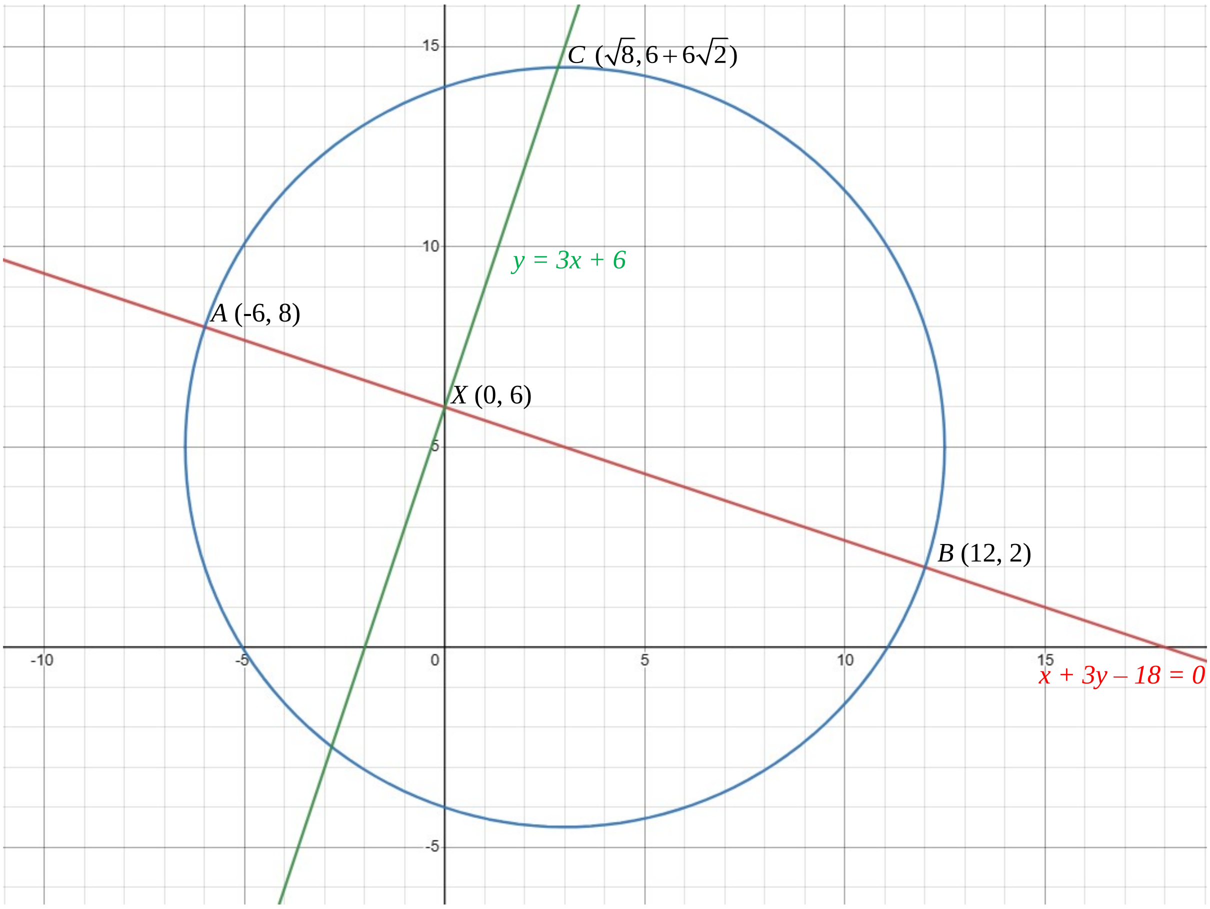
Question	Scheme	Marks
8(a)	$\left(\text{Grad}_{AB}\right)=\dfrac{2-8}{12-(-6)}$ oe $y-2="\left(-\dfrac{1}{3}\right)"(x-12)$ oe $\Rightarrow x+3y-18=0$ or $-x-3y+18=0$ oe	M1 dM1 (A1 on ePen) A1 [3]
(b)	Length $=\sqrt{(12--6)^2+(2-8)^2}=6\sqrt{10}$ eg $\sqrt{360}$ oe	M1A1 [2]
(c)	$\left(\left(\dfrac{2\times-6+1\times12}{1+2},\dfrac{2\times8+1\times2}{1+2}\right)\right)$ $(0,6)$ or $m=0, n=6$	B1 B1 (M1A1 on ePen) [2]
(d) **Mark parts (i) and (ii) together**	Gradient of $CA=\dfrac{q-8}{p+6}$ **or** gradient of $CB=\dfrac{q-2}{p-12}$ oe $\dfrac{q-8}{p+6}=-\dfrac{1}{\dfrac{q-2}{p-12}}=-\left(\dfrac{p-12}{q-2}\right)\Rightarrow q^2-10q+16=-p^2+6p+72$ Gradient of $XC=-\dfrac{1}{"-\frac{1}{3}"}\Rightarrow-\dfrac{1}{"-\frac{1}{3}"}=\dfrac{q-6}{p-0}(\Rightarrow q=3p+6)$ $('3p+6')^2-10('3p+6')+16=-p^2+6p+72\Rightarrow10p^2-80=0$ $10p^2-80=0\Rightarrow p=\sqrt{8}$ oe $q=3\times\sqrt{8}+6=6+6\sqrt{2}$ oe	M1 M1 B1ft ddM1A1 M1A1 [7]
ALT1 **Mark parts (i) and (ii) together**	$(\text{midpoint of } AB=)\left(\dfrac{-6+12}{2},\dfrac{8+2}{2}\right)$ $(=(3,5))$ $(\text{radius of } C)=\dfrac{"6\sqrt{10}"}{2}$ oe or $\sqrt{(-6-3)^2+(8-5)^2}$ oe $y=\dfrac{-1}{"-\frac{1}{3}"}x+6$ $(y=3x+6)$ oe $(x-"3")^2+(y-"5")^2=\left(\dfrac{"6\sqrt{10}"}{2}\right)^2\Rightarrow(x-"3")^2+("3x+6"-"5")^2=\left(\dfrac{"6\sqrt{10}"}{2}\right)^2$ $10x^2=80$ oe $x=\sqrt{8}$ oe $p=\sqrt{8}, q=6+3\sqrt{8}$ oe	M1 M1 B1ft ddM1 A1 M1 A1 [7]

ALT2 Mark parts (i) and (ii) together	$\left((AC)^2 =\right)(p--6)^2+(q-8)^2$ or $(AC=)\sqrt{(p--6)^2+(q-8)^2}$ oe $\left((BC)^2 =\right)(p-12)^2+(q-2)^2$ or $(AC=)\sqrt{(p-12)^2+(q-2)^2}$ oe $q=3p+6$ $\left((AB)^2=(AC)^2+(BC)^2\right)$ $\left("6\sqrt{10}"\right)^2=(p--6)^2+("3p+6"-8)^2+(p-12)^2+("3p+6"-2)^2$ $10p^2=80$ oe $p=\sqrt{8}$ oe $p=\sqrt{8}, q=6+3\sqrt{8}$ oe	M1 M1 B1ft ddM1A1 M1 A1 [7]
(e)	Length CX = $\sqrt{\left("\sqrt{8}"-"0"\right)^2+\left("6+6\sqrt{2}"-"6"\right)^2}\ \left(=4\sqrt{5}\right)$ Area of triangle ABC = $\frac{1}{2}\times"4\sqrt{5}"\times"6\sqrt{10}"=60\sqrt{2}$ oe	M1 dM1 A1 [3]
ALT	$\frac{1}{2}\begin{vmatrix}-6 & 12 & "\sqrt{8}" & -6\\ 8 & 2 & "6+3\sqrt{8}" & 8\end{vmatrix}$ oe $\frac{1}{2}\left[\left(-6\times2+12\times"(6+3\sqrt{8})"+"\sqrt{8}"\times8\right)-\left(-6\times"(6+3\sqrt{8})"+"\sqrt{8}"\times2+12\times8\right)\right]$ oe $60\sqrt{2}$ oe	M1 dM1 A1 [3]
		Total 17 marks

Part	Mark	Notes
(a)	**M1**	For correctly finding the gradient of *AB* in unsimplified form.
	dM1 (A1 on ePen)	For a full and correct attempt to find the equation of the line using their gradient. No simplification is required. If using $y = mx + c$, a value for c must be found.
	A1	For a correct equation in the required form.
(b)	**M1**	For using a correct method to find the length of line segment *AB*, in unsimplified form.
	A1	For the correct exact length.
(c)	**B1 (M1 on ePen)**	For either coordinate correct.
	B1 (A1 on ePen)	For both correct coordinates (0, 6) For part c the values of m and n can be explicitly identified or written in a coordinate.
In part (d) allow *p* to be interchangeable with *x*, *q* to be interchangeable with *y* throughout		
(d)	**M1**	For a correct statement of the gradient for either *CA* or *CB*
	M1	For using the negative perpendicular of either gradient and equating the gradients to form an equation in terms of p and q only.
	B1ft	For finding the negative reciprocal of their gradient of *XC* and placing this equal to a correct expression in p and q as shown.
	ddM1	For correctly substituting their **linear** expression for p or q into a **quadratic** equation in q or p to obtain an equation in one variable. Must use their $q = 3p + 6$ and is dependent on both previous method marks.
	A1	For the correct two term quadratic..
	M1	For **correctly solving their** quadratic to find a value for either p or q
	A1	For **both** p and q correct.
ALT1 (d)	**M1**	For the correct method to find the midpoint of *AB*
	M1	For the correct method to find the radius of *C*, ft their answer from part b if used.
	B1ft	For the equation of the line, unsimplified, ft their gradient of *AB*
	ddM1	For correctly substituting their $y = 3x + 6$ into the equation of a circle, using their midpoint of *AB* and their radius of *C* Must use their $y = 3x + 6$ and is dependent on both previous method marks.
	A1	Correct equation
	M1	For **correctly solving their** quadratic to find a value for either x or y
	A1	For **both** p and q correct.
ALT2 (d)	**M1**	For the correct method to find the length of *AC* or $(AC)^2$
	M1	For the correct method to find the length of *BC* or $(BC)^2$
	B1ft	For the equation of the line, unsimplified, ft their gradient of *AB*
	ddM1	For correctly substituting their lengths and their $q = 3p + 6$ into a correct Pythagorean equation, dependent on both previous method marks.
	A1	Correct equation
	M1	For **correctly solving their** quadratic to find a value for either p or q
	A1	For **both** p and q correct.

(e)	**M1**	For using a correct method to find the length of the perpendicular from *AB* to *C*
	dM1	For using their results from part (b) and their length of perpendicular from *AB* to *C* with the correct formula for the area of a triangle. Dependent on previous method mark.
	A1	For the correct area of $60\sqrt{2}$ or $30\sqrt{8}$ (units^2)
ALT	**M1**	For a correct statement for the area such as the one shown, using their values of *p* and *q*
	dM1	For the correct evaluation of their determinant Dependent on previous method mark.
	A1	For the correct area of $60\sqrt{2}$ or $30\sqrt{8}$ (units^2)

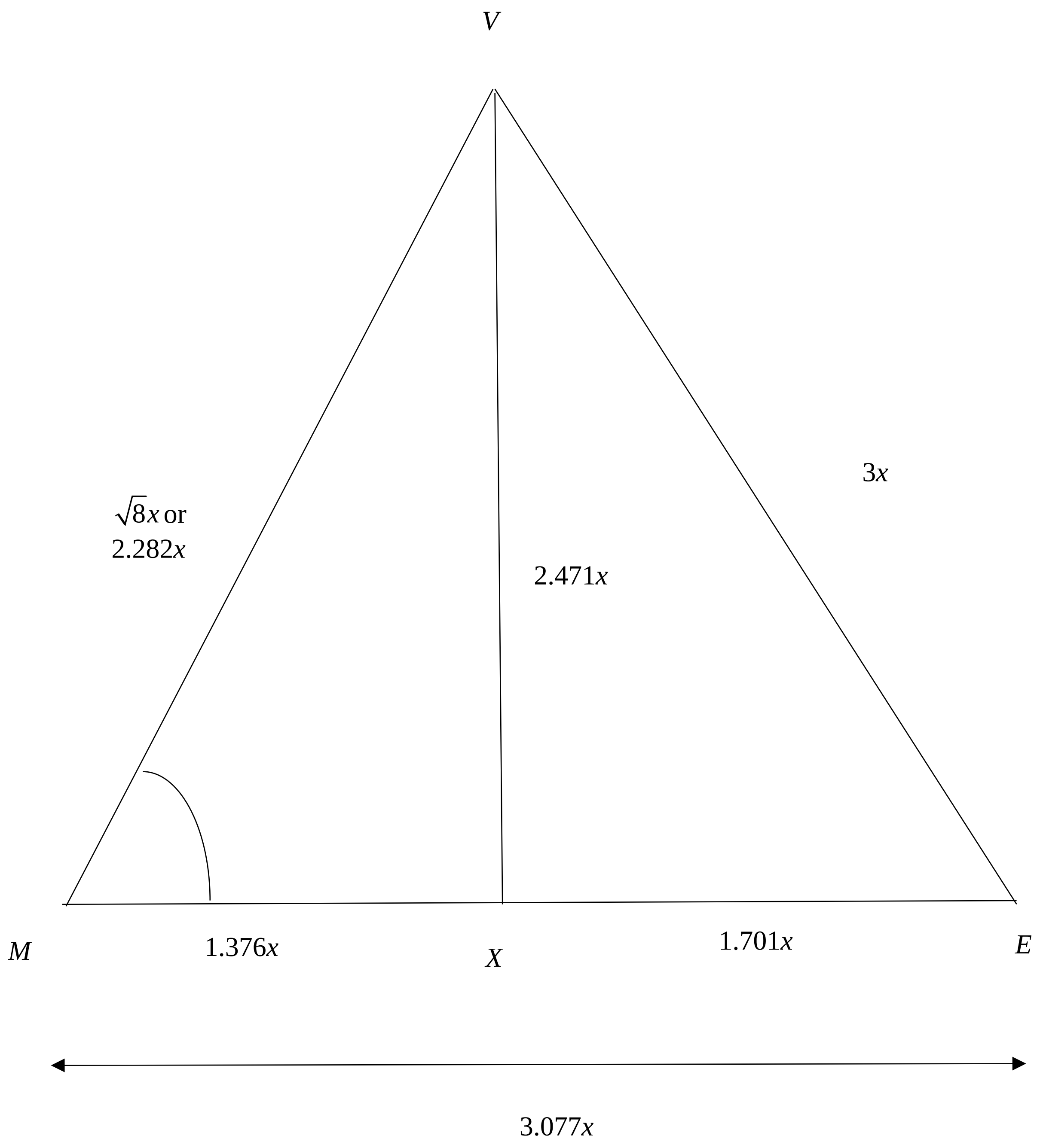
Useful Sketch for Parts c/d – look for any working on or near a sketch.



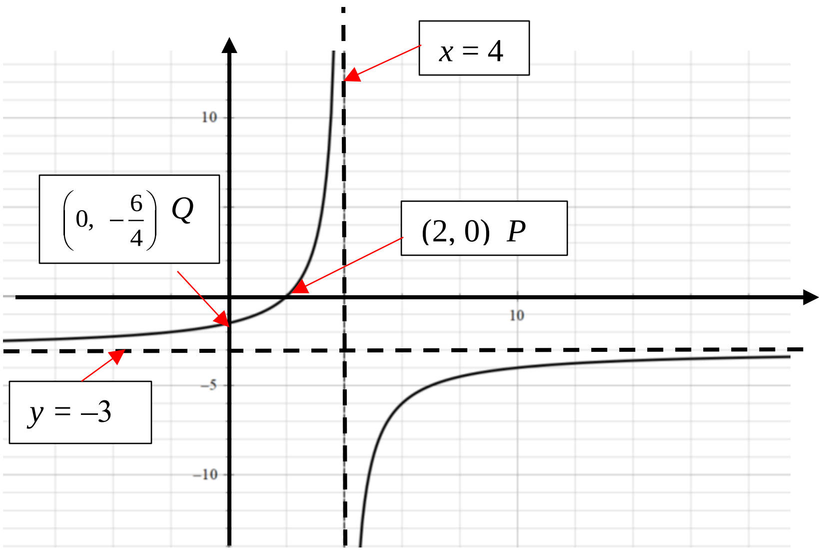
Question	Scheme	Marks
9	**Be careful to look on the sketch to award marks for the question and for any equivalent calculations or alternative methods.** **IF IN DOUBT SEND TO REVIEW** *X* is the point directly below *V* positioned on the base *ABCDE* Perpendicular from the mid-point of *BC* (*M*) to point *X*	
	$(\angle BXC =)72^{\circ}$ or $(\angle ABC =)108^{\circ}$	B1
	Length of *BX* eg $(BX =)\dfrac{x}{\sin 36^{\circ}}(=1.701x)$ or $\dfrac{\sin 54}{\sin 72}\times 2x$ or $\dfrac{x}{\cos 54}$ oe	M1
	$(VX =)\sqrt{(3x)^2-(\text{"}1.701x\text{"})^2}=2.471x$ or $(VM =)\sqrt{(3x)^2-(x)^2}=\sqrt{8}x(=2.828x)$	M1
	$(MX =)\sqrt{(\text{"}1.701x\text{"})^2-x^2}=1.376x$ or $\dfrac{x}{\tan 36}$ oe or $x\tan 54$	M1
	Required angle is $\angle VMX$ or $\angle VME$ $(\tan\angle VMX =)\dfrac{2.471x}{1.376x}\Rightarrow(\angle VMX = 60.888...^{\circ})$ oe $(\cos\angle VME =)\dfrac{(\text{"}\sqrt{8}x\text{"})^2+(\text{"}1.701\text{"}x+\text{"}1.376\text{"}x)^2-(3x)^2}{2\times\text{"}\sqrt{8}x\text{"}\times(\text{"}1.701\text{"}x+\text{"}1.376\text{"}x)}$ oe	dddM1
	$\Rightarrow$ awrt 60.9° or better	A1 [6]
ALT1	**May only be applied if they attempt to find *MX* in one step and attempt to find *VM* and use triangle *VMX*. If they find *MX* any other way, apply one of the other schemes**	
	$(\angle BXC =)72^{\circ}$ or $(\angle ABC =)108^{\circ}$	B1
	$(MX =)$ $\dfrac{x}{\tan 36}$ oe or $x\tan 54$	M2
	$(VM =)\sqrt{(3x)^2-(x)^2}=\sqrt{8}x(=2.828x)$	M1
	$(\cos\angle VMX =)\dfrac{\text{"}x\tan 54\text{"}}{\text{"}2\sqrt{2}x\text{"}}\Rightarrow(\angle VMX = 60.888...^{\circ})$	dddM1
	$\Rightarrow$ awrt 60.9° or better	A1 [6]

ALT2	$(\angle BXC =) 72^{o}$ or $(\angle ABC =) 108^{o}$	B1
	$(EB =)\sqrt{(2x)^2 + (2x)^2 - 2(2x)(2x)\cos 108}\,(= 3.236x)$ or $(EB =)\dfrac{2x}{\sin 36} \times \sin 108 (= 3.236x)$	M1
	$(VM =)\sqrt{(3x)^2 - (x)^2} = \sqrt{8}x (= 2.828x)$	M1
	$(EM =)\sqrt{("3.236x")^2 - x^2}\,(= 3.077x)$	M1
	$(\cos \angle VME =)\dfrac{("\sqrt{8}x")^2 + ("1.701"x + "1.376"x)^2 - (3x)^2}{2 \times "\sqrt{8}x" \times ("1.701"x + "1.376"x)}$	dddM1
	$\Rightarrow$ awrt 60.9^{o} or better	A1 [6]
		Total 6 marks

Mark	Notes
B1	For writing down or finding the angle $BXC = 72^{\text{o}}$ or $\angle ABC = 108^{\text{o}}$, any notation. Clear identification or implicit use of this angle in later working is acceptable. Note use of 36 degrees or 54 degrees implied this mark.
M1	Ignore missing x's throughout their solution. For **any** correct, suitable trigonometry to find the length BX
M1	Correct use of Pythagoras theorem (with a – sign) or suitable trigonometry to find the height VX of the pyramid or the length VM
M1	Correct use of Pythagoras theorem (with a – sign) or suitable trigonometry to find the length of the midpoint of BC to point X
dddM1	For any suitable trigonometry to find the size of the required angle. Dependent on all 3 previous method marks.
A1	For the correct angle, awrt 60.9^{o}
ALT1 **B1**	For writing down or finding the angle $BXC = 72^{\text{o}}$ or $\angle ABC = 108^{\text{o}}$, any notation. Clear identification or implicit use of this angle in later working is acceptable. Note use of 36 degrees or 54 degrees implied this mark.
M2	Ignore missing x's throughout their solution. For **any** correct method to find the length MX
M1	Correct method to find the length VM
dddM1	For any suitable trigonometry to find the size of the required angle. Dependent on all 3 previous method marks.
A1	For the correct angle, awrt 60.9^{o}
ALT2 **B1**	For writing down or finding the angle $BXC = 72^{\text{o}}$ or $\angle ABC = 108^{\text{o}}$, any notation. Clear identification or implicit use of this angle in later working is acceptable. Note use of 36 degrees or 54 degrees implied this mark.
M1	Ignore missing x's throughout their solution. For **any** correct method to find the length EB
M1	For any correct method to find VM
M1	For any correct method to find EM
dddM1	For any suitable trigonometry to find the size of the required angle. Dependent on all 3 previous method marks.
A1	For the correct angle, awrt 60.9^{o}
Where students are allowed to use their values, indicated by inverted commas, it MUST be clear from working or their sketch this is the correct use of the relevant side. Beware $\sqrt{8}x$ coming from incorrect working of $\sqrt{(2x)^2+(2x)^2}$	



V
$\sqrt{8}x$ or
$2.282x$
$3x$
$2.471x$
M
$1.376x$
X
$1.701x$
E
$3.077x$

Question	Scheme	Marks
10(a) **(i)**	When $y = 0$, $x = 2$ or $(2, 0)$	B1
(ii)	When $x = 0$, $y = -\frac{6}{4}$ oe or $\left(0, -\frac{6}{4}\right)$ oe	B1 [2]
	If not labelled part (i) and (ii), do not award marks unless the candidate has presented in the correct order or has made it clear which coordinate or pair of values is *P* and which is *Q*	
(b)	(i) $x = 4$ (ii) $y = -3$ **If not labelled part (i) and (ii), do not award marks unless the candidate has presented in the correct order or has identification of horizontal (or parallel to *x*-axis) and vertical (or parallel to *y*-axis) is clear.**	B1 B1 [2]
(c)		B1 B1 ft B1 ft [3]
(d)	$\left(\frac{dy}{dx} =\right)\frac{-3(x-4)-(6-3x)\times 1}{(x-4)^2} = \left[\frac{6}{(x-4)^2}\right]$ oe	M1A1A1
	where $x = 2$ $\frac{dy}{dx} = \frac{6}{(2-4)^2} = \frac{6}{4} \Rightarrow$ Gradient of normal $= -\frac{4}{6}$ oe	M1
	Equation of normal: $y - 0 = "-\frac{4}{6}"(x-2) \Rightarrow [3y = -2x + 4]$ oe	dM1A1 [6]
(e)	$\frac{6-3x}{x-4} = \frac{-2x+4}{3} \Rightarrow 2x^2 - 21x + 34 = 0$	M1
	$\Rightarrow (2x-17)(x-2) = 0$	M1
	At $R \Rightarrow x = \frac{17}{2}$, (2)	A1 [3]
		Total 16 marks

Part	Mark	Notes
(a) (i)	**B1**	Must state $y = 0$, $x = 2$ or (2, 0) or clearly stating $x = 2$
(ii)	**B1**	Must state $x = 0$, $y = -\frac{6}{4}$ oe or $\left(0, -\frac{6}{4}\right)$ oe or clearly stating $y = -\frac{6}{4}$
(b)(i)	**B1**	For $x = 4$
(ii)	**B1**	For $y = -3$
(c)	**B1**	For a negative reciprocal curve drawn anywhere in the grid – there **must** be two branches present, they **must not** cross any asymptotes drawn and **must not** obviously 'bend back' on themselves. Mark intention.
	B1ft	For the asymptotes drawn, follow through their (b)(i) and (ii). There **must** be at least one branch of a negative reciprocal curve present in the correct place for their work in (a) and (b), which **must** not cross or obviously bend back from the asymptotes. The asymptotes **must** be labelled with their equation or shown as passing through 4 on the x-axis and – 3 on the y-axis, both clearly labelled.
	B1ft	For at least one branch of a negative reciprocal curve in the correct place for their work in (a) and (b), passing through their intercepts on the axes. The intercepts should be correctly labelled with the coordinates or the axes labelled correctly with $-\frac{6}{4}$ (oe) and 2 (or their ft values) but condone labelling to be P and Q.
(d)	**M1**	For an expression of the form. $\frac{-a(x-4)-(6-3x)\times b}{(x-4)^2}$ oe
	A1	For an expression of the form. $\frac{-3(x-4)-(6-3x)\times b}{(x-4)^2}$ or $\frac{-a(x-4)-(6-3x)\times 1}{(x-4)^2}$ .oe
	A1	Fully correct – need not be simplified.
	M1	For substituting $x = 2$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$ and finding the gradient of the normal. This is not a dependent method mark, but the substitution must be into a **changed** function. If their expression does not allow substitution of $x = 2$, this mark cannot be awarded.
	dM1	For a complete and correct method to find the equation of the normal using their (changed gradient), $y = 0$ and $x = 2$. Dependent on the previous mark. If $y = mx + c$ is used they must find a value for c.
	A1	For any correct equation. **This can be in any form and may be left unsimplified.**
(e)	**M1**	For equating their equation of the normal to C and attempting to form a 3TQ. The attempt must involve correctly removing both denominators of the equation as a minimum and any attempt to collect terms.
	M1	For a minimally acceptable (see general guidance) and complete attempt to solve their quadratic equation, leading to a value of x..
	A1	For the x coordinate of point R $x = \frac{17}{2}$

Question	Scheme	Marks
11 (a)(i)	$\left(\alpha-\beta=2\sqrt{6}\Rightarrow(\alpha-\beta)^2=24\right)\Rightarrow(\alpha+\beta)^2-4\alpha\beta=24$ oe **(1)** $\alpha^2+\beta^2=30\Rightarrow(\alpha+\beta)^2-2\alpha\beta=30$ oe **(2)** **(2) – (1)** $6=2\alpha\beta\Rightarrow\alpha\beta=3$ cso	M1 M1 dM1A1* [4]
ALT1	$\left((\alpha-\beta)^2=\alpha^2+\beta^2-2\alpha\beta\Rightarrow\right)2\alpha\beta=\alpha^2+\beta^2-(\alpha-\beta)^2$ oe $30-\left(2\sqrt{6}\right)^2=6$ $2\alpha\beta=6\Rightarrow\alpha\beta=3$	M1 M1 dM1A1 [4]
(ii)	$30=(\alpha+\beta)^2-2\times3\Rightarrow(\alpha+\beta)^2=36\Rightarrow\alpha+\beta=6$ $[\alpha>\beta>0]$ cso	M1A1* [2]
ALT2 (i)	$\alpha-\beta=2\sqrt{6}\to\alpha=2\sqrt{6}+\beta$ $\alpha^2+\beta^2=30\to\left(2\sqrt{6}+\beta\right)^2+\beta^2=30$ $2\beta^2+4\sqrt{6}\beta-6=0$ oe eg $\beta^2+2\sqrt{6}\beta-3=0$ $(\beta=)\dfrac{-2\sqrt{6}\pm\sqrt{\left(2\sqrt{6}\right)^2-4(1)(-3)}}{2}$ oe $\to\beta\left(=3-\sqrt{6}\right)$ $\alpha=3+\sqrt{6}$ $\alpha\beta=\left(3+\sqrt{6}\right)\left(3-\sqrt{6}\right)=9+6\sqrt{3}-6\sqrt{3}-6=3$	M1 M1 dM1 A1* [4]
(ii)	$\alpha+\beta=3+\sqrt{6}+3-\sqrt{6}=6$	M1A1 [2]
(b)(i)	$\alpha^4+\beta^4=\left(\alpha^2+\beta^2\right)^2-2(\alpha\beta)^2=30^2-2\times3^2=882$	M1A1 [2]
(b) ii)	$\alpha^4-\beta^4=\left(\alpha^2+\beta^2\right)\left(\alpha^2-\beta^2\right)=\left(\alpha^2+\beta^2\right)(\alpha-\beta)(\alpha+\beta)$ $\alpha^4-\beta^4=30\times6\times2\sqrt{6}=360\sqrt{6}$	M1 A1 [2]
(c)	$\left(\alpha^4+\beta^4\right)+\left(\alpha^4-\beta^4\right)=2\alpha^4=882+360\sqrt{6}\Rightarrow\alpha^4=441+180\sqrt{6}$	M1A1 [2]
ALT1	$\alpha-\beta=2\sqrt{6},\alpha+\beta=6\Rightarrow\alpha=3+\sqrt{6}$ $\alpha^4=\left(3+\sqrt{6}\right)^4$ $\alpha^4=441+180\sqrt{6}$	M1 A1 [2]
ALT2	$\alpha-\beta=2\sqrt{6},\alpha+\beta=6\Rightarrow\beta=3-\sqrt{6}$ $\alpha^4=\beta^4+360\sqrt{6}=\left(3-\sqrt{6}\right)^4+360\sqrt{6}$ $\alpha^4=441+180\sqrt{6}$	M1 A1 [2]
		Total 12 marks

Part	Mark	Notes
(a) (i)	**M1**	For forming an equation of the form $(\alpha+\beta)^2 \pm p\alpha\beta = 24$ oe or $(\alpha+\beta)^2 \pm t\alpha\beta = 30$ oe
	M1	For this equation being fully correct. Both marks may be implied by early substitution of values.
	dM1	For correctly solving their simultaneous equations in $\alpha+\beta$ and $\alpha\beta$ to find a value for $\alpha\beta$, dependent on 1st method mark.
	A1*cso	For the correct value of $\alpha\beta$
ALT1	**M1**	For forming an equation of the form $q\alpha\beta = \alpha^2+\beta^2-(\alpha-\beta)^2$ oe
	M1	For this equation being fully correct. Both marks may be implied by early substitution of values.
	dM1	For correct substitution into their equation, dependent on 1st method mark.
	A1*cso	For the correct value of $\alpha\beta$
(a) (ii)	**M1**	Correctly uses either of their equations (must be of the required form) to substitute the given value of $\alpha\beta$ and obtains a value for $\alpha+\beta$
	A1*cso	For the correct value of $\alpha+\beta$
ALT2 (i)	**M1**	For an attempt to eliminate α or β and arrive at an unsimplified quadratic equation in one variable. Allow one error
	M1	For the correct quadratic equation
	dM1	For a **fully correct method** to solve their quadratic equation in α or β, dependent on the 1st method mark
	A1*cso	For correctly finding α and β and showing the minimum steps shown to find $\alpha\beta$
	M1	For finding $\alpha+\beta$ with their values
	A1*cso	For finding $\alpha+\beta$ with the correct values, minimum steps as shown.
(b) (i)	**M1**	For the correct algebra $\alpha^4+\beta^4=(\alpha^2+\beta^2)^2-2(\alpha\beta)^2$, if they use more complex algebra, it must be correct and fully ready for substitution of given values. Do not allow $\alpha^4+\beta^4=(\alpha^2+\beta^2)^2-2\alpha^2\beta^2$ unless recovered
	A1	For substituting the given values to find the correct value for $\alpha^4+\beta^4$
(b) (ii)	**M1**	For the correct algebra to write $\alpha^4-\beta^4$ in terms of $(\alpha^2+\beta^2),(\alpha-\beta)$ and $(\alpha+\beta)$
	A1	For the correct value of $\alpha^4-\beta^4$
(c)	**M1**	For adding together $\alpha^4+\beta^4$ and $\alpha^4-\beta^4$ to eliminate β^4 and reach α^4 either as an expression or implied by adding their values together and dividing by 2. If students subtract $\alpha^4+\beta^4$ and $\alpha^4-\beta^4$ to eliminate α^4 and reach β^4, they must then reach α^4 as an expression or implied by subtracting their values, using one of the expressions from part b and dividing by 2.
	A1	For the correct value of $\alpha^4 = 441+180\sqrt{6}$
ALT1	**M1**	For $\alpha^4=(3+\sqrt{6})^4$
	A1	For the correct value of $\alpha^4 = 441+180\sqrt{6}$
ALT2	**M1**	For $\alpha^4=\beta^4+360\sqrt{6}=(3-\sqrt{6})^4+360\sqrt{6}$
	A1	For the correct value of $\alpha^4 = 441+180\sqrt{6}$

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