Please check the examination details below before entering your candidate information

| Candidate surname | Other names |
|---|---|
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Centre Number | Candidate Number

# Pearson Edexcel International GCSE

## Friday 17 November 2023

Morning (Time: 2 hours)

Paper reference **4PM1/02**

# Further Pure Mathematics

## PAPER 2

**Calculators may be used.**

Total Marks

## Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

## Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

*Turn over* ▶

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Pearson

## International GCSE in Further Pure Mathematics Formulae sheet

### Mensuration

**Surface area of sphere** = $4\pi r^2$

**Curved surface area of cone** = $\pi r \times$ slant height

**Volume of sphere** = $\frac{4}{3}\pi r^3$

### Series

**Arithmetic series**

Sum to $n$ terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

**Geometric series**

Sum to $n$ terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

**Binomial series**

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

### Calculus

**Quotient rule (differentiation)**

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

### Trigonometry

**Cosine rule**
In triangle $ABC$: $a^2 = b^2 + c^2 - 2bc\cos A$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$ $\quad$ $\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$ $\quad$ $\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$ $\quad$ $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$

### Logarithms

$\log_a x = \frac{\log_b x}{\log_b a}$

**Answer all ELEVEN questions.**

**Write your answers in the spaces provided.**

**You must write down all the stages in your working.**

**1** The equation $kx^2 + 8x + 3k = 0$ where $k$ is a constant, has real unequal roots.

Find the set of values of $k$ giving your answer in an exact simplified form.

**(5)**

**(Total for Question 1 is 5 marks)**

**2** In triangle $ABC$, $AB = 3x$ cm, $BC = 5x$ cm and $\angle ABC = 110°$

(a) Find, in degrees to one decimal place, the size of $\angle BCA$

**(4)**

The area of triangle $ABC$ is 24 cm$^2$

(b) Find, to 3 significant figures, the value of $x$

**(3)**

**Question 2 continued**

**(Total for Question 2 is 7 marks)**

**3** A particle $P$ moves along the $x$-axis.

At time $t$ seconds $(t \geqslant 0)$ the acceleration, $a$ m/s$^2$, of $P$ is given by $a = 6t - 16$

When $t = 0$, $P$ is at the origin and is moving with velocity 12 m/s.

(a) Find an expression in terms of $t$ for

(i) the velocity of $P$ at time $t$ seconds

(ii) the displacement of $P$ at time $t$ seconds.

**(4)**

(b) Hence find the time at which $P$ first returns to the origin.

**(3)**

**Question 3 continued**

**(Total for Question 3 is 7 marks)**

**4** (a) On the axes opposite, draw the line with equation

(i) $y = -x - 1$ (ii) $y - 3x + 8 = 0$ (iii) $2y = x + 8$

**(3)**

(b) Show, by shading on your graph, the region $R$ defined by the inequalities

$y \geqslant -x - 1$ and $y \geqslant 3x - 8$ and $2y \leqslant x + 8$

**(1)**

For all points in $R$, with coordinates $(x, y)$

$$P = 2y - 3x$$

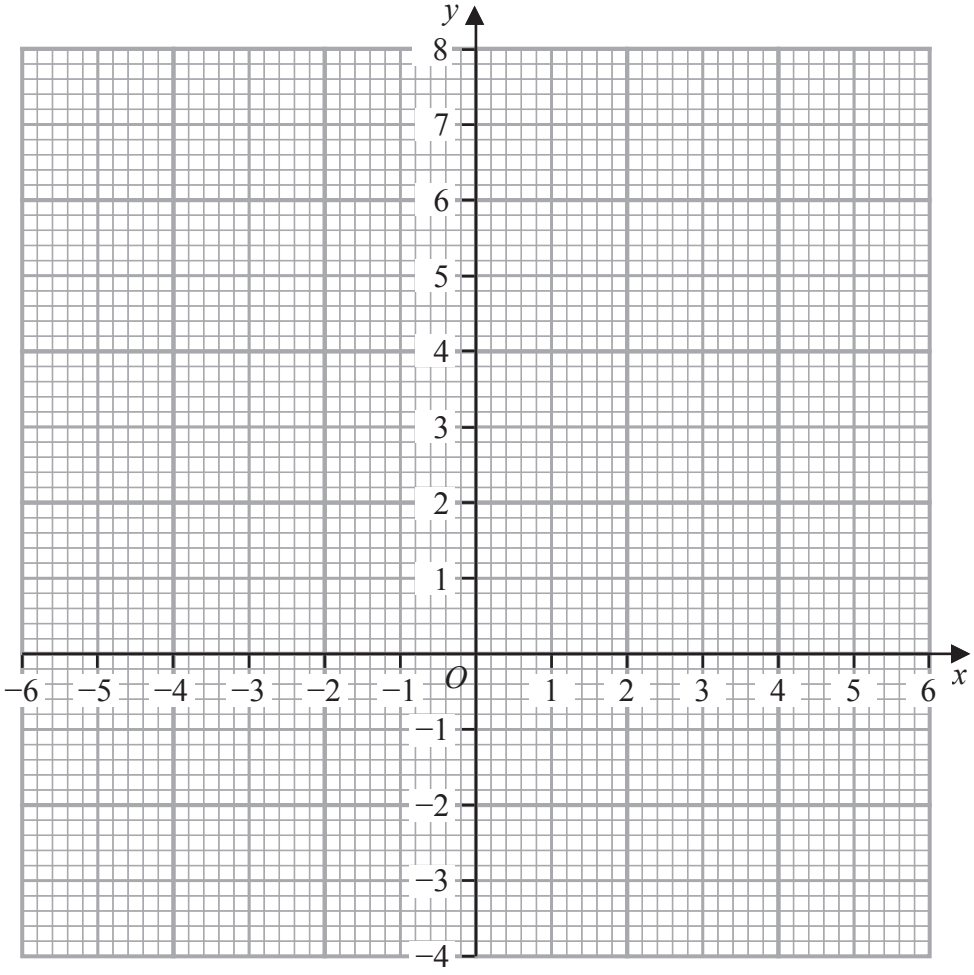
(c) Find

(i) the greatest value of $P$

(ii) the least value of $P$

**(4)**

**Question 4 continued**




**Turn over for a spare grid if you need to redraw your graph.**

**Question 4 continued**

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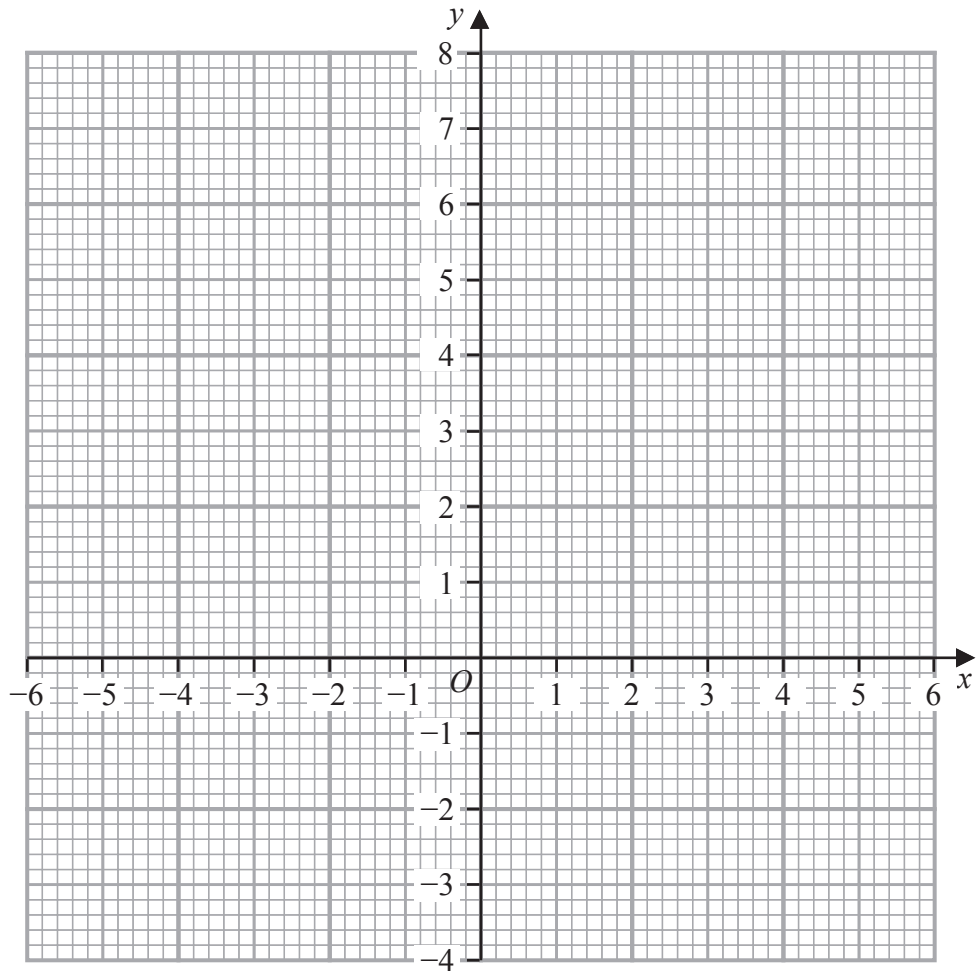
**Question 4 continued**

**Only use this grid if you need to redraw your graph.**




**(Total for Question 4 is 8 marks)**

**5**

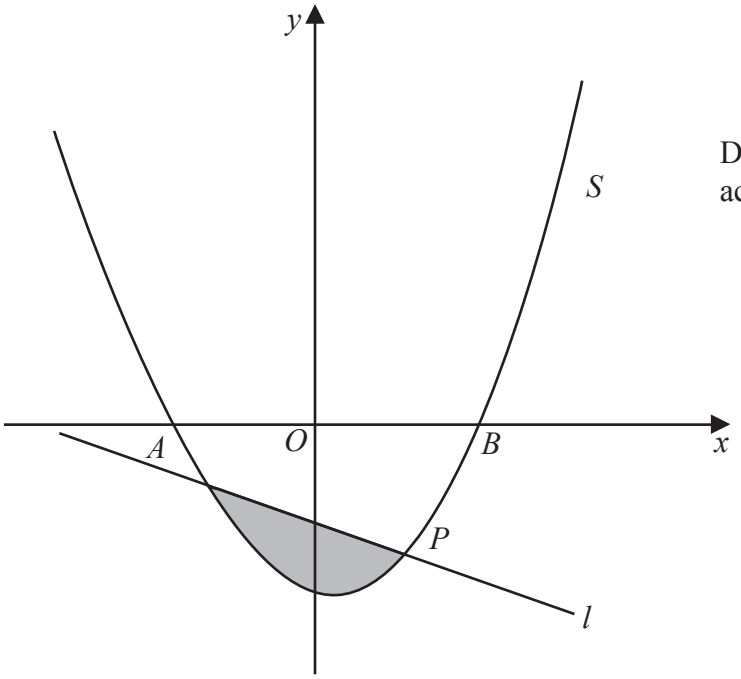



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**Figure 1**

Figure 1 shows part of the curve $S$ with equation $y = px^2 + qx + r$ where $p$, $q$ and $r$ are constants.

The points $A$, $B$ and $P$ with coordinates $(-2, 0)$, $(6, 0)$ and $(4, -6)$ respectively lie on $S$

(a) Show that an equation of $S$ is $y = \frac{x^2}{2} - 2x - 6$

**(3)**

The line $l$ is the normal to $S$ at the point $P$

(b) Show that an equation of $l$ is $2y + x + 8 = 0$

**(5)**

The finite region shown shaded in Figure 1 is bounded by S and $l$

(c) Use algebraic integration to find the exact area of the shaded region.

**(7)**

**Question 5 continued**

**Question 5 continued**

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**Question 5 continued**

**(Total for Question 5 is 15 marks)**

**6** The volume of oil in a container is $V$ cm$^3$ when the height of the oil is $h$ cm.
Oil is pouring into the container at a constant rate of 12 cm$^3$/s.
Given that $V = 3h^3$

find the exact rate, in cm/s, at which the height of the oil is increasing
when $V = 1536$ cm$^3$

**(7)**

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**Question 6 continued**

**(Total for Question 6 is 7 marks)**

**7** Two numbers $x$ and $y$ are such that $3x - y = 4$

$S = 5x^3 + y^2$

(a) Show that $S = 5x^3 + 9x^2 - 24x + 16$

**(2)**

Given that $x$ can vary,

(b) use calculus to find the value of $x$ for which $S$ is a minimum, justifying that this value of $x$ gives a minimum value of $S$

**(5)**

(c) Find the minimum value of $S$

**(2)**

**Question 7 continued**

**(Total for Question 7 is 9 marks)**

**8** The sum to $n$ terms of an arithmetic series $A$ is $S_n$

The sum of the first four terms of $A$ is 42 and the fifth term of $A$ is 23

(a) Show that $S_n = \sum_{r=1}^{n}(Pr - Q)$ where $P$ and $Q$ are prime numbers.

**(6)**

$S_{2n} - 3U_n = 1062$ where $U_n$ is the $n$th term of $A$

(b) Find the value of $n$

**(4)**

**Question 8 continued**

**(Total for Question 8 is 10 marks)**

**9**

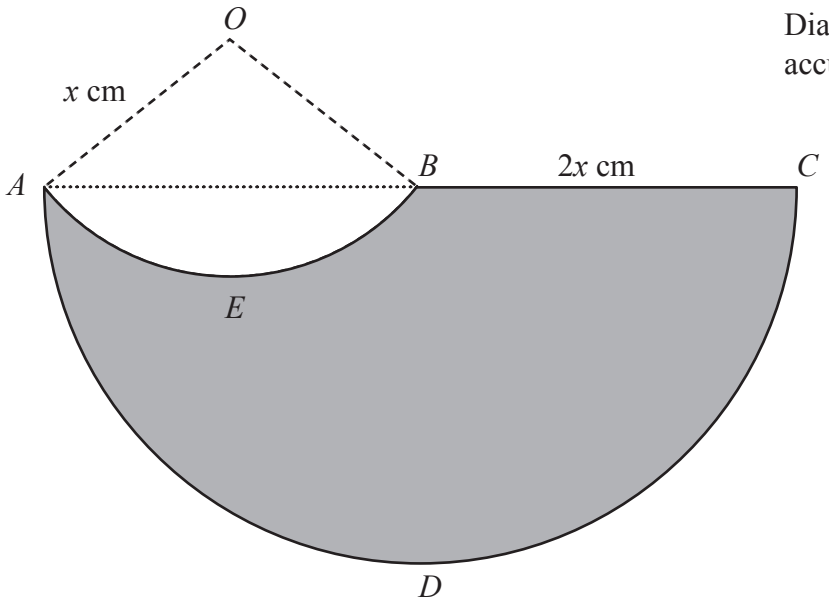


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**Figure 2**

A logo, *AEBCD*, is shown shaded in Figure 2.

The straight line *ABC* is the diameter of the semicircle *ADC*
*AEB* is an arc of a circle with centre *O*
All angles are measured in radians.

- $BC = 2x$ cm
- $OA = OB = x$ cm
- length of arc $AEB = 1.8x$ cm

The perimeter of the logo is $P$

(a) Show that $P = ax(\pi + \pi \sin 0.9 + b)$ where $a$ and $b$ are constants to be found.

**(7)**

Given that $x = 10$ cm,

(b) find, in cm$^2$ to 3 significant figures, the area of the logo.

**(6)**

**Question 9 continued**

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**Question 9 continued**

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**Question 9 continued**

**(Total for Question 9 is 13 marks)**

**10** The roots of a quadratic equation are $\alpha$ and $\beta$ where

$$\alpha + \beta = -\frac{5}{2} \text{ and } \alpha^3 + \beta^3 = \frac{115}{8}$$

(a) Show that $\alpha\beta = 4$

**(3)**

(b) Form a quadratic equation with integer coefficients, that has roots

$$\frac{\alpha^2 + 1}{\beta} \text{ and } \frac{\beta^2 + 1}{\alpha}$$

**(7)**

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**Question 10 continued**

**Question 10 continued**

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**Question 10 continued**

**(Total for Question 10 is 10 marks)**

**11**

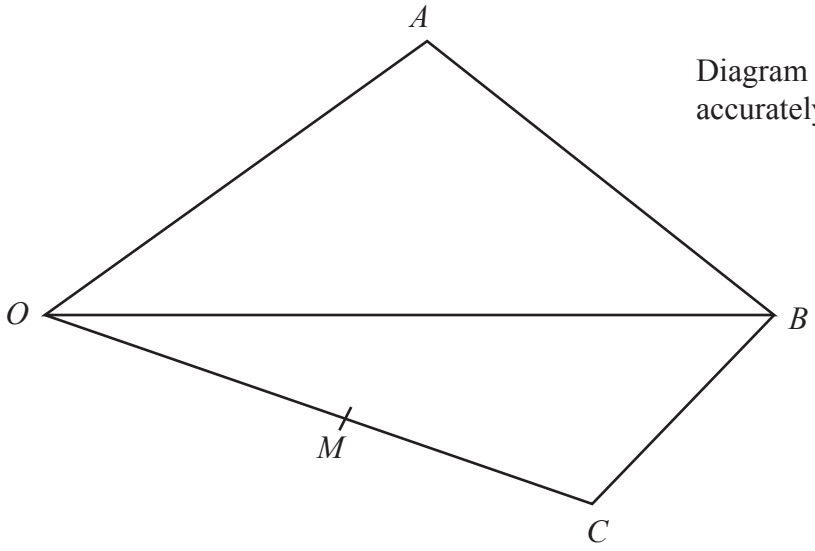


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**Figure 3**

Figure 3 shows quadrilateral $OABC$ where

$$\overrightarrow{OA} = 4\mathbf{p} + 5\mathbf{q} \qquad \overrightarrow{OB} = 3\mathbf{p} + \mathbf{q} \qquad \overrightarrow{OC} = 2\mathbf{p} - 4\mathbf{q}$$

The point $M$ is the midpoint of $OC$

(a) Find $\overrightarrow{MA}$ as a simplified expression in terms of **p** and **q**

**(3)**

The point $N$ lies on $OB$ such that $M$, $N$ and $A$ are collinear.

(b) Find the ratio $MN : NA$

**(6)**

**Question 11 continued**

**Question 11 continued**

**(Total for Question 11 is 9 marks)**

**TOTAL FOR PAPER IS 100 MARKS**

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