Please check the examination details belo	ow before entering your candidate information
Candidate surname	Other names
Pearson Edexcel Interi	
Friday 17 November	2023
Morning (Time: 2 hours)	Paper reference 4PM1/02
Further Pure Matl	nematics
Calculators may be used.	Total Marks

## **Instructions**

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer all questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  - there may be more space than you need.
- You must NOT write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

# Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  - use this as a guide as to how much time to spend on each question.

## **Advice**

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶





## **International GCSE in Further Pure Mathematics Formulae sheet**

#### Mensuration

**Surface area of sphere** =  $4\pi r^2$ 

Curved surface area of cone =  $\pi r \times \text{slant height}$ 

Volume of sphere =  $\frac{4}{3}\pi r^3$ 

#### Series

### **Arithmetic series**

Sum to *n* terms,  $S_n = \frac{n}{2} [2a + (n-1)d]$ 

## Geometric series

Sum to *n* terms, 
$$S_n = \frac{a(1-r^n)}{(1-r)}$$

Sum to infinity,  $S_{\infty} = \frac{a}{1-r} |r| < 1$ 

#### **Binomial series**

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots$$
 for  $|x| < 1, n \in \mathbb{Q}$ 

#### **Calculus**

## **Quotient rule (differentiation)**

$$\frac{\mathrm{d}}{\mathrm{d}x} \left( \frac{\mathrm{f}(x)}{\mathrm{g}(x)} \right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{\left[\mathrm{g}(x)\right]^2}$$

## **Trigonometry**

### Cosine rule

In triangle ABC:  $a^2 = b^2 + c^2 - 2bc \cos A$ 

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

## Logarithms

$$\log_a x = \frac{\log_b x}{\log_b a}$$



# Answer all ELEVEN questions.

# Write your answers in the spaces provided.

	You must write down all the stages in your working.	
1	The equation $kx^2 + 8x + 3k = 0$ where k is a constant, has real unequal roots.	
	Find the set of values of $k$ giving your answer in an exact simplified form.	
		(5)
	/T / 10 C	1. 3
	(Total for Question 1 is 5	marks)



2	In triangle ABC, $AB = 3x$ cm, $BC = 5x$ cm and $\angle ABC = 110^{\circ}$	
	(a) Find, in degrees to one decimal place, the size of $\angle BCA$	
		(4)
	The area of triangle $ABC$ is 24 cm <sup>2</sup>	
	(b) Find, to 3 significant figures, the value of x	
		(3)





3	A particle $P$ moves along the $x$ -axis. At time $t$ seconds ( $t \ge 0$ ) the acceleration, $a$ m/s <sup>2</sup> , of $P$ is given by $a = 6t - 16$ When $t = 0$ , $P$ is at the origin and is moving with velocity 12 m/s.	
	(a) Find an expression in terms of t for	
	(i) the velocity of <i>P</i> at time <i>t</i> seconds	
	(ii) the displacement of $P$ at time $t$ seconds.	(4)
	(b) Hence find the time at which <i>P</i> first returns to the origin.	(3)





(a) On the axes opposite, draw the line with equation

(i) 
$$y = -x - 1$$

(i) 
$$y = -x - 1$$
 (ii)  $y - 3x + 8 = 0$ 

(iii) 
$$2y = x + 8$$

(3)

(b) Show, by shading on your graph, the region R defined by the inequalities

$$y \geqslant -x-1$$
 and  $y \geqslant 3x-8$  and  $2y \leqslant x+8$ 

(1)

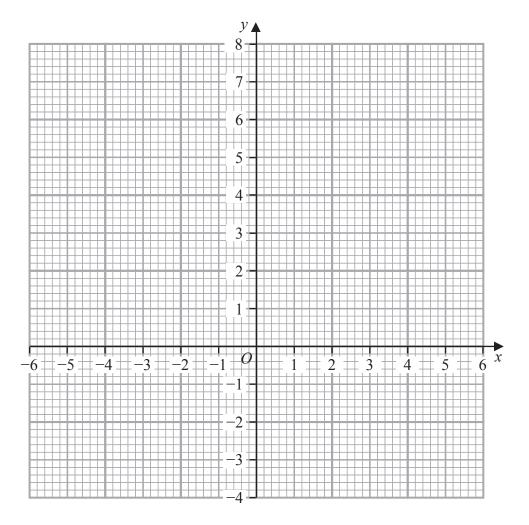
For all points in R, with coordinates (x, y)

$$P = 2y - 3x$$

- (c) Find
  - (i) the greatest value of P
  - (ii) the least value of P

**(4)** 


# **Question 4 continued**



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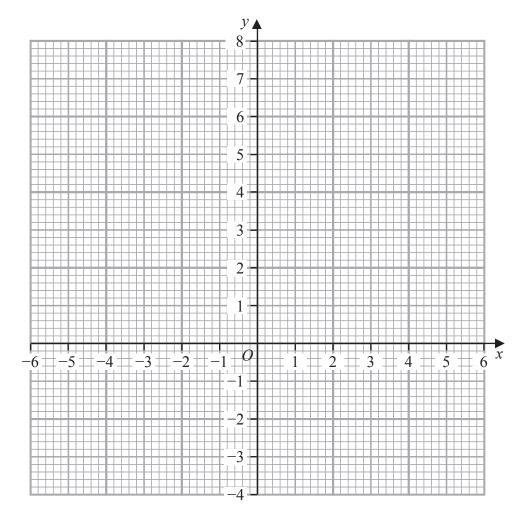
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Question 4 continued	



# **Question 4 continued**

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(Total for Question 4 is 8 marks)

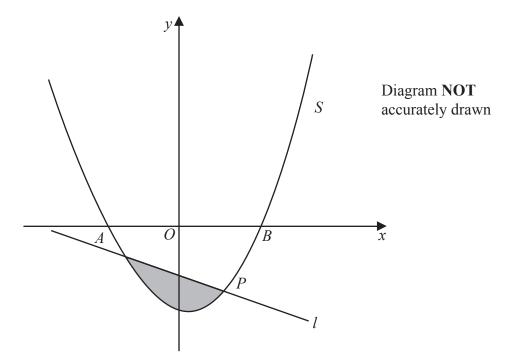


Figure 1

Figure 1 shows part of the curve S with equation  $y = px^2 + qx + r$  where p, q and r are constants.

The points A, B and P with coordinates (-2, 0), (6, 0) and (4, -6) respectively lie on S

(a) Show that an equation of S is 
$$y = \frac{x^2}{2} - 2x - 6$$
 (3)

The line l is the normal to S at the point P

(b) Show that an equation of *l* is 2y + x + 8 = 0

(5)

The finite region shown shaded in Figure 1 is bounded by S and l

(c) Use algebraic integration to find the exact area of the shaded region.

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Question 5 continued





6	Oil is pouring into the container at a constant rate of $12 \text{ cm}^3/\text{s}$ . Given that $V = 3h^3$			
	find the exact rate, in cm/s, at which the height of the oil is increasing when $V = 1536 \text{ cm}^3$			
		(7)		





7 Two numbers x and y are such that 3x - y = 4

$$S = 5x^3 + y^2$$

(a) Show that  $S = 5x^3 + 9x^2 - 24x + 16$ 

(2)

Given that x can vary,

(b) use calculus to find the value of x for which S is a minimum, justifying that this value of x gives a minimum value of S

(5)

(c) Find the minimum value of S

(2)






**(6)** 

(4)

8	The sum to $n$ terms of an arithmetic series $A$ is $S_n$
	The sum of the first four terms of $A$ is 42 and the fifth term of $A$ is 23
	(a) Show that $S_n = \sum_{r=1}^n (Pr - Q)$ where $P$ and $Q$ are prime numbers.
	$S_{2n} - 3U_n = 1062$ where $U_n$ is the <i>n</i> th term of $A$
	(b) Find the value of n

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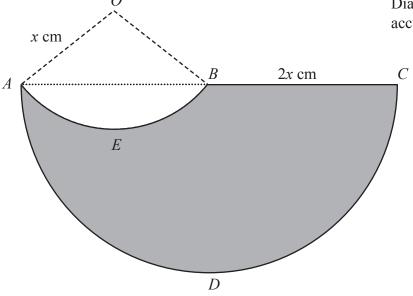


Figure 2

A logo, AEBCD, is shown shaded in Figure 2.

The straight line *ABC* is the diameter of the semicircle *ADC AEB* is an arc of a circle with centre *O* All angles are measured in radians.

- BC = 2x cm
- OA = OB = x cm
- length of arc AEB = 1.8x cm

The perimeter of the logo is P

(a) Show that  $P = ax(\pi + \pi \sin 0.9 + b)$  where a and b are constants to be found.

**(7)** 

Given that x = 10 cm,

(b) find, in cm<sup>2</sup> to 3 significant figures, the area of the logo.

**(6)** 



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Question 9 continued	





10 The roots of a quadratic equation are  $\alpha$  and  $\beta$  where

$$\alpha + \beta = -\frac{5}{2}$$
 and  $\alpha^{3} + \beta^{3} = \frac{115}{8}$ 

(a) Show that  $\alpha\beta = 4$ 

(3)

(b) Form a quadratic equation with integer coefficients, that has roots

$$\frac{\alpha^2 + 1}{\beta}$$
 and  $\frac{\beta^2 + 1}{\alpha}$  (7)



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Question 10 continued	





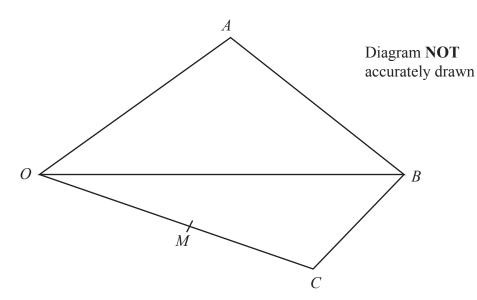


Figure 3

Figure 3 shows quadrilateral OABC where

$$\overrightarrow{OA} = 4\mathbf{p} + 5\mathbf{q}$$
  $\overrightarrow{OB} = 3\mathbf{p} + \mathbf{q}$   $\overrightarrow{OC} = 2\mathbf{p} - 4\mathbf{q}$ 

The point M is the midpoint of OC

(a) Find  $\overrightarrow{\mathit{MA}}$  as a simplified expression in terms of **p** and **q** 

(3)

The point N lies on OB such that M, N and A are collinear.

(b) Find the ratio MN: NA

**(6)** 



Question 11 continued		
(Total for Question 11 is 9 marks)		

TOTAL FOR PAPER IS 100 MARKS

